Hard-to-Interpret Signals*

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Abstract

Decisions under uncertainty are often made with information whose interpretation is uncertain because multiple interpretations are possible. Individuals may perceive and handle uncertainty about interpretation differently and in ways that are not directly observable to a modeler. This paper identifies and experimentally examines behavior that can be interpreted as reflecting an individual's attitude towards such uncertainty.

Keywords: ambiguous information, Bayes, updating, martingale, uncertain inference

JEL: C91, D81, D83, D91

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1 Introduction

1.1 Objective

The two cornerstones of Bayesian theory are the subjective prior and Bayesian updating. Ellsberg (1961) demonstrates the behavioral limitations of the assumption of a prior through his celebrated thought experiments, which have the clear intuition that a probability measure does not permit a role for limited information and confidence underlying beliefs. In this paper, we present a parallel critique of the updating component consisting of both a thought experiment and a laboratory experiment that provides supporting empirical evidence.

The importance attached to Ellsberg's experiments is due to the presumption that in many instances of decision-making under uncertainty in the field, information may be lacking to justify sharp beliefs. Clearly, this presumption does not require that in all, or even most, such cases there is no information at all. Our motivation begins with the presumption that often there is a great deal of information, but the difficulty for the decision-maker is that its interpretation may not be clear in the sense that the inferences to be drawn from them are uncertain. In such cases we refer to *hard-to-interpret, or ambiguous, signals*. Fed policy communication and qualitative corporate news are two typical examples. They also illustrate the broader class of situations where complete information is complex and thus where information often comes in the form of summaries, open to multiple interpretations (and inferences), rather than in the form of detailed reports.

A timely example was provided by the COVID-19 pandemic. A policymaker had to choose an action to combat the potential new pandemic due to the novel virus (and more recently - its variants). A critical unknown was the probability that an infected individual in the population will suffer serious health consequences (require hospitalization or, at the extreme, die). There was information about the effects of the virus in other jurisdictions: how many patients required hospitalization, and most importantly, the case-fatality rate.[1] However, it was not clear at the time what inferences to make about the risk of suffering serious health outcomes - a point that was well

[1] The case-fatality rate (CFR) is the number of deaths of individuals who tested positive divided by the number of individuals who were confirmed as infected. Importantly, CFR is to be distinguished from the infection-fatality rate (IFR), which is the proportion of all infections that result in death.

understood by epidemiologists and received considerable attention in the media. For example, even if the policy-maker knew how many people were tested for the virus and how many of them tested positive, she could not know how many people were infected but were not tested (because they had only mild symptoms or because they died without having been tested). Consequently, an observed high rate of serious health outcomes may have indicated a high infection rate and that an infected individual was likely to experience serious health outcomes, or alternatively, it may have reflected that only individuals who were symptomatic and severely ill were tested. Similarly, an observed low rate of serious health outcomes admits more than one interpretation. How would such hard-to-interpret information affect the policy-maker's choice of action?

We identify choices in an Ellsberg-style binary setting that can be understood as revealing that uncertainty about how to interpret signals matters for behavior (Sections 2), and later (Appendix A), provide behavioral definitions of the attitude (aversion, affinity or indifference) to signal ambiguity in a much more general setting. Sensitivity to signal ambiguity, which is our focal hypothesis, is conceptually distinct from sensitivity to prior ambiguity (for example, about the composition of an urn or the hazard of serious health outcomes in an evolving pandemic). Accordingly, while Ellsberg pointed to the limitation of modeling prior beliefs by an additive probability measure, our analysis points to the limitation of modeling updating in such a way that updating conforms to the *martingale property* of beliefs (that is, prior beliefs is an average of the set of posteriors). At the functional form level, additivity of prior beliefs and the martingale property are the two distinct fundamental properties of the Bayesian model. Importantly, the martingale property is extended here to a property of preference (rather than probabilistic beliefs) which need satisfy only mild nonparametric restrictions.

Studying the behavioral meaning of hard-to-interpret signals is motivated in part by the inherent interest in such a fundamental notion; see below for references to papers where ambiguity in signals plays a role. We believe that although its importance is evident from applications such as the example above, it has not been investigated systematically in decision theory where prior ambiguity plays the predominant role. In addition, the way in which hard-to-interpret signals are treated by decision-makers reflects on the potential importance of the existing literature on ambiguity. Specifically, if such signals themselves reduce confidence in beliefs then there is reason to believe that, at least in some circumstances, ambiguity might persist rather than being only a short-run phenomenon.

We present a thought experiment that demonstrates our proposed definition of sensitivity to ambiguous signals and we substantiate its relevance to observed behavior in a controlled experiment. We elicit probability equivalents to an event, both unconditionally and conditionally on two complementary signals. The experimental design is guided by the fact that even when available information takes the form of noisy (or risky) signals, individuals often fail to update their beliefs as specified by Bayes rule. We therefore employ a between-subject design that compares deviations from Bayesian updating when signals are noisy (the control) to when they are ambiguous. We find that ambiguous signals significantly increase deviations from updating that is consistent with Bayes rule. In addition, we find a significant association between reduction of compound lotteries (and indirectly - indifference to prior ambiguity) and updating of beliefs that is consistent with Bayes rule (both when signals are risky and ambiguous).

The paper proceeds as follows. The rest of this introduction considers related literature. In Section 2, we present the thought experiment and define the behavior that is the focus of the current paper. The experimental implementation is described in Section 3. Section 4 looks more deeply at the focal behavior, and then describes its implications for some existing models of preference. Appendix A extends and formalizes our framework. Online appendices provide additional details.

1.2 Related literature

Two very recent experimental studies investigate updating when information is (in some sense) ambiguous. Neither includes behavioral definitions for different attitudes to signal ambiguity, or highlights the relevance of the martingale property. In a contemporaneous project, Liang (2022) elicits certainty equivalents (not probability equivalents) for many bets (including uncertain) and information structures (including uncertain). In the absence of behavioral definitions, it is not clear how to identify attitudes. In addition, though there is overlap in motivation, there is no overlap in design as Liang (2022) does not include our main treatment with uncertain prior and ambiguous information. Therefore, he cannot compare the effects of risky and ambiguous information when the prior is uncertain, which is the focus of our investigation. Shishkin and Ortoleva (2023) study how ambiguous information affects the valuation of bets. They are not concerned with new behavioral definitions or with the special role of the martingale property. Rather, their focus is on testing for the presence of dilation (Good, 1974; Wasserman and Seidenfeld, 1993), where ambiguity increases, and valuations fall, for every possible signal realization. The relation of dilation to our proposed behavioral definition (2.5) is explained in Section 4.1.

Ambiguous signals are considered, implicitly or explicitly, in a number of applied studies. Ambiguous communication is shown to arise endogenously from maximizing behavior in a range of strategic settings (Bose and Renou 2014; Blume and Board 2014; Kellner and Le Quement 2017, 2018; Beauchene, Li and Li 2019; and Kellner, Le Quement and Riener 2022 for an experimental counterpart). Levy and Razin (2016) study settings with group communication in which communication, the signal in their model, creates ambiguity. They consider several applications including to jury deliberations and common-value auctions. In the context of human capital accumulation, Giustinelli and Pavoni (2017) document that when foreign-born students enrol in the educational system in Italy and receive information about it, their ambiguity about the general curricula increases over time. Ambiguous signals have been studied also in macro/finance models (Epstein and Schneider 2010; Ilut 2012; Ilut, Kehrig and Schneider 2018; Yoo 2019). The distinction in Daniel and Titman (2006) between tangible and intangible information is suggestive of the distinction between noisy and ambiguous signals. In all of these studies, preferences and/or the form taken by updating are assumed known to the modeler and interpretations of the model are based on functional form appearance or what seems "natural." This paper is complementary in that it takes behavior alone to be observable and asks, for example, "what behavior would reveal an aversion to ambiguity in signals?"

Fryer, Harms and Jackson (2019) study the relation between signals that are open to interpretation and polarization. They posit a particular updating rule and study its implications for polarization. In contrast, we ask what can be learned about updating from behavior with an objective of using identified behavior to distinguish between alternative models of updating. Their online experiment is designed to study polarization, while our experiment is designed to examine whether uncertainty about signal interpretation is revealed by behavior.

Updating under ambiguity has been studied in axiomatic decision theory (see, for example, Gilboa and Schmeidler 1993; Pires 2002; Epstein and Seo 2010; Gul and Pesendorfer 2021). The martingale property is studied axiomatically in Cripps (2018), and is at the heart of the tests of Bayesian updating discussed by Shmaya and Yariv (2016) and Augenblick and Rabin (2021); in all these cases, it is assumed that beliefs are represented by a single (Savage) prior. Gajdos et al (2008) and Hayashi and Wada (2010) incorporate imprecise information into models of preference. They assume that information comes in the form of an objective (observable) set of probability measures over the state space, but they do not address updating, as it is usually understood, because their model does not include both ex ante and conditional stages. Riedel et al (2018) extend this line of work to a dynamic framework and investigates conditions that deliver dynamic consistency. None of these papers address the specific questions studied here.

Epstein and Schneider (2007, 8, 10) pay explicit attention to the behavioral meaning of functional form specifications and they introduce and discuss the notion of ambiguous signals. In particular, they distinguish between noisy and ambiguous signals, and correspondingly point to a new dimension of information quality, distinct from the usual notion of the precision of a noisy signal, that pertains to the ease/difficulty of its interpretation. They also describe a thought experiment which we build upon here. An important difference is that our thought experiment "leads to" and illustrates a general model (Appendix A), while such a general analysis is not apparent in the previous work. Moreover, the current paper is the first to document empirically (in an experimental setting) the behavioral relevance of these theoretical distinctions.

2 A thought experiment

This section builds on Ellsberg's two-urn experiment, and on Epstein and Schneider (2007,8), and suggests a thought experiment to give behavioral meaning to sensitivity to hard-to-interpret signals. A more general and formal treatment is provided in Section 4.1 and Appendix A.

2.1 The choice problems

Consider a "payoff urn" that contains 10 colored balls, each of which is either red or black, with at least one of each color. One ball will be drawn from the urn and the decision-maker (DM) is asked to evaluate bets on its color. A correct bet pays \$100, while an incorrect bet pays \$0. Elicit *probability equivalents* in two *ordered* scenarios.

1. *Unconditional choice*: Let f_R and f_B denote bets on red and black, respectively, being drawn from the payoff urn. Elicit *unconditional* probability equivalents $p_{0,R}$ and $p_{0,B}$, where

   $$f_R \sim_0 (100, p_{0,R}) \text{ and } f_B \sim_0 (100, p_{0,B}).$$

   where $(100, p)$ is the bet that pays \$100 with probability p, and \$0 with probability $1 - p$, and the relation $\sim_0$ denotes indifference at the unconditional stage. The intuitive behavior highlighted by Ellsberg in his two-urn experiment corresponds to $p_{0,R}, p_{0,B} < \frac{1}{2}$, but this is not necessary for what follows.

2. *Conditional choice*: The DM is now told about a second "signal urn" that is constructed by adding an equal number (N) of red and black balls to the payoff urn. The total number ($2N$) of balls added is not specified. Then a ball is drawn from the *signal urn* and its color is revealed: $\sigma \in \mathbf{\Sigma} = \{\sigma_R, \sigma_B\}$, where σ denotes the color of the ball drawn from the signal urn. Once again, consider bets on the color to be drawn from the *payoff urn* and elicit *conditional* probability equivalents $p_{\sigma,R}$ and $p_{\sigma,B}$ for each signal $\sigma \in \{\sigma_R, \sigma_B\}$:

   $$\begin{array}{lll} f_R & \sim_\sigma & (100, p_{\sigma,R}) \\ f_B & \sim_\sigma & (100, p_{\sigma,B}). \end{array} \tag{2.1}$$

   where $\sim_\sigma$ denotes indifference at the conditional stage (after a signal is observed).

Below we assume

$$(p_{\sigma_R,R} - p_{\sigma_B,R}) \cdot (p_{\sigma_R,B} - p_{\sigma_B,B}) < 0, \tag{2.2}$$

a property that we call *signal diversity.* Most importantly, it excludes the case where the same signal is viewed as (weakly) better for both bets. For example, a special case that is natural for the present setting is that the DM views σ_R *as a better signal for the bet on red than is* σ_B, *and the reverse for the bet on black*, that is,

$$p_{\sigma_R,R} > p_{\sigma_B,R} \text{ and } p_{\sigma_R,B} < p_{\sigma_B,B}. \tag{2.3}$$

We assume that risk preferences (the ranking of lotteries) are unaffected by signal realizations and are *monotone* in the sense that

$$p' > p \implies (100, p') \succ_0 (100, p).$$

Given that lotteries have only two possible outcomes, monotonicity implies the Independence axiom and hence expected utility theory. However, the thought experiment and the laboratory investigation that follow are immune to documented descriptive violations of Independence, (the Allais paradox, for example), because all such evidence concerns choice between lotteries having at least three outcomes.

2.2 Behavior: the symmetric case

It is convenient to adopt the following notation: the payoff-relevant state space is $S = \{R, B\}$, the set of prizes is $X = \{100, 0\}$, and the signal space is $\mathbf{\Sigma} = \{\sigma_R, \sigma_B\}$. Conditional and unconditional preferences are defined on bets and lotteries, that is, on $\{f_R, f_B\} \cup \Delta(X)$, where $\{f_R, f_B\}$ are bets on red and black from the payoff urn and $\Delta(X)$ are objective lotteries over X.

Because information about both payoff and signal urns is color-symmetric, one would expect a "rational" individual to satisfy also the following *symmetry* condition:

$$p_{0,R} = p_{0,B},\; p_{\sigma_R,R} = p_{\sigma_B,B},\; p_{\sigma_R,B} = p_{\sigma_B,R}. \tag{2.4}$$

We assume (2.4) throughout this section, in Section 4.1, and also in the laboratory experiment. See Appendix A for the more general case where symmetry is not imposed, and for other generalizations of the above choice problems whereby S, $\mathbf{\Sigma}$ and X can be any finite sets, and bets on colors can be replaced by arbitrary Savage acts from S into X.

Assuming (2.4), our focal behavior corresponding to *(strict) aversion to signal ambiguity* is:

$$p_{0,R} > \tfrac{1}{2}p_{\sigma_R,R} + \tfrac{1}{2}p_{\sigma_B,R}. \tag{2.5}$$

In the rest of this section, we describe some intuition for (2.5). Weak aversion, strict and weak affinity, and indifference or neutrality are defined by the obvious modifications of (2.5) and can be motivated similarly. (All these inequalities refer explicitly only to bets on red, corresponding inequalities for bets on black follow immediately from symmetry.) Though all forms of non-indifference (inequality in (2.5)) are of equal interest, as is common in the literature our discussion focuses on strict aversion.

The intuition we suggest for (2.5) centers on uncertainty about the number of balls added to the signal urn and hence about how to interpret a signal. To explain, note first that the unconditional probability equivalent

p_0 reflects the attitude towards the uncertain composition of the payoff urn, as in Ellsberg's experiment, but is not affected by uncertainty about signal interpretation because even the possibility of signals is presumably unknown at the unconditional stage. However, uncertainty about signal interpretation is relevant for conditional probability equivalents. For example, a red draw (σ_R) is a strong signal in favor of a bet on red (and against a bet on black) if only a small number of balls were added in constructing the signal urn, but it is only a weak signal for red (and against black) if a large number of balls were added. A conservative decision-maker facing this uncertainty might interpret σ_R as a weak positive signal when evaluating a bet on red, (corresponding to large N), hence leading to a small probability equivalent $p_{\sigma_R,R}$ for betting on a red ball drawn from the payoff urn. But similar uncertainty applies when interpreting the implication of observing a black ball drawn from the signal urn, and a conservative attitude would lead to viewing it as a strong negative signal for a bet on red (corresponding to N small), and hence to a small probability equivalent $p_{\sigma_B,R}$. This suggests how aversion to signal ambiguity might explain (2.5).

There is a parallel with the Ellsberg-based approach to prior ambiguity. Given symmetry (2.4), and hence $p_{\sigma_B,R} = p_{\sigma_R,B}$, then (2.5) can be described as saying that a given signal (σ_R) is interpreted as providing weak support for *both* an event (drawing red) *and* its complement (drawing black). This is a counterpart of the essence of Ellsberg's two-urn experiment, namely that both an event and its complement are deemed unlikely.

To illustrate (2.5), consider a numerical example in which DM has the following additional information regarding the payoff and signal urns: All 8 unknown balls of the payoff urn are red (black) if a fair coin toss gives heads (tails); and the signal urn is constructed by adding N balls of each color, where $N = 0$ or 45. Thus, if DM calculates objective probabilities correctly (satisfies the Reduction of Compound Lotteries axiom, ROCL), presumably $p_{0,R} = \frac{1}{2}$ as there is no prior ambiguity.[2] The posterior probabilities satisfy

$$\Pr(R \mid \sigma_R) \in \{.53,\ .82\} \text{ and } \Pr(R \mid \sigma_B) \in \{.18,\ .47\}.$$

Then aversion to uncertainty about signal interpretation (the signal is strong

[2] Extensive research has documented that ROCL is not a good behavioral assumption in this case, and that behavior over compound lotteries is associated with ambiguity attitude. In the experimental design we will both control for this behavior, and use the measured violation of ROCL to approximate ambiguity attitudes (see Section 3.1).

if $N = 0$ but weak if $N = 45$) plausibly leads to probability equivalents $p_{\sigma_R,R} < .67$ and $p_{\sigma_B,R} < .33$, and hence to (2.5).

We focus on (2.5) in our experiment. Therefore, in order to further justify its interpretation, Section 4.1 deepens the intuition for this condition, while still maintaining the assumptions specified above. Appendix A goes much further towards describing a general theory and drops the restrictions of binariness and symmetry.

3 A laboratory experiment

This section describes the design and results of a lab experiment whose goal is to evaluate the empirical applicability of the signal-sensitive behavior proposed in (2.5). There are a few major practical challenges that a lab experiment must overcome. First, subjects may not be Bayesian even when the accuracy of the signal is known (see Grether 1980, for an early example in Economics, and Benjamin 2019 for a current comprehensive survey). Second, they might not reduce objective compound lotteries, a behavior that has been shown to be empirically associated with sensitivity to ambiguity. Third, they may not satisfy expected utility even when dealing with objective probabilities. Fourth, even if all the above are non-issues, and subjects are sensitive to signal ambiguity as we suggest, they may use the elicitation system to hedge such ambiguity. In the following subsection we detail how we dealt with these challenges and provide the details of the experimental design.

3.1 Experimental Design

In order to decrease the cognitive load on subjects, we adopted a simple between-subject design, where the control group received a known risky signal, while the treatment group received an ambiguous signal. Our focus is on the differential effect of ambiguity versus risk on updating.

The environment is similar to the numerical example presented in Section 2.2. The payoff urn (for both groups) consisted of 10 balls, that contained either 9 red balls (and 1 black ball) or 1 red ball (and 9 black balls), each with probability .5, as in the left panel of Figure 3.1. Hence, the probability of drawing a red ball is either .1 or .9, each with probability .5. Four considerations motivated us to eliminate (pure) prior ambiguity from the payoff

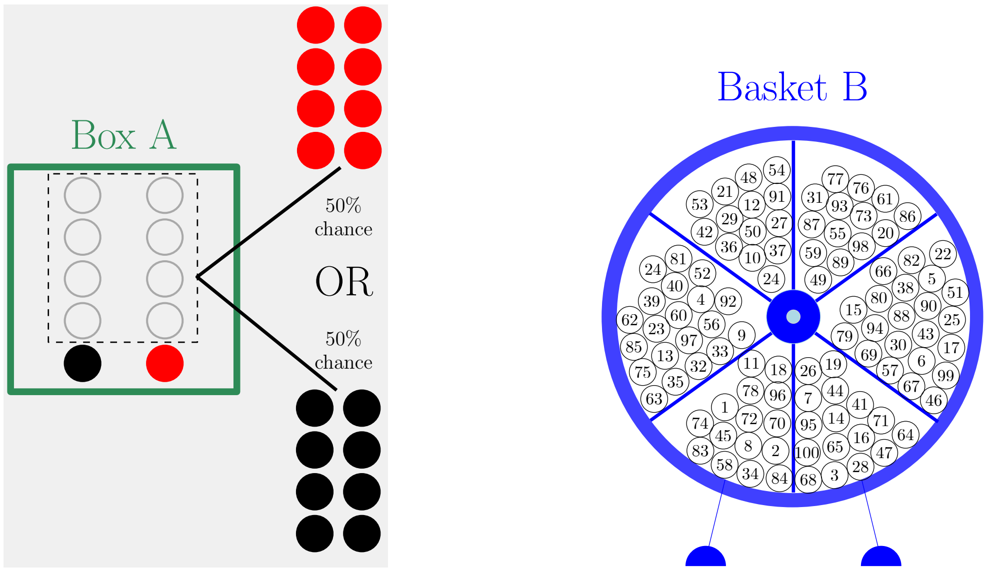

Figure 3.1: The payoff urn (left) and the basket used to elicit probability equivalents (right)

urn and instead employ a compound risky urn. First, there exist now strong empirical evidence that many subjects do not distinguish between symmetric ambiguous environments and similar environments with compound risk (Halevy 2007; Dean and Ortoleva 2019; Gillen, Snowberg and Yariv 2019; and especially Chew, Miao and Zhong 2017). The payoff urn is a special case of Chew et al's two-point compound-risk,[3] which they show is similar to two-point ambiguity. We therefore expect many (ambiguity averse) subjects to prefer a one-stage lottery with a winning probability of .5 to a bet on either color from the payoff urn. Second, symmetry between a bet on red and a bet on black is universal in such a case, while if the compositions were symmetric but included prior ambiguity, then some subjects may have had non-symmetric belief - which would complicate the identification of signal-ambiguity. We provide a theoretical identification result for this case in Appendix A. Third, theoretical identification relies on subjects isolating

[3] "Two-point compound risk" refers to two-stage lottery with symmetric two possible second-stage lotteries and uniform first-stage, of the form: $\left((x, q; 0, 1-q), \frac{1}{2}; (x, 1-q; 0, q), \frac{1}{2}\right)$ where $0 \leq q \leq 1$. "Two-point ambiguity" refers to the corresponding ambiguous scenario, where the second-stage is determined by the physical environment, and the first-stage by the decision maker's belief.

their responses and not using the random incentive system to hedge the prior ambiguity. Baillon, Halevy and Li (2022b) document strong empirical evidence against this isolation assumption. By using a compound-risky payoff urn and offering the subject to choose a color to bet on (red or black) we eliminated any hedging opportunities in the unconditional choice.[4] Fourth, as a by-product of this design choice, we can evaluate the association between reduction of compound lotteries and Bayesian updating, even when the signal is risky (in a framework with no subjective uncertainty or ambiguity). To the best of our knowledge, there exist no empirical evidence on this association. The limitation of this design choice is that we cannot measure directly the association between attitudes to prior and signal ambiguities. However, the unconditional PE serves as a control for the extent of violation of ROCL, and indirectly - a measure of attitude to prior ambiguity.

The elicitation of probability equivalents (PEs) was implemented as in Freeman, Halevy and Kneeland (2019). Subjects were presented with a basket containing 100 balls numbered from 1 to 100 (as in the right panel of Figure 3.1), and on each line of a choice list they were asked to choose between their bet on the payoff urn (option A) and a bet that the ball drawn from the basket has a number that is smaller or equal to the line number (option B), so the latter increases when the subject moves down the list. In the initial choice list the step was 10 percent, and then subjects were presented with a zoom-in list were the resolution was 1 percent.[5,6] One may worry that

[4]Under reduction of compound lotteries, there should not be a concern for hedging here, as the payoff urn is compound risk and not ambiguous. However, if a subject identifies the two, and reacts to ambiguity using hedging, she may hedge here as well. As a result, she may report the probability equivalent .5 for bets on both red and black from the payoff urn, even if her true probability equivalent is smaller than .5 (Baillon, Halevy and Li 2022a).

[5]If subjects switched more than once in a choice list, a pop-up explained to them the logic of monotonic preferences. However, if they wished to switch multiple times - they were allowed to do so. In other words, we did not impose a single crossing, but tried to make sure subjects understood their choices. This technique was first used in Freeman et al (2019).

[6]We calculated the probability equivalent as the average of the last line in which Option A was chosen and the first line in which Option B was chosen. For subjects whose choices are consistent with monotone and transitive preferences, these lines will be consecutive (single switching point, e.g. if a subject switched to B at .5 then the PE would be .495). If a subject switches multiple times between A and B (reported in Appendix B.3.1), this is the midpoint in the range of switching (so if the last line A was chosen is .6 and the first line B was chosen was .5, the reported PE would be .55).

if subjects have non-expected utility preferences, the elicitation of PEs is not incentive compatible. As demonstrated in Freeman et al (2019) and again in Freeman and Mayraz (2019), this poses a challenge only when the constant alternative in the choice list is certain, while in our case it is uncertain.

The control group facing a risky signal was then introduced to a signal urn constructed by adding 5 red and 5 black balls to the payoff urn (left panel of Figure 3.2). The signal urn therefore included 20 balls, which were equally likely to be 14 red (and 6 black) or 6 red (and 14 black). Conditional PEs were elicited, that is – the PE of the chosen bet on the payoff urn conditional on each color being drawn from the signal urn. By Bayes rule, if the prior probability that the payoff urn contains a single red ball is p, and if the signal urn contains N additional balls of each color, then the posterior probability of drawing a red ball from the payoff urn conditional on a red ball being drawn from the signal urn is:

$$P\left(R|\sigma_R, N\right) = \frac{9\left(9+N\right)\left(1-p\right)+\left(1+N\right)p}{10\left[\left(9+N\right)\left(1-p\right)+\left(1+N\right)p\right]} \tag{3.1}$$

Applied to the risky signal urn ($N = 5$), the Bayesian updates of $P\left(R\right)$ are .66 and .34 for a favorable and unfavorable signal, respectively. We did not expect subjects to calculate Bayes rule exactly. In order to facilitate a reasonable approximation to Bayes rule, and inspired by Gigerenzer and Hoffrage (1995), we presented to subjects the two possible signal urn compositions (right panel of Figure 3.2), which suggest that the probabilities of drawing the chosen color from the *signal* urn are .7 or .3 depending on the composition of the *payoff* urn.

The treatment group faced an ambiguous signal urn, constructed by adding N balls of each color to the payoff urn, where N was either 0 or 45 (left panel of Figure 3.3). That is, the signal urn contained either 10 balls (with a composition of 9R1B or 1R9B) or 100 balls (with a composition of 54R46B or 46R54B). If $N = 0$ the signal is much more informative than if $N = 45$; accordingly, if $p = .5$, then $P\left(R|\sigma_R, N=0\right) = .82$ while $P\left(R|\sigma_R, N=45\right) = .532$. As done for the control group, we presented subjects with images of the possible compositions of the signal urn (right panel of Figure 3.3) in order to facilitate their intuitive reasoning when eliciting conditional PEs.

We chose the risky and ambiguous signal urns such that if the likelihood in the ambiguous treatment is symmetric then the Bayesian posterior for

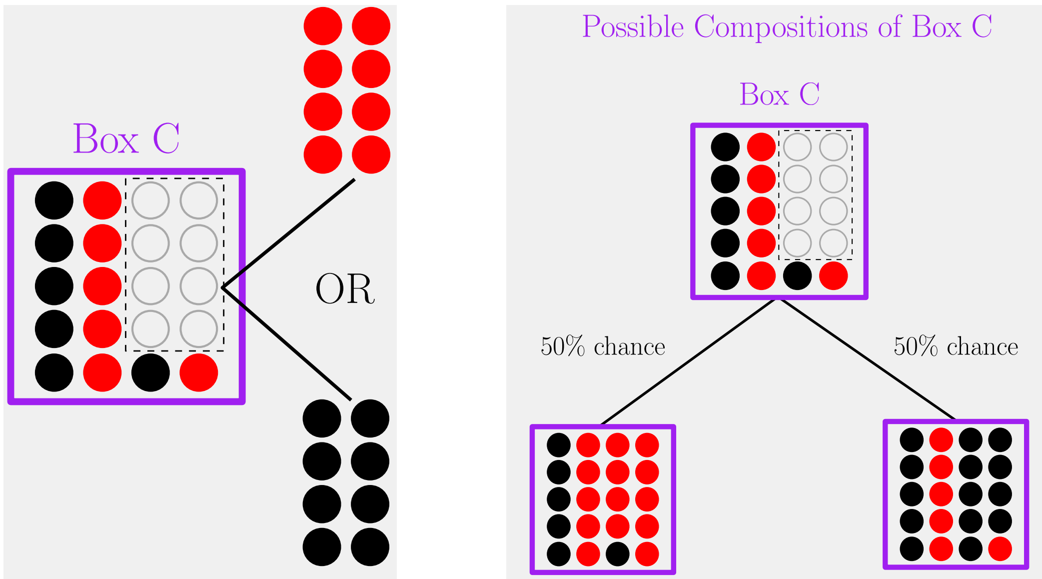

Figure 3.2: The signal urn: the risk control

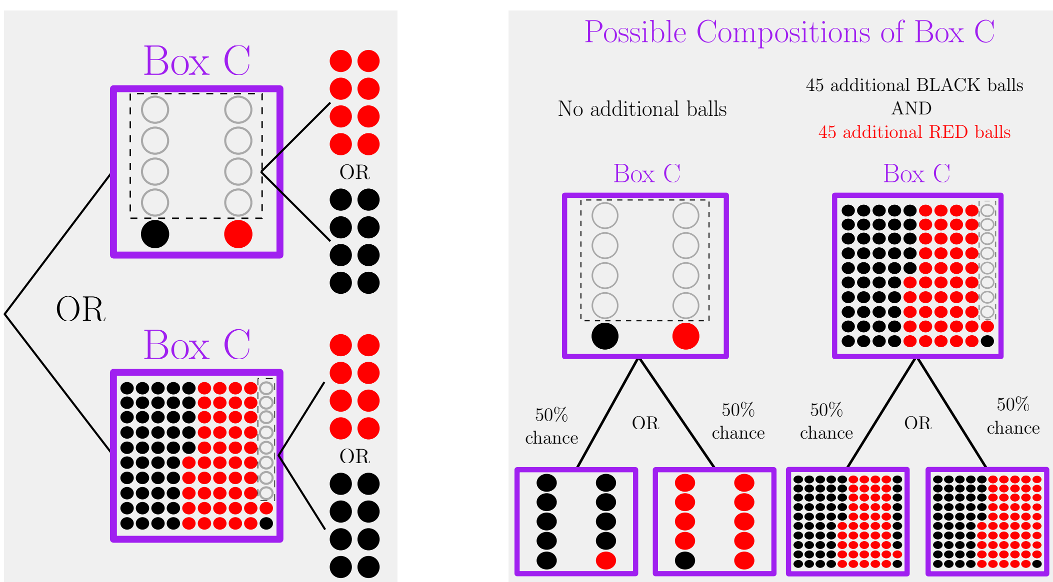

Figure 3.3: The signal urn: the ambiguous treatment

the risky signal is approximately equal to the average of the two possible Bayesian posteriors for the ambiguous signals.[7]

One might be concerned that the elicitation of PE is noisy, in the sense that the reported PE equals the DM's true PE plus some measurement error. As such, a skeptical reader may worry that the elicitation of two (conditional) PEs generates more noise compared to the single (unconditional) PE, and hence mechanically introduces a non-neutral attitude to signal ambiguity, even if the DM is Bayesian. However, this is not the case as random noise would tend to cancel out, and the average conditional PEs would be close to the prior. Hence, the noise (or measurement error) hypothesis would work against finding non-neutrality to signal ambiguity. More crucially, the control (risky) group reported two conditional PEs as well, and our focus is on the differential effect of ambiguous signals relative to this control. Moreover, even if the noise hypothesis is extended to the mental calculation of PEs and the comparison between the risky (control) and the ambiguous (treatment) signals, it will (again) work in the direction of the null hypothesis. If subjects update using Bayes rule, the ambiguous signal treatment involves an additional calculation relative to the risky signal control – subjects must calculate the probability equivalent for each composition of the signal urn and then take a weighted average of the two numbers. If the mental calculation of PE is noisy, then taking an average of the two PEs would lower the measured noise and one would observe even smaller deviations from Bayesian updating in the ambiguous signal urn than in the risk control.

As we elicited PEs conditional on both red and black balls being drawn from the signal urn and paid only one choice, we expose the experimental design in the conditional stage to the theoretical possibility of hedging. That is, if a subject is ambiguity averse, she may use the incentive system to hedge part of the ambiguity concerning the inferences made based on the signal urn. Although this is a theoretical possibility (and a concern had the payoff urn been ambiguous), we find it highly improbable that subjects will be sophisticated enough to hedge in this way. In any case, the resulting bias would be that ambiguity averse subjects who do not have probabilistic beliefs about the structure of the signal will behave *as if* they are Bayesian, which is the null hypothesis in the current investigation.

[7] A design that uses the average of the two possible signal urns (for example with $N = 22$) will result in a Bayesian posterior of approximately .56 that is very close to the less informative signal ($N = 45$).

To sum up, the decisions made by subjects were straightforward. Each subject chose only three PEs, namely, assuming the subject chose to bet on red: $p_{0,R}$ – the unconditional PE; $p_{\sigma_R,R}$ and $p_{\sigma_B,R}$ – the PEs conditional on drawing a favorable (red) and unfavorable (black) signal from the signal urn. Since symmetry is built into the payoff urn using the compound lottery, Bayesian updating implies that the unconditional PE is the (equally weighted) average of the conditional PEs (see Lemma 1). Therefore, our interest is in measuring the deviation from this benchmark

$$(0.5p_{\sigma_R,R} + 0.5p_{\sigma_B,R}) - p_{0,R}. \tag{3.2}$$

However, since we do not expect subjects to be exactly Bayesian even in the case of a risky signal, we compare this measure for hard-to-interpret signals with the corresponding measure for risky signals.

Before moving to results, one may wonder what would be the effect of replacing the ambiguous signals with two equally likely possible signals (that is, $N = 0$ and $N = 45$, each with probability of .5). As argued above, there is now considerable empirical evidence that many DMs identify ambiguous environments with compound but risky environments in which they do not reduce compound lotteries. This evidence suggests that modeling ambiguity as a compound object has sound behavioral support, and contributes to the understanding of new dimensions of ambiguity.

The experiment was conducted at the Toronto Experimental Economics Laboratory in March 2018. Subjects had to answer 12 comprehension questions, and were incentivized by \$0.25/question to answer each correctly on their first attempt (they had to answer it correctly before moving to the next question/stage). The experiment was programmed in zTree (Fischbacher 2007). The potential prize in the experiment was \$20 plus a show-up payment of \$7 and a maximum of \$3 as payment for answering the comprehension question correctly. We recruited 154 subjects: 68 for the risk control and 86 for the ambiguous signal treatment. The instructions as well as the experimental interface are included in Appendix B.4.

3.2 Results

This section reports results for 129 subjects (about 84% of all subjects): 60 in the risky signal control and 69 in the ambiguous signal treatment, all of whom satisfied the following two weak conditions. They did not switch more than once in the choice lists (so their choices are consistent with monotone and transitive preferences), and their "favorable" conditional PE (the color of the ball drawn from the signal urn matches the color they chose to bet on in the payoff urn) is not lower than their "unfavorable" conditional PE (the color of the ball drawn from the signal urn does not match the color they chose to bet on in the payoff urn). The latter condition requires that subjects respond in a way consistent with understanding what are "favorable" and "unfavorable" signals (as the signal is always informative), a key feature of the experimental environment. We believe that the choices of the excluded subjects reflect confusion rather than deliberate choice.[8,9,10]

Throughout this section we consider subjects who deviate by up to 2.5% from standard behavior (reduction of compound lotteries and Bayesian updating) as exhibiting "approximate" standard behavior.

3.2.1 Unconditional probability equivalents

48% of subjects (62 out of 129) have unconditional PE of approximately .5, 45% (58 subjects) have unconditional PE lower than .475 and the remaining 7% (9 subjects) higher than .525. These are standard results for 2-point ambiguity and compound risk attitude (Halevy 2007; Chew et al 2017), and justify our behavioral approach of using compound lotteries to mirror two-point ambiguity. As expected, there is no treatment effect when measuring unconditional PE (p-value for Fisher exact test is .925).

[8] The assignment into the risky and ambiguous treatments is random: 4 out of 68 and 6 out of 86 subjects in the risky and ambiguous signal treatments, respectively, have multiple switching in the unconditional PE stage (which is identical). The difference is insignificant (p-value of Fisher exact test is 1). Moreover, even if one considers *all* excluded subjects (8 in the risk control and 17 in the ambiguous-signal treatment), the difference between the treatments is not significant (p-value of .196 in a Fisher exact test).

[9] In a previous version of the paper we used a misguided criterion that was "too strict" in classifying subjects. In particular, subjects for whom both conditional PEs were lower than the unconditional PE, as predicted by (4.2), were classified as if they have updated in the "wrong" direction.

[10] Appendix B.3.1 reports the results for all subjects, which are qualitatively similar to those reported below.

3.2.2 Bayesian updating

In the risky-signal control 50% of subjects are approximately Bayesian[11] in the sense that their unconditional PE is approximately the average of their conditional PE, while in the ambiguous-signal treatment the proportion falls to approximately 32%. The increase in the incidence of non-Bayesian behavior as a response to hard-to-interpret signals is significant at the 5% level (p-values of one-sided proportion test and one sided Fisher exact test are .018 and .028, respectively).

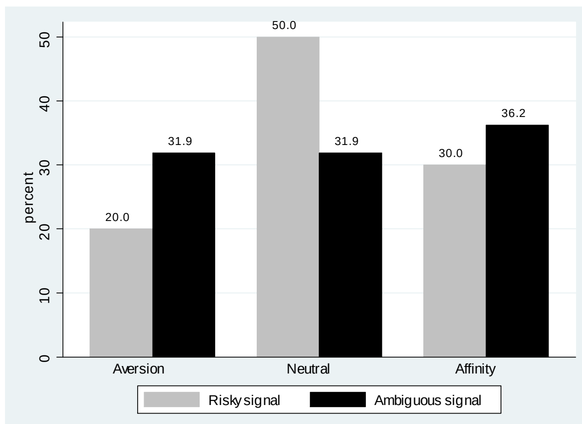

Figure 3.4: Bayesian behavior and attitude to signal ambiguity

Figure 3.4 demonstrates that two-thirds of the increase in non-Bayesian behavior is obtained among subjects who exhibit aversion to signal ambiguity (the average of the conditional PEs is lower than the unconditional PE), where the proportion of subjects increases by more than 50% relative to the risky-signal control (from 20% to almost 32%).

3.2.3 Reduction of compound lotteries and Bayesian updating

Table 3.1 explores the relation between reduction of compound lotteries

[11]See Appendix B.3.2 for the CDFs of the difference, demonstrating that the 5% approximation is inconsequential for our results

Risky Signal	ROCL		
Bayesian	Yes	No	Total
Yes	21	9	30
No	9	21	30
Total	30	30	60

Ambiguous Signal	ROCL		
Bayesian	Yes	No	Total
Yes	16	6	22
No	16	31	47
Total	32	37	69

Table 3.1: Frequencies in the risky signal control (left) and ambiguous signal treatment (right). Bayesian (rows) is approximately satisfying Bayes rule (left) and being approximately neutral to signal ambiguity (right). ROCL (columns) is having unconditional PE of approximately 0.5. The p-value of Fisher exact test is .004 in both.

and Bayesian updating, and (indirectly[12]) between attitudes towards prior-ambiguity and towards signal-ambiguity. We find that the association between approximately reducing compound lotteries and being approximately Bayesian in the risky-signal control is significant at the 1% level. Moreover, the association between reduction of compound lotteries (and indirectly prior-ambiguity neutrality) and being neutral to signal ambiguity in the ambiguous-signal treatment is significant at the 1% level, though the proportion of Bayesian subjects in this treatment is much lower (32% in the treatment versus 50% in the control).

3.2.4 Favorable and unfavorable signals

Although not the main focus of our study, in the spirit of the literature surveyed in Benjamin (2019) it is interesting to note how subjects respond to favorable and unfavorable signals when the signal is risky and when it is ambiguous, while assuming an unconditional prior of .5. Figure 3.5 plots the distribution of responses that are consistent or deviate from the Bayesian benchmark in the risky control and the ambiguous treatment, for favorable and unfavorable signals separately.

In the risky signal control, 13% of subjects had favorable PE that was approximately the Bayesian update of .5 (.66 +/- .025), while 63% of the remaining had PE that was below this. In the ambiguous signal treatment, and assuming equally likely signals, only a single subject had PE that was approximately the Bayesian update of .5 (.676 +/- .025), while 75% of the

[12] As discussed above, we have only an indirect measure of prior-ambiguity, since using unconditional PE measures attitude to 2-point compound-risk that has been shown to be strongly associated with attitude to two-point ambiguity (Halevy, 2007; Chew et al 2017).

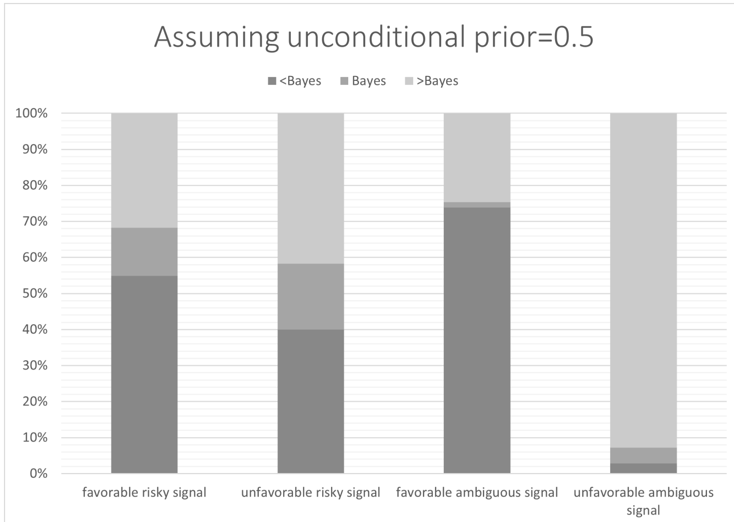

Figure 3.5: Distribution of updating in response to risky/ambiguous favorable/unfavorable signals

others had PE that was below the Bayesian benchmark of .5. For the unfavorable signal, the treatment effect is even starker: 18% of subjects in the risk control are close to the Bayesian update of .5 (.34 +/- .025), and the remainder are almost equally split between those whose unfavorable PE is higher and lower than the benchmark. In the ambiguous signal treatment, only 3 out of 69 are approximately Bayesian (.324 +/- .025, assuming equally likely signals and a prior of .5), and almost 97% (64 subjects) of the others have PE that is higher than the Bayesian benchmark.

Three comments are in order regarding these observations. First, as noted above, most subjects do not start from an unconditional PE of .5 (it is typically lower). Although one can adjust for the prior over the composition of the payoff urn based on individual unconditional PE and calculate the Bayesian benchmark based on this imputed prior – as is done in Figure 3.6, it is important to note that this approach is inconsistent with Bayesian rationality. For example, if a subject chose to bet on red from the payoff urn and her unconditional PE is smaller than .5, one would impute that she believes that the payoff urn is more likely to contain 9 black balls than 9 red balls. But had this been the case, she should have chosen to bet on black (and not red) from the payoff urn. Second, even if the average behavior in the risky

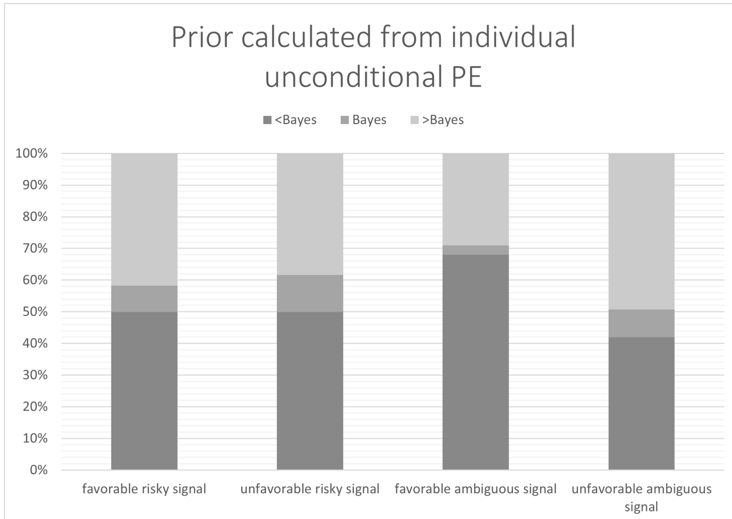

Figure 3.6: Distribution of updating in response to risky/ambiguous favorable/unfavorable signals

signal treatment is not too far from the Bayesian update of .5, there exists a huge heterogeneity at the individual level. Third, the tendency to underreact to an unfavorable ambiguous signal can be rationalized by the belief that the signal is more likely to be less informative (it is more likely that the signal urn contains 100 balls rather than 10 balls). Indeed, this is consistent with the tendency to underreact to a favorable signal as well.[13] This demonstrates the necessity to measure *individual* behavior using (3.2), as it ties together the conditional (favorable and unfavorable) PEs and unconditional PE, allows for non-neutral attitude to prior ambiguity (compound-risk), and answers the crucial question if there exists a prior over the possible signals that can rationalize the unconditional/conditional PEs via Bayesian updating? (see discussion in 4.3.1).

[13]We believe that an alternative approach that assumes a symmetic likelihood (as assumed above) and attempts to rationalize the favorable/unfavorable conditional PE relative to the prior of .5, based on heterogeneous ambiguity attitude cannot tie together the three PEs in a consistent way.

4 Theoretical perspectives

4.1 Aversion to signal ambiguity: further discussion

Here we provide additional theoretical motivation and perspective for (2.5).

Recall that in both the thought experiment and in the experimental implementation, because of the specification of two ordered scenarios, signals are not anticipated when unconditional choices are made. To clarify the relevance of "(un)anticipated signals," suppose that a choice between prospects is made ex-ante, before realization of a signal but *with the expectation* that before the state of the world is realized, a signal about the state will be forthcoming. Though choice cannot be made contingent on the signal, its *mere anticipation* can still affect the ex-ante evaluation of prospects – for example, if the DM backward inducts from anticipated conditional rankings. In that case, unconditional choices would be "contaminated" by the signal structure, which would leave unclear how to isolate the behavioral implications of the signal structure. It follows that the behavior described throughout this paper should not be seen as describing dynamic choice, but rather as choices in two different (with and without signals), but related, settings (the payoff urn is common).[14]

In order to provide additional motivation for (2.5), we adapt the usual practice in the literature on unconditional ambiguity-sensitive preferences, where behavior in the "ambiguous" domain (bets on Ellsberg's unknown urn) is compared with behavior in the risk domain (bets on Ellsberg's known urn). Behavior in the risk domain is assumed to be "standard" (expected utility), and differences in behavior across the domains are attributed to non-indifference to ambiguity. Because our focus is on updating behavior, we go further in these respects. In our case, the comparison risk domain includes also risky ("noisy") signals and in addition, updating in the risk domain is assumed to conform to Bayes' rule.[15] Then, for each color in turn, we compare bets on that color from the payoff urn versus from a risky urn

[14] Alternatively, if the environment is dynamic but the decision maker is myopic or cannot anticipate possible signals, similar behavior may arise. Since it may be challenging to identify myopia, we have concentrated (both theoretically and experimentally) on the setting of two ordered scenarios.

[15] This discussion applies to the thought experiment. In the experiment itself, however, we do not make such an assumption, but compare the updating behavior under risk and ambiguity (see Section 3.1.)

(constructed below) *both unconditionally, and then also conditionally* after realization of both ambiguous and noisy signals.

Accordingly, consider a hypothetical risky signal structure featuring two noisy signals, denoted also σ_R and σ_B, with probabilities given by some $\alpha \in \Delta(\mathbf{\Sigma})$. Let the ex ante risk counterpart of a bet on red be the objective lottery $(100, \alpha_{\sigma_R} p_{\sigma_R,R} + \alpha_{\sigma_B} p_{\sigma_B,R})$. Assuming Bayesian updating in the risk domain, the "posterior lotteries" are $(100, p_{\sigma,R})$ if the α_σ-signal is realized, where $\sigma = \sigma_R, \sigma_B$. Therefore, for each signal σ, the DM would be indifferent between f_R and $(100, p_{\sigma,R})$, (since $p_{\sigma,R}$ is the probability equivalent for f_R according to the conditional preference $\succeq_\sigma$). Signal ambiguity is absent at the unconditional stage and also in the risk domain, while aversion to it is reflected in the conditional probability equivalents $p_{\sigma,R}$, thus suggesting that $f_R \succ_0 (100, \alpha_{\sigma_R} p_{\sigma_R,R} + \alpha_{\sigma_B} p_{\sigma_B,R})$. Similarly for the corresponding statements regarding the bet on B. Since any α is plausible as the hypothetical signal likelihoods, one is led to the following definition of aversion to signal ambiguity: *There exists* $\alpha = (\alpha_{\sigma_R}, \alpha_{\sigma B}) \in \Delta(\Sigma)$, such that

$$\begin{aligned} p_{0,R} &> \alpha_{\sigma_R} p_{\sigma_R,R} + \alpha_{\sigma_B} p_{\sigma_B,R} \text{ and} \\ p_{0,B} &> \alpha_{\sigma_R} p_{\sigma_R,B} + \alpha_{\sigma_B} p_{\sigma_B,B}. \end{aligned} \tag{4.1}$$

Under symmetry (2.4), clearly (2.5) implies (4.1). Note that $\alpha = \left(\frac{1}{2}, \frac{1}{2}\right)$ is not implied logically by the symmetry condition (2.4), nor is it "natural" given its role as defining a hypothetical (shadow) risky signal structure - any α would do. However, the next Lemma shows, assuming also diversity (2.2), that fixing $\alpha = \left(\frac{1}{2}, \frac{1}{2}\right)$ is without loss of generality and hence that the two definitions are equivalent. Therefore, the above intuition applies also to (2.5).

Lemma 1 *Assuming signal diversity, there exists $\alpha \in \Delta(\Sigma)$ satisfying (4.1) if and only if (2.5) is satisfied.*

Proof. Given symmetry (2.4), condition (2.5) implies (4.1) with $\alpha_\sigma = \frac{1}{2}$ for all σ. Conversely, suppose there exists α as indicated but that $p_{0,R} \leq \frac{1}{2} p_{\sigma_R,R} + \frac{1}{2} p_{\sigma_B,R}$. Then, for both $s = R, B$:

$$\begin{aligned} \Sigma_\sigma \alpha_\sigma p_{\sigma,s} &< p_{0,s} \leq \tfrac{1}{2} p_{\sigma_R,s} + \tfrac{1}{2} p_{\sigma_B,s} \Longrightarrow \\ 0 &< \left(\tfrac{1}{2} - \alpha_{\sigma_R}\right) \left(p_{\sigma_R,s} - p_{\sigma_B,s}\right). \end{aligned}$$

But this is impossible given signal diversity (2.2). ■

Further perspective is provided by modifying the existential quantifier "there exists α" in (4.1) to "for all α," thus requiring that the behavior indicated in (4.1) be exhibited *for all* signal structures. This condition is equivalent (given symmetry) to requiring that

$$p_{0,R} > p_{\sigma_R,R} \text{ and } p_{0,R} > p_{\sigma_B,R},$$

or, more explicitly in terms of preferences,

$$f_R \succ_0 (100, p_{\sigma,R}) \ \text{ for } \sigma = \sigma_R, \sigma_B. \tag{4.2}$$

This says roughly that, for each σ, $\succeq_\sigma$ is *more ambiguity averse than* $\succeq_0$ in the sense of the comparative notion widely adopted in the decision theory literature and built on the following intuition: given that $f_R \sim_\sigma (100, p_{\sigma,R})$, that the lottery is ambiguity-free, and that $\succeq_0$ is less averse to ambiguity, then $f_R \succ_0 (100, p_{\sigma,R})$ follows. Thinking of ambiguous signals as increasing ambiguity aversion beyond what prevails ex ante, one might take (4.2) as the behavioral meaning of signal ambiguity aversion. Clearly, (4.2) is strictly more demanding than (2.5). Moreover, it is too strong in our view.

For further perspective, note that (4.2) is strictly weaker than dilation, which, in a maxmin framework corresponds to the case where, for every signal realization, the set of posteriors enlarges (includes) the set of priors. (See the literature cited in the Introduction.) A fortiori it is also too strong to capture what we have in mind, as we now explain.

A signal σ may reduce utility because of uncertainty about its interpretation, but, in general, a signal also contains information about the state space that could render the bet under consideration more attractive, thus raising utility. Think of two dimensions of a signal – its "mean informational content" and "uncertainty about that content" – that may affect utility in opposite directions. The condition (4.2) identifies as ambiguous only signals for which the uncertainty effect dominates for all signal realizations. In contrast, the behavior that we propose can identify uncertainty about a signal's interpretation even if, for some realizations, its mean informational content dominates and results in an overall increase in utility. To see how, suppose that contrary to (4.2), $(100, p_{\sigma,R}) \succ_0 f_R$, and that there is nevertheless uncertainty about the interpretation of σ_R. The indicated strict ranking reveals that σ_R is a very strong favorable signal in the mean dimension for drawing red, strong enough to more than offset difficulties with interpretation. But then it is also a very unfavorable signal for drawing black. Thus σ_R makes the bet on black unattractive because of *both* uncertainty about interpretation *and* because of its negative mean informational content. This can lead to its probability equivalent $p_{\sigma,B}$ being sufficiently small that, assuming symmetry (2.4), $f_R \succ_0 (100, \frac{1}{2}p_{\sigma_R,R} + \frac{1}{2}p_{\sigma_B,R})$ is satisfied.

The preceding suggests that (4.2) is too restrictive to be taken as the behavioral manifestation of aversion to signal ambiguity. Though the behavior we suggest may be too broad, in that it might admit other rationales unrelated to signals being hard-to-interpret,[16] an advantage of our proposed liberal approach is that a model that precludes (2.5) is more readily dismissed as being unable to capture aversion to signal ambiguity.

4.2 Relation between theory and experimental design

Identifying violation of the martingale property (2.5) with sensitivity to signal ambiguity is based, theoretically and experimentally, on comparative statics of updating under risk and under ambiguity. Our experimental design parallels the theoretical approach of comparing PE conditional on risky (as control) and ambiguous (as treatment) signals to unconditional PE, though in a between-subject experimental design. This design allows the experimenter to control for deviations from the standard assumptions – Bayesian updating and reduction of compound objective lotteries, which are present also in the risky signal control, and concentrate solely on the marginal effect of ambiguous signals. Hence, only the marginal effect of ambiguity in deviations from the martingale property should be attributed to signal ambiguity. To minimize confusion, it is crucial to acknowledge that while we experimentally employ a compound objective lottery to generate the payoff urn (for the reasons discussed in Section 3.1 - mainly to impose symmetry and eliminate the potential for hedging), we do not provide here a decision-theoretic model that can simultaneously accommodate ambiguity aversion – which is related to violations of reduction of compound objective lotteries, and sensitivity to signal ambiguity. Moreover, we are not aware of a formal model that can account for the documented association between violation of ROCL (as observed in the payoff urn) and departure from Bayesian updating under risk (as measured by violations of the martingale property in the risk control).

[16] For example, Cripps (2018) presents an axiomatic model of non-Bayesian updating of objective probability distributions where prior beliefs need not equal the average of posteriors because of under- or over-reaction to new information.

Therefore, in interpreting the results, we take the association between violation of ROCL and ambiguity aversion as an empirical fact, and use it as an indirect measure of ambiguity aversion.

4.3 Models

Section 3.2 showed that sensitivity to signal ambiguity is common, but not universal. Decision-makers vary in their attitude to hard-to-interpret signals: some are close to the Bayesian benchmark, others are averse, while the remainder like signal ambiguity. The goal of this section (and Appendix B.1) is to demonstrate (in a non-exhaustive way) how some popular models accommodate the various patterns of behavior and the associations among behaviors documented in the experiment.

As emphasized, we explore choice given two different information structures – no signals (unconditional), and then a particular signal structure as defined above (conditional). Existing static models of ambiguity-sensitive preference restrict attention to one fixed (implicit) information structure and thus do not apply directly. Put another way, one could apply any of these models separately to model $\succeq_0$ and each conditional order $\succeq_\sigma$. Applied in this way, received theories would not address updating in that they would not restrict how unconditional and conditional preferences are related, rendering (2.5), as well as many other patterns of unconditional and conditional choices, rationalizable. We view this approach as conceding that received theories are orthogonal to the issues considered here. We proceed instead by examining whether *extensions* of these models that include plausible and/or commonly used updating rules can accommodate signal ambiguity. Another point to emphasize is that our treatment of models is intended to be illustrative rather than exhaustive. After examining the benchmark Bayesian model, we focus on the maxmin model (Gilboa and Schmeidler 1989) with two alternative updating rules. (See also Appendix B.2 for an examination of the smooth ambiguity model.)

For all models, preferences are defined on a set $\mathcal{F}$ of Savage acts over the state space S with outcomes in X. We maintain the binary structure, assuming for the most part that $S = \{R, B\}$, $X = \{100, 0\}$, $\mathcal{F} = \{f_R, f_B\}$, and that the signal space is $\mathbf{\Sigma} = \{\sigma_R, \sigma_B\}$, but arguments extend readily to the general setup treated in Appendix A. Risk preferences are expected utility with vNM index u normalized by

$$u(100) = 1,\ u(0) = 0.$$

Utility functions on $\mathcal{F}$, denoted $V_\sigma(\cdot)$, for $\sigma \in \{0\} \cup \mathbf{\Sigma}$ for unconditional and conditional preferences respectively, are defined by probability equivalents:

$$V_\sigma(f_R) = p_{\sigma,R} \text{ and } V_\sigma(f_B) = p_{\sigma,B}.$$

Symmetry (2.4) and signal diversity (2.2) are assumed throughout. We examine the capacity of models to accommodate (2.5) and its signal-ambiguity seeking counterpart where the inequality is reversed.

4.3.1 Models with "Bayesian updating"

In the Bayesian model, unconditional utility has the subjective expected utility (SEU) form with respect to prior belief m_0,

$$V_0(f) = \int_S u(f)\, dm_0(s),\quad f \in \mathcal{F}.$$

Conditional utility $V_\sigma(\cdot)$ is given by SEU with respect to the posterior $m(\cdot \mid \sigma)$ which is derived by Bayesian updating using a likelihood function $\ell(\sigma \mid s)$. Exclude the degenerate case where signals are uninformative and assume that

$$\ell(\sigma_R \mid R) \neq \ell(\sigma_R \mid B); \tag{4.3}$$

this implies signal diversity. The well-known implication of this model is that

$$\begin{aligned} m_0(\cdot) &= \sum_\sigma L(\sigma)\, m(\cdot \mid \sigma) \text{ and} \\ V_0(f) &= \sum_\sigma L(\sigma)\, V_\sigma(f),\ f \in \mathcal{F}, \end{aligned} \tag{4.4}$$

where

$$L(\sigma) \equiv \int_S \ell(\sigma \mid s)\, dm_0(s).$$

Therefore, by Lemma 1, indifference to signal ambiguity (that is, equality in (2.5)) is implied.

Note that the preceding applies to *any likelihood function* ℓ (and more generally, for any L consistent with (4.4)), just as the Ellsberg paradox is robust to which prior is assumed. In particular, it applies to two variants of the above Bayesian model that have been explored in the literature. To capture uncertainty about interpretation of signals, and hence about the true likelihood function, Acemoglu, Chernozhukov and Yildiz (2016) assume that updating of m_0 is done using an average likelihood $\overline{\ell}$ of the form

$$\overline{\ell}\left(\sigma \mid s\right) = \int \ell\left(\sigma \mid s\right) d\lambda_s\left(\ell\right),$$

where, for each s, $\lambda_s \in \Delta\left(\Delta\left(\{\sigma_R, \sigma_B\}\right)\right)$ is a subjective distribution over likelihoods. Conclude that this specification does not model hard-to-interpret signals in the sense of the behavior we have identified. In another variant, it is assumed that the Bayesian agent uses the "wrong" likelihood function, specifically, one in which signals are taken to be more precise than they really are. Such agents are often called "overconfident" (Daniel, Hirshleifer and Subrahmanyam 1998). We see that such overconfidence is behaviorally distinguishable from an affinity to signal ambiguity.

Indifference to signal ambiguity is implied also in models that can rationalize (unconditional) Ellsbergian ambiguity aversion if a suitable "Bayesian-like" updating rule is added. See Epstein and Seo (2015) for one such model.

4.3.2 Maxmin utility

Following Gilboa and Schmeidler (1989), ambiguity about S is represented by a subjective set $\mathcal{M}_0 \subset \Delta(S)$, and unconditional utility is given by

$$V_0\left(f\right) = \min_{m \in \mathcal{M}_0} \int u\left(f\right) dm.$$

In the alternative scenario, the individual is informed that a signal will be realized. Thus she contemplates uncertainty about $\{\sigma_R, \sigma_B\} \times S$, which is modeled by a subset $\mathcal{M}$ of $\Delta\left(\{\sigma_R, \sigma_B\} \times S\right)$. We assume that $\mathcal{M}$ is consistent with $\mathcal{M}_0$ in the sense that $\mathcal{M}_0$ equals the set of all S-marginals of measures in $\mathcal{M}$, that is,

$$\mathcal{M}_0 = \left\{mrg_S\ m : m \in \mathcal{M}\right\}. \tag{4.5}$$

After realization of the signal σ, the individual updates her set of priors to $\mathcal{M}_\sigma \subset \Delta(S)$ and she evaluates acts using the conditional maxmin utility function

$$V_\sigma\left(f\right) = \min_{m \in \mathcal{M}_\sigma} \int u\left(f\right) dm.$$

It remains to describe $\mathcal{M}$ and the sets $\mathcal{M}_\sigma$ in greater detail. We consider two widely used update rules: prior-by-prior Bayesian updating (also known as generalized Bayes' rule (GBR)), and maximum likelihood updating (ML), whereby only those priors that maximize the likelihood of the realized signal are retained and updated by Bayes' rule.[17] We further divide the discussion into two cases that highlight the main message regarding how to model signal ambiguity within the framework of maxmin utility. (See Appendix B.1 for supporting details.)

Single-likelihood: For each s, let $\ell(\cdot \mid s) \in \Delta(\{\sigma_R, \sigma_B\})$ describe the distribution of signals conditional on the true state satisfying (4.3). The critical feature of this special case is that this conditional distribution, or likelihood function, is unique as in Bayesian modeling. To incorporate this sharp view of likelihoods, let $\mathcal{M}$ consist of all measures m on $\{\sigma_R, \sigma_B\} \times S$ for which the S-marginal lies in $\mathcal{M}_0$ and the S-conditional is ℓ. Then, *for both of the noted updating rules, affinity to signal ambiguity* is implied (and the affinity is strict if $\mathcal{M}_0$ is not a singleton).

Multiple-likelihoods: To sharpen the contrast with the preceding case, suppose that unconditional beliefs about S are represented by the single (full support) prior m_0, that is, $\mathcal{M}_0 = \{m_0\}$. Multiplicity arises at the level of conditional distributions or likelihoods: let $\mathcal{L}$ denote a subjective set of possible likelihood functions ℓ, where $\ell(\cdot \mid s) \in \Delta(\{\sigma_R, \sigma_B\})$ for every s. Define $\mathcal{M}$ to be the set of all measures m on $\{\sigma_R, \sigma_B\} \times S$ for which the S-marginal is m_0 and, for which the S-conditional is an element of $\mathcal{L}$. Then *signal ambiguity aversion is implied for both updating rules.* The intuition is apparent at the functional form level: the multiplicity of likelihoods captures uncertainty about how to interpret a given signal and permits the adopted interpretation to vary with the bet being evaluated. For example, when evaluating the bet on red (black), a signal σ is interpreted conservatively in the way least (most) favorable to red being drawn. This acts to reduce conditional utility levels for each bet and each signal, consistent with (2.5).

Note that unconditional ambiguity aversion (satisfied by single-likelihood but not by multiple-likelihood as defined above) and signal ambiguity aver-

[17] Pires (2002) provides axiomatic foundations for GBR; Gilboa and Schmeidler (1993) axiomatize ML in the special case where the maxmin model of preference with set $\mathcal{M}$ also conforms with Choquet expected utility (Schmeidler 1989).

sion (satisfied by multiple-likelihood but not by single-likelihood) are independent properties. Simultaneous aversion to both kinds of ambiguity can be achieved by perturbing initial beliefs in the multiple-likelihoods model slightly and taking $\mathcal{M}_0$ to be a small neighborhood of m_0.

5 Conclusion

We provide a counterpart to Ellsberg's experiments, which focus on prior ambiguity, by considering the response to information in environments where information is available but is compatible with different interpretations and hence inferences. We suggest that such decision environments are common, including, for example, the recent COVID-19 pandemic. After identifying revealed sensitivity to hard-to-interpret information with failure of the martingale property of belief, we document experimentally that many subjects respond in such a way to ambiguous signals.

As noted in the Introduction, attitude to signal ambiguity has attracted significant attention in applied work. Many of the cited papers are motivated by introspection and interpretation suggested by the appearance of a functional form, which may be tempting at first glance but are well-known to be unreliable. Consistent with standard practice in decision theory, we offer definitions for attitudes towards signal ambiguity based on behavior that is (in principle) observable and hence verifiable by an observer. The current paper is the first attempt to provide such a definition, and is complemented by an experimental study that demonstrates how to operationalize and identify the proposed behavior in a lab setting, and that documents its pervasiveness among the population of subjects.

This study utilizes two related, but distinct, choice environments on which unconditional and conditional preferences are defined. The definition of sensitivity to signal ambiguity relies on the DM not anticipating the signals at the unconditional stage. This leaves the important research question of dynamic perception of signal ambiguity: when the DM anticipates receiving the ambiguous signals, and the anticipation may contaminate the unconditional choice – both theoretically (e.g., through recursive evaluation), and behaviorally (if the agent is not myopic). One can even imagine a dynamic setting in which the dynamic structure itself is ambiguous – when the likelihood of receiving a signal is ambiguous. In these settings, it is challenging to separate the updating component from prior beliefs. The current study proposes the first benchmark to identify sensitivity to signal ambiguity in abstract setting, but we expect more work on new dimensions of ambiguity of information structures.

We believe that the behavior identified in the current study is natural in interactive settings where inference is often made from the actions of other agents whose rationality is uncertain. Though precise experimental identification is challenging, the single agent environment studied here suggests that non-Bayesian behavior in the form of deviations from the martingale property could be a good starting point to measure agents' lack of confidence in others' reasoning.

Lastly, though not the main focus of the current study, the association documented between ROCL and Bayesian updating in the risk control calls for further study, both experimental and theoretical. We focused on the marginal effect of signal ambiguity, but much of the literature on failure of Bayesian reasoning deals with objective environments. The evidence herein suggests new avenues to understand the vast behavioral literature on non-Bayesian updating, drawing potential new connections to other choice domains; for example, the form in which information is revealed (one-shot or gradual), deviations from expected utility (both under risk and under uncertainty), and preferences for the timing of resolution of uncertainty.

Moreover, most of the models we discussed in this paper did not account for the relation between attitude to ambiguity and compound objective lotteries. If such a relation is a common element in behavioral responses to ambiguity and updating, then it calls for a model that can account for the three behaviors simultaneously.

A Appendix: A more general analysis

A.1 Primitives

- S: finite (payoff relevant) state space
- $\mathbf{\Sigma}$: finite set of signals
- X: set of real-valued outcomes with largest and smallest elements (say 100 and 0)
- Acts f map S into X; $\mathcal{F}$ is a *(fixed) finite subset* of acts

- $\Delta(X)$: the set of all (simple) lotteries P
- Preferences $\succeq_0$ and $\{\succeq_\sigma\}_{\sigma\in\mathbf{\Sigma}}$ on $\mathcal{F}\cup\Delta(X)$

Adopt the following basic assumptions on preferences:

Pref0 All preferences are complete and transitive.

Pref1 All conditional preferences agree with $\succeq_0$ in the ranking of lotteries.

Pref2 $\succeq_0$ restricted to lotteries conforms to expected utility theory.

Pref3 $\succeq_0$ is strictly FOSD-monotone on lotteries

Pref4 For each $\sigma\in\{0\}\cup\mathbf{\Sigma}$, and act f, $\exists$ *probability-equivalent* $p_{\sigma,f}$, such that $P_{\sigma,f}=(100,p_{\sigma,f})\sim_\sigma f$

Pref1 expresses the assumption that signals are unrelated to the objective prospects (lotteries). Pref2 is almost universal in the decision theory literature focussing on ambiguity. Pref3 and Pref4 are self-explanatory and common. These assumptions permit construction of utility functions $V_\sigma(\cdot)$ for $\succeq_\sigma$, $\sigma\in\{0\}\cup\mathbf{\Sigma}$, where,

$$V_\sigma(f)=p_{\sigma,f},\ \text{ for all } f\in\mathcal{F},$$

and, for all $P\in\Delta(X)$,

$$V_\sigma(P)=p,\text{ where } P\sim_0(100,p).$$

These utility functions render meaningful the comparison of utility levels unconditionally and across different signals. In particular, the inequality

$$V_{\sigma'}(f)>V_\sigma(f),\ \text{ for given } \sigma'\neq\sigma\in\mathbf{\Sigma},$$

is equivalent to the preference statement

$$[f\sim_{\sigma'}P'\text{ and } f\sim_\sigma P]\implies P'\succ_0 P,$$

It is interpreted to mean that σ' *is a better signal for* f *than is* σ.

Remark 2 *Finiteness of the set of acts $\mathcal{F}$ is not typical in axiomatic studies but is entirely appropriate in underpinnings for experiments where one elicits risk equivalents of only finitely many acts. The attitudes towards signal ambiguity defined below depend on the empirically relevant set $\mathcal{F}$.*

Refer to *signal diversity* (relative to $\mathcal{F}$) if for some $\sigma_1 \in \Sigma$: For every disjoint subsets $\Sigma_I, \Sigma_{II} \subset \Sigma \backslash \{\sigma_1\}$, at least one nonempty, $\exists f \in \mathcal{F}$ s.t.

$$\begin{aligned} p_{\sigma,f} &> p_{\sigma_1,f} \text{ if } \sigma \in \Sigma_I \\ p_{\sigma,f} &< p_{\sigma_1,f} \text{ if } \sigma \in \Sigma_{II}, \end{aligned}$$

that is, σ is better (worse) than σ_1 for f if $\sigma \in \Sigma_I$ (Σ_{II}). If $\mathbf{\Sigma} = \{\sigma_1, \sigma_2\}$ is binary, then signal diversity reduces to: $\exists g, h \in \mathcal{F}$ s.t.

$$(p_{\sigma_2,g} - p_{\sigma_1,g})(p_{\sigma_2,h} - p_{\sigma_1,h}) < 0$$

that is, σ_1 is better for one act and σ_2 is better for the other, as in (2.2).

A.2 Attitudes: definitions and characterizations

Define attitudes to signal ambiguity as follows (strict notions can be defined in the obvious way).

Definition 3 *Weak aversion: There exists $\alpha \in \Delta(\mathbf{\Sigma})$ s.t.*

$$V_0(f) \geq \sum_{\sigma} \alpha_\sigma V_\sigma(f) \text{ for all } f \in \mathcal{F}. \tag{A.1}$$

Weak affinity: There exists $\alpha \in \Delta(\mathbf{\Sigma})$ s.t.

$$V_0(f) \leq \sum_{\sigma} \alpha_\sigma V_\sigma(f) \text{ for all } f \in \mathcal{F}. \tag{A.2}$$

Indifference: There exists $\alpha \in \Delta(\mathbf{\Sigma})$ such that

$$V_0(f) = \sum_{\sigma} \alpha_\sigma V_\sigma(f) \text{ for all } f \in \mathcal{F}. \tag{A.3}$$

Intuition for these definitions is similar to that for (4.1). In the SEU framework, when updating conforms to Bayes' rule, (A.3) reduces to the familiar martingale condition relating prior and posterior beliefs. Our intention here is to identify it as a meaningful condition more generally where preferences over acts are not necessarily SEU and beliefs are not necessarily representable by probability measures.

The presence of the existential quantifiers $\exists\alpha$ raises two questions about these definitions. First, is indifference equivalent to the conjunction of weak aversion and weak affinity? Second, and more practically, can the defining conditions be verified? The next theorem addresses these concerns.

Theorem 4 *(i) There is weak aversion to signal ambiguity iff*

$$\min_{\sigma \in \boldsymbol{\Sigma}} \left(\int V_\sigma (f) \, d\beta \right) \leq \int V_0 (f) \, d\beta \quad \textit{for all } \beta \in \Delta (\mathcal{F}) . \tag{A.4}$$

(ii) There is weak affinity to signal ambiguity iff

$$\int V_0 (f) \, d\beta \leq \max_{\sigma \in \boldsymbol{\Sigma}} \left(\int V_\sigma (f) \, d\beta \right) \quad \textit{for all } \beta \in \Delta (\mathcal{F}) . \tag{A.5}$$

(iii) There is indifference to signal ambiguity iff $\forall \beta \in ba (\mathcal{F})$,[18]

$$\min_{\sigma \in \boldsymbol{\Sigma}} \left(\int V_\sigma (f) \, d\beta \right) \leq \int V_0 (f) \, d\beta \leq \max_{\sigma \in \boldsymbol{\Sigma}} \left(\int V_\sigma (f) \, d\beta \right) . \tag{A.6}$$

Assuming signal diversity, then: (a) indifference is also equivalent to the conjunction of weak aversion and affinity; and (b) $\alpha = (\alpha_\sigma)$ *in the martingale condition is unique.*

In each case, the corresponding equivalent statement replaces the existential quantifiers for α with more customary and preferable universal quantifier (see Section A.3 for how the reformulation aids verifiability). The condition (A.6) can be simplified since the left-hand inequality is redundant given that β is not restricted in sign. However, it is strictly stronger than the act-by-act condition

$$\min_{\sigma \in \boldsymbol{\Sigma}} V_\sigma (f) \leq V_0 (f) \leq \max_{\sigma \in \boldsymbol{\Sigma}} V_\sigma (f) \text{ for all } f \in \mathcal{F},$$

which would be sufficient if in (A.3) we allowed α to vary with f. The conjunction of (i) and (ii) is weaker than (A.3), because the former asserts only existence of two measures, one for (A.1) and another, generally distinct, measure for (A.2), while (A.3) asserts that there is a single measure satisfying both inequalities. Accordingly, the characterization (A.6) is stronger than the conjunction of (A.4) and (A.5) because the βs are not restricted to be probability measures. However, under signal diversity, the conjunction of weak aversion and weak affinity is equivalent to indifference.[19]

[18] $ba (\mathcal{F})$ is the set of signed measures on $\mathcal{F}$. Given finiteness of $\mathcal{F}$, it is isomorphic to $\mathbb{R}^{|\mathcal{F}|}$.

[19] The proof is elementary. For example, assume that (A.1) and (A.2) are satisfied with α and α' respectively. Then $\sum_{\sigma \neq \sigma_1} (\alpha_\sigma - \alpha'_\sigma) (V_\sigma (f) - V_{\sigma_1} (f)) \leq 0$ for all f, which contradicts signal diversity unless $\alpha = \alpha'$ (take $\Sigma_I = \{\sigma \neq \sigma_1 : \alpha_\sigma > \alpha'_\sigma\}$ and $\Sigma_{II} = \{\sigma \neq \sigma_1 : \alpha_\sigma < \alpha'_\sigma\}$).

Signal diversity also guarantees other desirable properties. For example, define strict attitudes by the obvious strict inequality counterparts of (A.1) and (A.2). Then, for example, strict aversion (affinity) and weak affinity (aversion) are disjoint if signal diversity is satisfied.

Part (iii) of the theorem can be interpreted as providing an *axiomatization* for the property (A.3) of updating and doing so for a very broad class of preferences.[20] There is arguably a rough parallel with Machina and Schmeidler (1992). They generalize SEU and axiomatize probabilistically sophisticated preferences – those for which underlying beliefs can be represented by a probability measure; and they do so without unduly restricting other aspects of preference. We generalize the other main component of the Bayesian model, namely Bayesian updating, and we axiomatize those collections $\{V_\sigma\}_{\sigma \in \{0\} \cup \mathbf{\Sigma}}$ of preferences that satisfy the key martingale property of Bayesian updating; and we do so without assuming maxmin or any other parametric class of preferences, and without specifying a particular updating rule beyond what is implicit in (A.3) or (A.6). Another parallel is that just as probabilistic sophistication defines a benchmark for modeling sensitivity to unconditional ambiguity of the sort highlighted by Ellsberg, we propose (A.3) as a benchmark for modeling sensitivity to signal ambiguity.

A.3 Verifiability

Here we show that the alternative characterizations (A.4) and (A.5) in Theorem 4 provide a tractable way to check whether a given data set is consistent with weak aversion or weak affinity. By "data," we mean probability equivalents elicited along the lines of our thought (and laboratory) experiments. Utility values for each act are equal to the corresponding probability equivalents–hence, it merits emphasis that the utility values appearing in the theorem are observable. When a similar procedure is applied to check for strict aversion, (using the obvious strict counterpart of the theorem), one obtains a generalization of the inequality (2.5) which is the focus of the text. That presumed a binary environment and the symmetry expressed by (2.4), while these restrictions are not needed in Theorem 4.

Consider the practical value of the characterization (A.4) for verifying

[20] Condition (A.6) is a full-fledged axiom because the utility values $V_\sigma(f)$ are probability equivalents and hence observable. Its interpretation is not clear however.

(A.1): The former can be written as

$$\max_{\beta\in\Delta(\mathcal{F})}\Phi(\beta)\leq 0,$$

where $\Phi(\beta)=\min_{\sigma\in\mathbf{\Sigma}}\left(\int V_{\sigma}(f)\,d\beta\right)-\int V_{0}(f)\,d\beta$. Finding a maximizer is a matter of linear programming because Φ is piecewise linear.

To illustrate, consider the thought experiment and revert to earlier notation. Then

$$\begin{aligned}\Phi(\beta) &= \min\left\{\beta p_{\sigma_R,R}+(1-\beta)\,p_{\sigma_R,B},\beta p_{\sigma_B,R}+(1-\beta)\,p_{\sigma_B,B}\right\} \\ &\quad -(\beta p_{0,R}+(1-\beta)\,p_{0,B})\end{aligned}\tag{A.7}$$

The maximum is achieved at $\beta^{*}=0,1$, or β^{c},

$$\beta^{c}=\frac{1}{1+\frac{p_{\sigma_R,R}-p_{\sigma_B,R}}{p_{\sigma_B,B}-p_{\sigma_R,B}}}.$$

β^{c} is that weight β for which the two terms inside the minimization in (A.7) are equal:

$$\beta^{c}p_{\sigma_R,R}+(1-\beta^{c})\,p_{\sigma_R,B}=\beta^{c}p_{\sigma_B,R}+(1-\beta^{c})\,p_{\sigma_B,B}.\tag{A.8}$$

Thus weak aversion is *equivalent* to $\Phi(\beta)\leq 0$ at these three values of β and hence (by brute calculation) to:

$$\begin{aligned}&\beta^{*}p_{0,R}+(1-\beta^{*})\,p_{0,B} \\ \geq\ &\min\left\{\beta^{*}p_{\sigma_R,R}+(1-\beta^{*})\,p_{\sigma_R,B},\beta^{*}p_{\sigma_B,R}+(1-\beta^{*})\,p_{\sigma_B,B}\right\},\end{aligned}\tag{A.9}$$

where

$$\beta^{*}=\begin{cases}0 & p_{0,R}-p_{0,B}>p_{\sigma_R,R}-p_{\sigma_R,B}\\ 1 & p_{0,R}-p_{0,B}<p_{\sigma_B,R}-p_{\sigma_B,B}\\ \frac{1}{1+\frac{p_{\sigma_R,R}-p_{\sigma_B,R}}{p_{\sigma_B,B}-p_{\sigma_R,B}}} & p_{\sigma_B,R}-p_{\sigma_B,B}\leq p_{0,R}-p_{0,B} \\ & \quad\leq p_{\sigma_R,R}-p_{\sigma_R,B}\end{cases}\tag{A.10}$$

Under the intuitive assumption

$$p_{\sigma_B,R}-p_{\sigma_B,B}<p_{0,R}-p_{0,B}=0<p_{\sigma_R,R}-p_{\sigma_R,B},\tag{A.11}$$

(A.9)-(A.10) are equivalent to the single inequality

$$p_{0,R}\geq\frac{p_{\sigma_B,B}-p_{\sigma_R,B}}{\left(p_{\sigma_B,B}-p_{\sigma_R,B}\right)+\left(p_{\sigma_R,R}-p_{\sigma_B,R}\right)}p_{\sigma_R,R}+\frac{p_{\sigma_R,R}-p_{\sigma_B,R}}{\left(p_{\sigma_B,B}-p_{\sigma_R,B}\right)+\left(p_{\sigma_R,R}-p_{\sigma_B,R}\right)}p_{\sigma_R,B}.$$

If (A.11) is strengthened to symmetry (2.4), then $\beta^{*}=\frac{1}{2}$ and one obtains the weak inequality form of (2.5).

A.4 Proof of Theorem 4

For vector inequalities, adopt the notation

$$\begin{aligned} x &\gg y\text{: } x_i > y_i \text{ all } i \\ x &> y\text{: } x_i \geq y_i \text{ all } i \text{ and } x \neq y \\ x &\geq 0\text{: } x_i \geq y_i \text{ all } i \end{aligned}$$

All vectors are column vectors unless transposed by superscript $^\intercal$.

We use Tucker's Theorem of the Alternative (Mangasarian 1969): Exactly one of the following systems of inequalities has a solution:

$$\begin{aligned} (1)\ Bx &> 0,\ Cx \geq 0,\ Dx = 0 \quad (B \text{ nonvacuous}) \\ (2)\quad 0 &= B^\intercal y_2 + C^\intercal y_3 + D^\intercal y_4,\ y_2 \gg 0,\ y_3 \geq 0. \end{aligned}$$

Purely for notational simplicity, let $\mathbf{\Sigma} = \{\sigma_1, \sigma_2\}$ and $\mathcal{F} = \{g, h\}$ be binary; the reader will see that the argument is perfectly general.

Proof of (iii): If we denote by x the vector $(1, \alpha_{\sigma_1}, \alpha_{\sigma_2})^\intercal$, or as any positive scalar multiple thereof, then existence of solution α satisfying (A.3) can be restated as: $\exists x \in \mathbb{R}^3$ solving

$$Cx = 0,\ x > 0$$

where[21]

$$d^\intercal = \begin{bmatrix} 1 & -1 & -1 \end{bmatrix},\ C = \begin{bmatrix} A \\ d^\intercal \end{bmatrix},\ \text{ and}$$

$$A = \begin{bmatrix} A_g^\intercal \\ A_h^\intercal \end{bmatrix},\ A_f^\intercal = \begin{bmatrix} V_0(f) & -V_{\sigma_1}(f) & -V_{\sigma_2}(f) \end{bmatrix},\ \ f = g, h.$$

By Tucker's Theorem, the alternative is: $\exists y = (y^2, y^4)$ such that

$$y^2 + C^\intercal y^4 = 0,\ y^2 \gg 0,$$

or equivalently $C^\intercal y^4 << 0$, or equivalently (let $y^4 = \left(\beta_g, \beta_h, \beta_0\right)$):

$$\begin{bmatrix} A^\intercal & d \end{bmatrix} y^4 \ll 0,$$

[21]Note that $x > 0$ and $d^\intercal x = 0$ imply that $x_1 > 0$. Below keep in mind also that C is $3 \times (\Sigma + 1)$.

$$\begin{bmatrix} V_0(g) & V_0(h) & 1 \\ -V_{\sigma_1}(g) & -V_{\sigma_1}(h) & -1 \\ -V_{\sigma_2}(g) & -V_{\sigma_2}(h) & -1 \end{bmatrix} \begin{bmatrix} \beta_g \\ \beta_h \\ \beta_0 \end{bmatrix} << 0,$$

or equivalently: $\exists \left(\beta_g, \beta_h, \beta_0\right)$ s.t.

$$\begin{aligned} \Sigma_f \beta_f V_0(f) + \beta_0 &< 0 \quad \text{and} \\ \Sigma_f \beta_f V_\sigma(f) + \beta_0 &> 0 \text{ for all } \sigma \end{aligned}$$

which is true iff

$$\Sigma_f \beta_f V_0(f) < -\beta_0 < \Sigma_f \beta_f V_\sigma(f) \text{ for all } \sigma.$$

Conclude that the alternative is: $\exists \left(\beta_g, \beta_h\right)$ s.t.

$$\Sigma_f \beta_f V_0(f) < \Sigma_f \beta_f V_\sigma(f) \text{ for all } \sigma.$$

Therefore, (A.3) obtains iff: $\forall \left(\beta_g, \beta_h\right)$

$$\Sigma_f \beta_f V_0(f) \geq \Sigma_f \beta_f V_\sigma(f) \text{ for some } \sigma.$$

But taking $-\beta$, obtain also that: $\forall \left(\beta_g, \beta_h\right)$

$$\Sigma_f \beta_f V_0(f) \leq \Sigma_f \beta_f V_\sigma(f) \text{ for some } \sigma.$$

Combine to obtain: $\forall \beta$,

$$\min_\sigma \Sigma_f \beta_f V_\sigma(f) \leq \Sigma_f \beta_f V_0(f) \leq \max_\sigma \Sigma_f \beta_f V_\sigma(f).$$

Consider (iii.a). Assume that (A.1) and (A.2) are satisfied with α and α' respectively, $\alpha \neq \alpha'$. Then $\sum_{\sigma \neq \sigma_1} \left(\alpha_\sigma - \alpha'_\sigma\right)\left(V_\sigma(f) - V_{\sigma_1}(f)\right) \leq 0$ for all $f \in \mathcal{F}$. Obtain a contradiction by taking $\Sigma_I = \{\sigma : \alpha_\sigma - \alpha'_\sigma > 0\}$ and $\Sigma_{II} = \{\sigma : \alpha_\sigma - \alpha'_\sigma < 0\}$ in the definition of signal diversity,

Uniqueness follows similarly.

Proof of (i): Use notation from the preceding proof. x denotes the vector $(1, \alpha_{\sigma_1}, \alpha_{\sigma_2})^\intercal$, or any positive scalar multiple thereof. We want a solution to

$$x > 0,\ Ax \geq 0,\ d^\intercal x = 0.$$

By Tucker's Theorem, the alternative is:

$$\begin{aligned} y^2 + A^\intercal y^3 + d y^4 &= 0, \\ y^2 &\gg 0, y^3 \geq 0 \end{aligned}$$

or, letting $y^3 = (\beta_g, \beta_h)^\intercal \geq 0$,

$$\begin{bmatrix} V_0(g) & V_0(h) \\ -V_{\sigma_1}(g) & -V_{\sigma_1}(h) \\ -V_{\sigma_2}(g) & -V_{\sigma_2}(h) \end{bmatrix} \begin{bmatrix} \beta_g \\ \beta_h \end{bmatrix} + \begin{bmatrix} 1 \\ -1 \\ -1 \end{bmatrix} y^4 \ll 0.$$

Thus (adding 1st and 2nd components, then 1st and 3rd) the *alternative* to (A.1) is that $\exists\, (\beta_g, \beta_h)^\intercal \geq 0$ s.t.

$$\Sigma_{f=g,h}\beta_f V_0(f) < \Sigma_{f=g,h}\beta_f V_\sigma(f) \text{ for each } \sigma.$$

Conclude that (A.1) is equivalent to: $\forall\, (\beta_g, \beta_h)^\intercal \geq 0$,

$$\Sigma_{f=g,h}\beta_f V_0(f) \geq \min_\sigma \Sigma_{f=g,h}\beta_f V_\sigma(f).$$

The proof for (ii) is similar. ■

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B Online Appendix

B.1 Details for the maxmin model

We provide some supporting details for the maxmin model defined in Section 4.3.2. Accordingly, $S = \{R, B\}$ and $\mathbf{\Sigma} = \{\sigma_R, \sigma_B\}$. Both symmetry (2.4) and signal diversity (2.2) are assumed.

B.1.1 Maxmin with single-likelihood

We are given that $\ell(\cdot \mid s) \in \Delta(\mathbf{\Sigma})$ for each $s = R, B$, satisfying (4.3). Without loss of generality, renaming signals if necessary, suppose that

$$\ell(\sigma_R \mid R) > \ell(\sigma_R \mid B). \tag{B.1}$$

Let $\mathcal{M}_0 \subset \Delta(S)$ be compact and a non-singleton, and let $\mathcal{M} \subset \Delta(\mathbf{\Sigma} \times S)$ be constructed as in Section 4.3.2. Unconditional utilities are

$$\begin{aligned} V_0(f_B) &= \min_{m \in \mathcal{M}} m(B) = m^*(B), \text{ and} \\ V_0(f_R) &= \min_{m \in \mathcal{M}} m(R) = m^{**}(R). \end{aligned}$$

By symmetry, the probability interval $[m^{**}(R), 1 - m^*(B)]$ for red is symmetric about $\frac{1}{2}$ and

$$V_0(f_R) = m^{**}(R) < \tfrac{1}{2}.$$

For any given m in $\mathcal{M}$, its Bayesian update is

$$\begin{aligned} m(s \mid \sigma) &= [m(s)\,\ell(\sigma \mid s)] / L_m(\sigma), \text{ where} \\ L_m(\cdot) &\equiv \int \ell(\cdot \mid s')\, dm(s'). \end{aligned}$$

Prior-by-prior updating (GBR): Conditional utilities are given by, for each $\sigma = \sigma_B, \sigma_R$,

$$\begin{aligned} V_\sigma(f_B) &= \min_{m \in \mathcal{M}} \frac{m(B)\,\ell(\sigma \mid B)}{L_m(\sigma)} \\ &= \frac{m^*(B)\,\ell(\sigma \mid B)}{L_{m^*}(\sigma)}. \end{aligned}$$

Therefore,

$$\begin{aligned} V_0\left(f_B\right) &= m_0^*\left(B\right) = m_0^*\left(B\right)\ell\left(\sigma_B \mid B\right) + m_0^*\left(B\right)\ell\left(\sigma_R \mid B\right) \Longrightarrow \\ V_0\left(f_B\right) &= L_{m^*}\left(\sigma_B\right)V_{\sigma_B}\left(f_B\right) + L_{m^*}\left(\sigma_R\right)V_{\sigma_R}\left(f_B\right). \end{aligned} \tag{B.2}$$

Similarly for R,

$$V_0\left(f_R\right) = L_{m^{**}}\left(\sigma_B\right)V_{\sigma_B}\left(f_R\right) + L_{m^{**}}\left(\sigma_R\right)V_{\sigma_R}\left(f_R\right). \tag{B.3}$$

In addition, because $\max m\left(B\right) = m^{**}\left(B\right) > m^*\left(B\right) = \min m\left(B\right)$,

$$\begin{aligned} L_{m^*}\left(\sigma_B\right) &= m_0^*\left(B\right)\ell\left(\sigma_B \mid B\right) + m_0^*\left(R\right)\ell\left(\sigma_B \mid R\right) \\ &< m_0^{**}\left(B\right)\ell\left(\sigma_B \mid B\right) + m_0^{**}\left(R\right)\ell\left(\sigma_B \mid R\right) = L_{m^{**}}\left(\sigma_B\right). \end{aligned}$$

By (B.1),

$$V_{\sigma_B}\left(f_B\right) > V_{\sigma_R}\left(f_B\right) \text{ and } V_{\sigma_R}\left(f_R\right) > V_{\sigma_B}\left(f_R\right). \tag{B.4}$$

Therefore, from (B.2), (B.3), and (B.4),

$$\begin{aligned} V_0\left(f_R\right) &< L_{m^*}\left(\sigma_B\right)V_{\sigma_B}\left(f_R\right) + L_{m^*}\left(\sigma_R\right)V_{\sigma_R}\left(f_R\right), \text{ and} \\ V_0\left(f_B\right) &< L_{m^{**}}\left(\sigma_B\right)V_{\sigma_B}\left(f_B\right) + L_{m^{**}}\left(\sigma_R\right)V_{\sigma_R}\left(f_B\right). \end{aligned}$$

Combine these with the equalities (B.2) and (B.3) to obtain

$$V_0\left(f_s\right) < \sum_{\sigma} \alpha_\sigma V_\sigma\left(f_s\right),\ s = R, B,$$

where $\alpha_\sigma = \frac{1}{2}L_{m^*}\left(\sigma\right) + \frac{1}{2}L_{m^{**}}\left(\sigma\right)$. This proves strict signal ambiguity affinity.

Maximum likelihood updating (ML): Conditional on each realized signal σ, one retains only those measures in $\mathcal{M}$ that maximize the probability of σ. Each is updated by Bayes' rule and the minimum conditional probability of s, $s = R, B$, defines the conditional utilities $V_\sigma^{ML}(f_s)$. Since the minimum is taken over a smaller set than under GBR, it is immediate that, for each σ and s,

$$V_\sigma^{ML}(f_s) \geq V_\sigma(f_s). \tag{B.5}$$

Unconditional utilities are identical for the two updating rules. It follows that there is signal ambiguity loving also under ML.

B.1.2 Maxmin with multiple-likelihoods

We have $\mathcal{M}_0 = \{m_0\}$. By symmetry for unconditional utility,

$$m_0(R) = m_0(B) = \tfrac{1}{2}.$$

Let the nonsingleton set $\mathcal{L}$ be such that $\ell(\sigma \mid s) > 0$ for every σ, s, and $\ell \in \mathcal{L}$. Each likelihood is determined by a pair $(\ell(\sigma_R \mid R), \ell(\sigma_R \mid B)) \in [0,1]^2$. Thus $\mathcal{L}$ can be identified with a subset of the unit square (it is assumed compact).

Prior-by-prior updating (GBR): Aversion to signal ambiguity follows from (B.5) and the result below for ML.

Maximum likelihood updating (ML): The priors maximizing the likelihood of σ are obtained by combining m_0 with every ℓ in $\mathcal{L}_\sigma$,

$$\mathcal{L}_\sigma = \arg\max_{\ell \in \mathcal{L}} L_\ell(\sigma),\ L_\ell(\sigma) \equiv \Sigma_s \ell(\sigma \mid s)\, m_0(s).$$

Utilities are given by

$$\begin{aligned} V_\sigma(f_s) &= \tfrac{1}{2}\frac{\min_{\ell \in \mathcal{L}_\sigma} \ell(\sigma \mid s)}{L^*(\sigma)}, \quad s = R, B, \\ L^*(\sigma) &= \max_{\ell \in \mathcal{L}} L_\ell(\sigma). \end{aligned}$$

Therefore, using symmetry (2.4),

$$\begin{aligned} V_{\sigma_R}(f_R) + V_{\sigma_B}(f_R) &= V_{\sigma_R}(f_R) + V_{\sigma_R}(f_B) \\ &= \frac{1}{2L^*(\sigma_R)}\left[\min_{\ell \in \mathcal{L}_{\sigma_R}} \ell(\sigma_R \mid R) + \min_{\ell \in \mathcal{L}_{\sigma_R}} \ell(\sigma_R \mid B)\right] \\ &\leq \frac{1}{2L^*(\sigma_R)} \min_{\ell \in \mathcal{L}_{\sigma_R}} \left[\ell(\sigma_R \mid R) + \ell(\sigma_R \mid B)\right] \\ &= \frac{\min_{\ell \in \mathcal{L}_{\sigma_R}} \left[\ell(\sigma_R \mid R) + \ell(\sigma_R \mid B)\right]}{\max_{\ell \in \mathcal{L}} \left[\ell(\sigma_R \mid R) + \ell(\sigma_R \mid B)\right]} \leq 1, \end{aligned}$$

which implies the weak-inequality counterpart of (2.5). ■

B.1.3 A numerical example

The following numerical example may further clarify the intuition for our behavioral definitions and how they differ from interpretations suggested by the appearance of functional form.

Consider two possible joint distributions on $\Sigma \times S$, μ_1 and μ_2, given by

$$\begin{aligned} \mu_1(\sigma_R, R) &= \mu_2(\sigma_B, B) = \frac{3}{8}, \; \mu_1(\sigma_R, B) = \mu_2(\sigma_B, R) = \frac{1}{8} \\ \mu_1(\sigma_B, B) &= \mu_2(\sigma_R, R) = \frac{1}{2}, \; \mu_1(\sigma_B, R) = \mu_2(\sigma_R, B) = 0. \end{aligned}$$

Suppose that DM is a maxmin agent and that conditional on each signal realization she uses the posteriors implied by Bayesian updating, namely the set of posteriors for R equal to $\mathcal{M}_{\sigma_R} = \{\frac{3}{4}, 1\}$ and $\mathcal{M}_{\sigma_B} = \{0, \frac{1}{4}\}$ conditional on σ_R and σ_B respectively. Thus $p_{\sigma_R,R} = \frac{3}{4}$ and $p_{\sigma_B,R} = 0$. The joint distributions imply the set of priors $\mathcal{M}_0 = \{\frac{3}{8}, \frac{5}{8}\}$ for R. If this is the set of priors used by DM at the ex ante stage, then $p_{0,R} = \frac{3}{8}$ and, contrary to (2.5),

$$p_{0,R} = \tfrac{1}{2} p_{\sigma_R,R} + \tfrac{1}{2} p_{\sigma_B,R},$$

which we interpret as indifference to signal ambiguity (Appendix A). In contrast, the non-singleton sets of posteriors and the aversion to ambiguity built into the maxmin model would seem to suggest aversion to signal ambiguity.

We have two reactions to the critique of our martingale approach implied by the preceding. First, interpretations suggested by the appearance of functional forms may be tempting at first glance, but are well-known to be unreliable.

Second, we feel that the designation of signal ambiguity neutrality in the above example is intuitive. The set $\mathcal{M}_0$ of priors can be viewed as being constructed by backward induction, pasting together the indicated sets of posteriors with prior beliefs about signals (here each signal has probability $\frac{1}{2}$); similarly for the associated conditional and prior utilities of the bets on R and B. Thus the prior utilities of DM, modeled by the example, are identical to those that would apply to an individual who foresees the signal structure and uses backward induction reasoning. Given that (by assumption) our DM does not foresee the signal structure at the prior stage, the above specification seems less natural than in a dynamic setting. In fact, it is very special in that when applied to our DM, it implies that nevertheless her behavior is (as if) she could foresee. In other words, revealing the signal structure (including its ambiguity) to DM at the prior stage would not affect her prior probability equivalents $p_{0,R}$ and $p_{0,B}$, suggesting indifference to signal ambiguity.

B.2 Smooth ambiguity

Consider the smooth model (Klibanoff, Marinacci and Mukerji 2005) adapted as follows. For concreteness and simplicity only, adopt the setting in the experiment. Accordingly, take $S = \{R, B\}$ and $\mathbf{\Sigma} = \{\sigma_R, \sigma_B\}$. Denote by $n = 1$ or 9 the possible number of red balls in the payoff urn and by $m_n \in \Delta(S)$ the corresponding probability distribution for the color drawn from the payoff urn ($m_n(R) = n/10$). At the unconditional stage, before becoming aware of the signal structure, uncertainty about n is represented by $\mu_0 \in \Delta(\{1, 9\})$; since equal probabilities are announced to subjects, take $\mu_0(1) = \frac{1}{2}$, though any subjective prior would do equally. Her unconditional utility function is V_0 given by

$$\phi \circ V_0(f) = \phi(p_{0,f}) = \int_{\{1,9\}} \phi\left(\int_S u(f)\, dm_n\right) d\mu_0(n), \tag{B.6}$$

for $f = f_R, f_B$, where $\phi(\cdot)$ is (strictly) increasing. Unconditional ambiguity aversion is modeled by taking ϕ concave.

In the alternative scenario, the individual is informed about the signal structure, (where $N = 0$ or 45 balls of each color are added when constructing the signal urn), and that a signal, either σ_R or σ_B, has been realized. Inferences about the payoff urn composition depend on beliefs about both n and N represented by the measure $\mu \in \Delta(\{1, 9\} \times \{0, 45\})$. The only restriction on μ is that the marginal probability $\mu(n)$ satisfy

$$\mu(n) = \mu_0(n), \;\; n = 1, 9. \tag{B.7}$$

The likelihood of each signal σ given any pair (n, N) is well-defined (e.g. $L(\sigma_R \mid n = 9, N = 45) = L(\sigma_B \mid n = 1, N = 45) = 54/100$), which permits Bayesian updating to $\mu(\cdot \mid \sigma)$. Conditional utility is defined by

$$\phi \circ V_\sigma(f) = \phi(p_{\sigma,f}) = \int_{\{1,9\}\times\{0,45\}} \phi\left(\int_S u(f(s))\, dm_n(s)\right) d\mu(n, N \mid \sigma). \tag{B.8}$$

Remark 5 *Following Klibanoff, Marinacci and Mukerji (2009), one might replace $m_n(s)$ above by $m_n(s \mid \sigma, N)$. However, the natural assumption is that draws from the payoff and signal urns are independent conditional on (n, N). Therefore, $m_n(s \mid \sigma, N) = m_n(s \mid N) = m_n(s)$, and we are back to (B.8).*

The utility functions in (B.6) and (B.8), plus (B.7), constitute a version of the smooth model for our setting. Define $L^*(\sigma)$ by

$$L^*(\sigma) = \Sigma_{n,N} L(\sigma \mid n, N)\, \mu(n, N).$$

Then it follows from the martingale property of Bayesian updating that

$$\begin{aligned}
\phi(p_{0,f}) &= L^*(\sigma_R)\, \phi(p_{\sigma_R,f}) + L^*(\sigma_B)\, \phi(p_{\sigma_B,f}) \\
&\leq \phi\left(L^*(\sigma_R)\, p_{\sigma_R,f} + L^*(\sigma_B)\, p_{\sigma_B,f}\right) \Longrightarrow \\
p_{0,f} &\leq L^*(\sigma_R)\, p_{\sigma_R,f} + L^*(\sigma_B)\, p_{\sigma_B,f},
\end{aligned}$$

which, by Lemma 1 (and its obvious extension to weak inequalities), is equivalent to (weak) signal ambiguity loving. Conclude that unconditional (Ellsberg) ambiguity aversion implies signal ambiguity loving.

B.3 Experimental results

B.3.1 Experimental results for all subjects

	Risky Signal		*Ambiguous signal*		*Total*	
	#	%	#	%	#	%
$p_0 < .475$	30	44.1	42	48.84	72	46.75
$.475 \leq p_0 \leq .525$	32	47.1	37	43.02	69	44.81
$.525 < p_0$	6	8.8	7	8.14	13	8.44
Total	68	100	86	100	154	100

Table B.1: Unconditional probability equivalents to 2-point compound risk for all subjects

There is no evidence of differential assignment to treatments based on unconditional PE (Fisher exact test p-value is .831).
The increase in the incidence of non-Bayesian behavior as a response to hard-to-interpret signals is significant at the 5% level (p-values of one-sided proportion test and one sided Fisher exact test are .016 and .024, respectively).

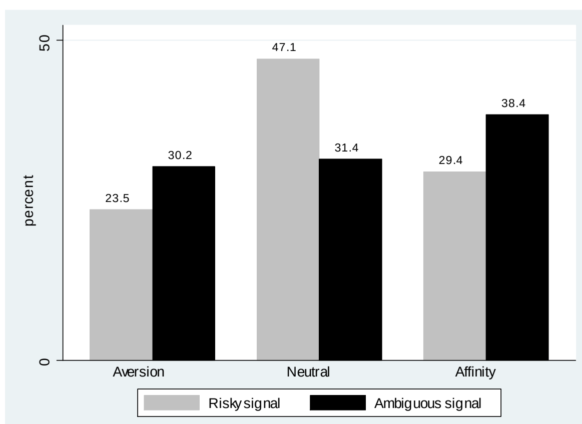


Figure B.1: Bayesian behavior and attitude to signal ambiguity - all subjects

Risky Signal	ROCL		
Bayesian	Yes	No	Total
Yes	21	11	32
No	11	25	36
Total	32	36	68

Ambiguous Signal	ROCL		
Bayesian	Yes	No	Total
Yes	19	7	26
No	18	42	60
Total	37	49	86

Table B.2: Frequencies in the risky signal control (left) and ambiguous signal treatment (right). Bayesian (rows) is approximately satisfying Bayes rule (left) and being approximately neutral to signal ambiguity (right). ROCL (columns) is having unconditional PE of approximately 0.5. p-value of Fisher exact test is .007 inthe risk control and smaller than .001 in the ambiguous treatment.

B.3.2 Approximate Bayesian

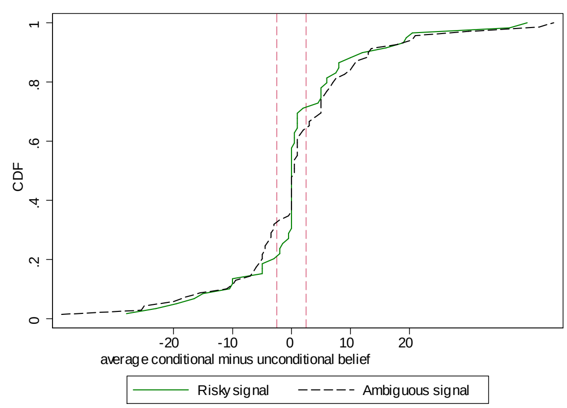
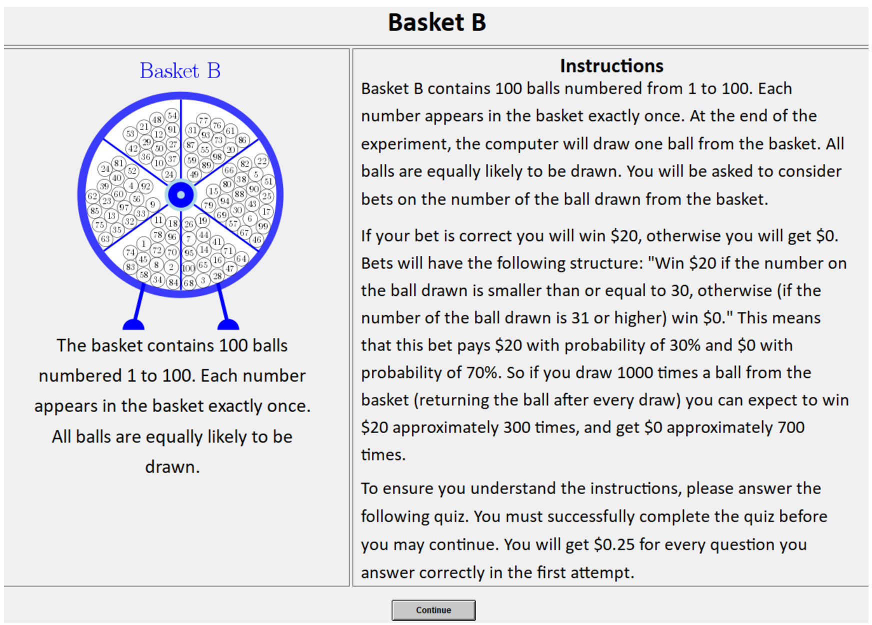


Figure B.2: CDF of Bayesian updating

Figure B.2 plots the difference in the distributions of our proposed measure for Bayesian behavior (3.2) between the risky-signal control (green full line) and the ambiguous-signal treatment (dashed line). A value of "0" on the horizontal axis implies that the unconditional PE equals the average of the two conditional PEs, that is – the subject is *exactly Bayesian*. The interval between the two vertical dashed lines is the 5% interval: $[-2.5\%, +2.5\%]$ in which we classify subjects as *approximate Bayesian*. Subjects that are to the left (right) of the interval have PEs that indicate that they are averse (seeking) to signal ambiguity. As can be easily seen from the figure, our findings reported in the paper are independent of the exact definition of "approximate Bayesian".

B.4 Experimental interface

B.4.1 Risk control



Quiz

1. A ball is drawn from Basket B

a. The ball with the number 11 has the same chance of being drawn as the ball with the number 85?

TRUE	☑	FALSE	☐

Your answer is correct

b. Which is more likely?

The ball drawn has a number less than or equal to 90	☑	The ball drawn has a number less than or equal to 20	☐

Your answer is correct

c. Which is more likely?

The ball drawn has a number less than or equal to 40	☐	The ball drawn has a number less than or equal to 60	☑

Your answer is correct

2. Suppose you choose the option "Win $20 if the number on the ball drawn from Basket B is smaller than or equal to 40; $0 otherwise."

a. What is your payment if the ball drawn is numbered 22?

$0	☐	$20	☑

Your answer is correct

b. What is your payment if the ball drawn is numbered 69?

$0	☑	$20	☐

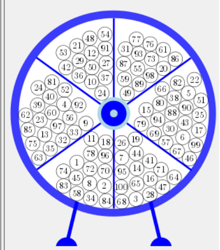
Your answer is correct

c. What is your payment if the ball drawn is numbered 40?

$0	☐	$20	☑

Your answer is correct

Basket B

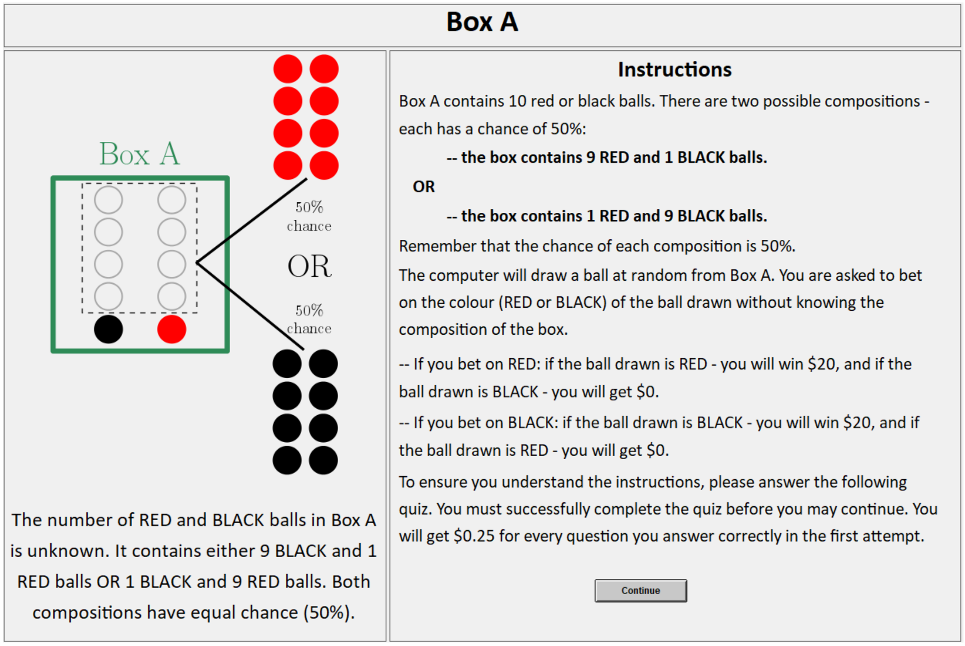


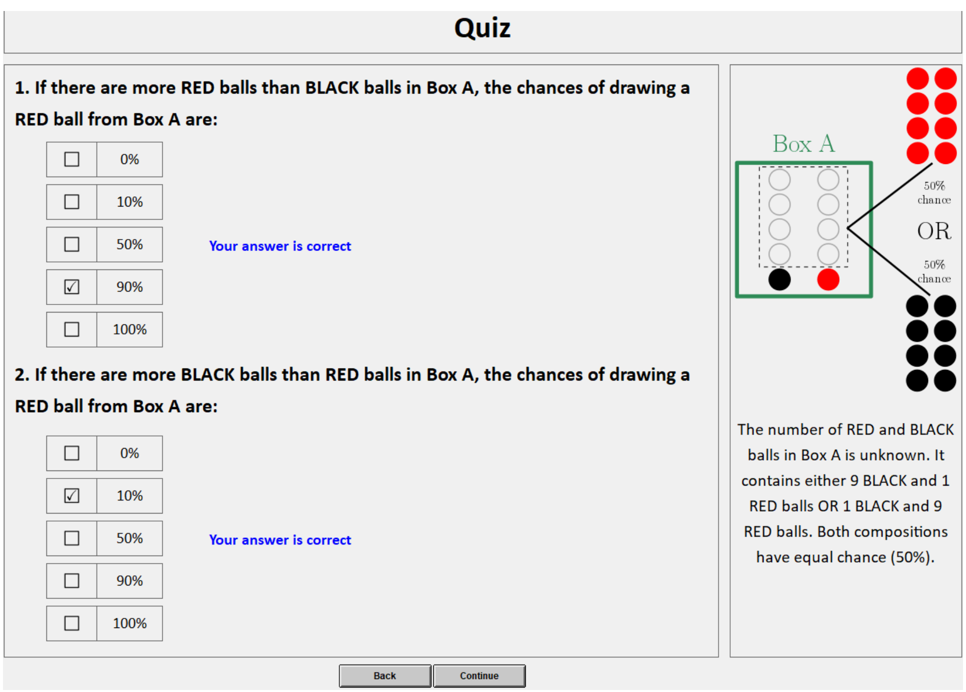
The basket contains 100 balls numbered 1 to 100.

Each number appears in the basket exactly once.

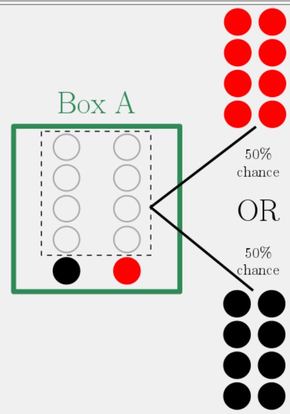
All balls are equally likely to be drawn.

Back Continue


Box A
Box A
50% chance
OR
50% chance
The number of RED and BLACK balls in Box A is unknown. It contains either 9 BLACK and 1 RED balls OR 1 BLACK and 9 RED balls. Both compositions have equal chance (50%).
Instructions
Box A contains 10 red or black balls. There are two possible compositions - each has a chance of 50%:
-- the box contains 9 RED and 1 BLACK balls.
OR
-- the box contains 1 RED and 9 BLACK balls.
Remember that the chance of each composition is 50%.
The computer will draw a ball at random from Box A. You are asked to bet on the colour (RED or BLACK) of the ball drawn without knowing the composition of the box.
-- If you bet on RED: if the ball drawn is RED - you will win $20, and if the ball drawn is BLACK - you will get $0.
-- If you bet on BLACK: if the ball drawn is BLACK - you will win $20, and if the ball drawn is RED - you will get $0.
To ensure you understand the instructions, please answer the following quiz. You must successfully complete the quiz before you may continue. You will get $0.25 for every question you answer correctly in the first attempt.
Continue


Quiz
1. If there are more RED balls than BLACK balls in Box A, the chances of drawing a RED ball from Box A are:
☐ 0%
☐ 10%
☐ 50%
☑ 90%
☐ 100%
Your answer is correct
2. If there are more BLACK balls than RED balls in Box A, the chances of drawing a RED ball from Box A are:
☐ 0%
☑ 10%
☐ 50%
☐ 90%
☐ 100%
Your answer is correct
Box A
50% chance
OR
50% chance
The number of RED and BLACK balls in Box A is unknown. It contains either 9 BLACK and 1 RED balls OR 1 BLACK and 9 RED balls. Both compositions have equal chance (50%).
Back
Continue

Box A



The number of RED and BLACK balls in Box A is unknown. It contains either 9 BLACK and 1 RED balls OR 1 BLACK and 9 RED balls. Both compositions have equal chance (50%).

Instructions

Box A contains 10 red or black balls. There are two possible compositions - each has a chance of 50%:

-- the box contains 9 RED and 1 BLACK balls.

OR

-- the box contains 1 RED and 9 BLACK balls.

Remember that the chance of each composition is 50%.

The computer will draw a ball at random from Box A. You are asked to bet on the colour (RED or BLACK) of the ball drawn without knowing the composition of the box.

-- If you bet on RED: if the ball drawn is RED - you will win \$20, and if the ball drawn is BLACK - you will get \$0.

-- If you bet on BLACK: if the ball drawn is BLACK - you will win \$20, and if the ball drawn is RED - you will get \$0.

Please select the colour you would like to bet on. The rest of the experiment will invoke bets on the colour you choose.

BLACK	☐	RED	☑

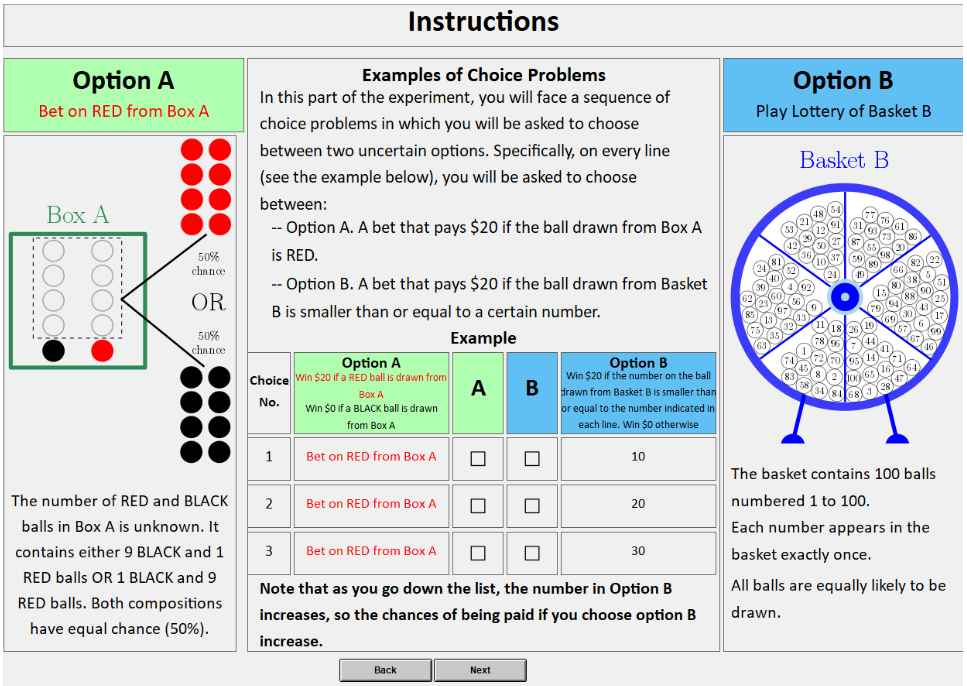
Submit your Bet

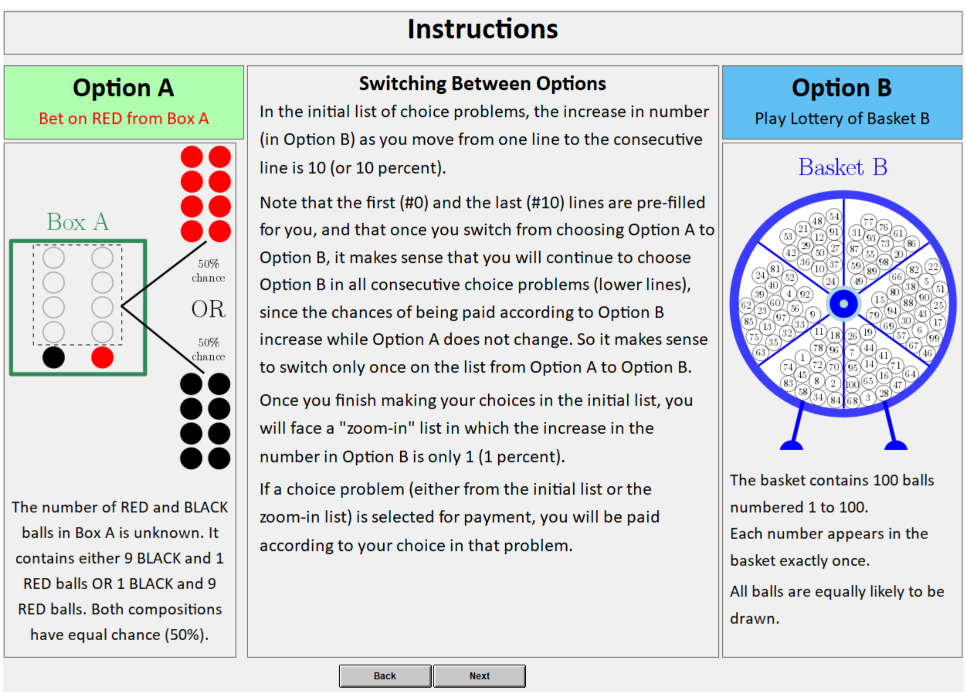
Instructions

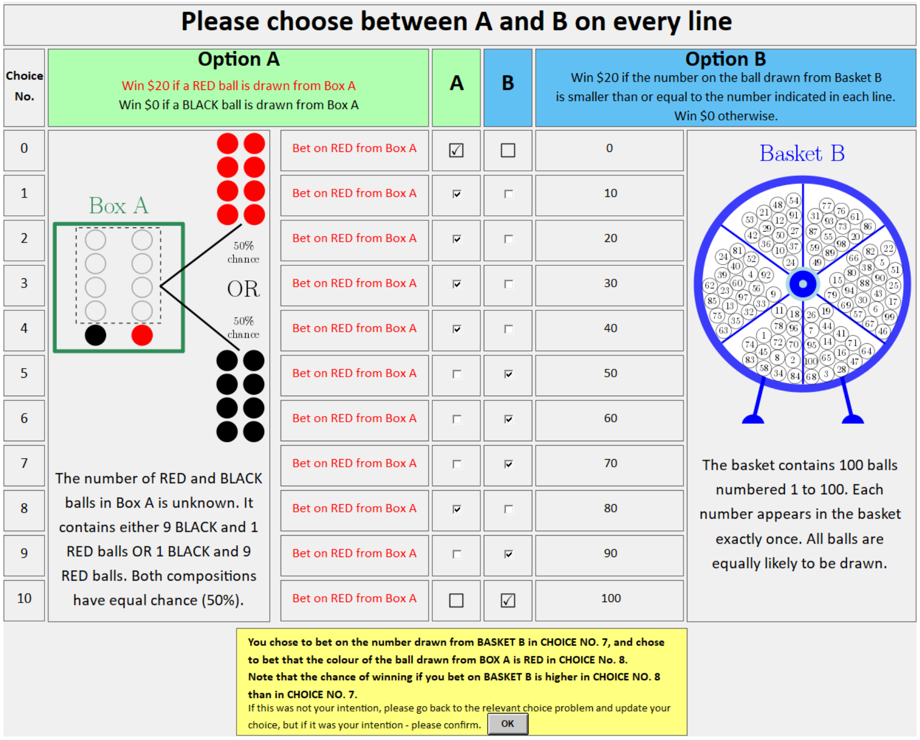
Following these instructions, you will be asked to make some choices. There are no correct choices. Your choices depend on your preferences and beliefs, so different participants will usually make different choices. You will be paid according to your choices, so read these instructions carefully and think before you decide.

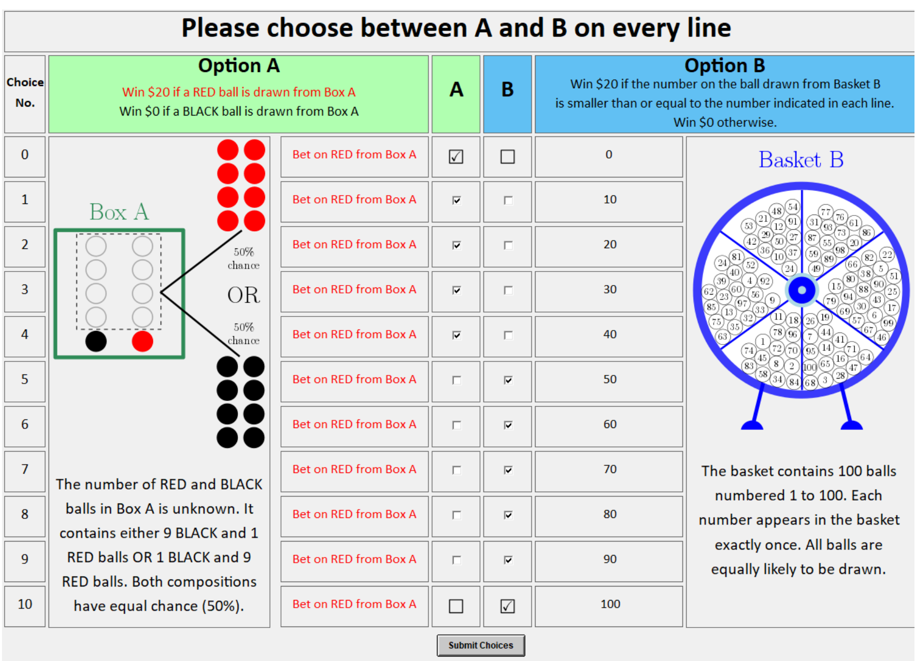
One of the choice problems will be selected at random, and your chosen option in that choice problem will determine your payment. This protocol of determining payments suggests that you should choose in each choice problem as if it is the only choice problem that determines your payment.

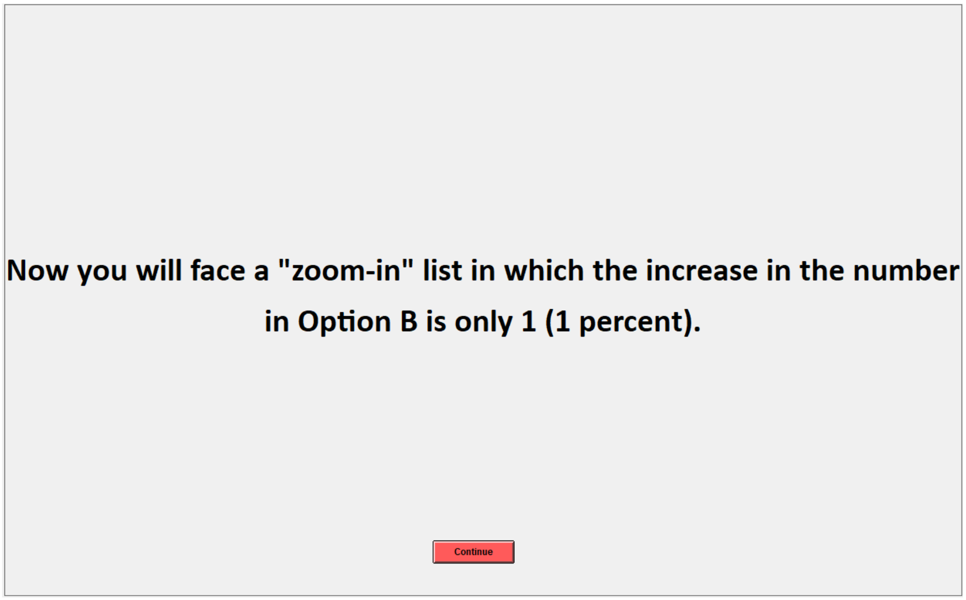
Next

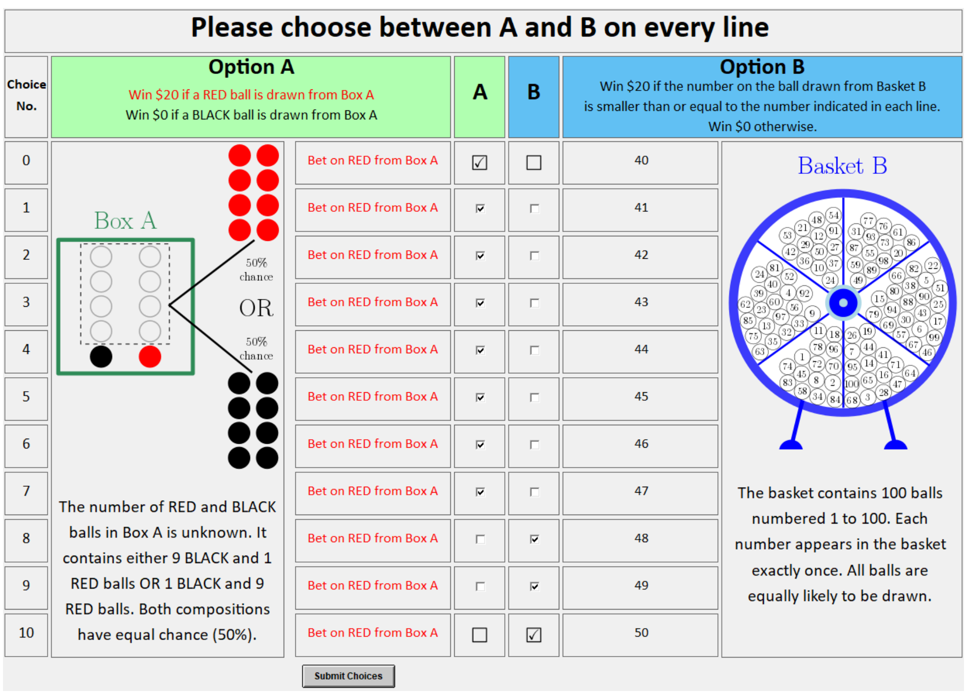

Instructions
Option A
Bet on RED from Box A
Box A
50% chance
OR
50% chance
The number of RED and BLACK balls in Box A is unknown. It contains either 9 BLACK and 1 RED balls OR 1 BLACK and 9 RED balls. Both compositions have equal chance (50%).
Examples of Choice Problems
In this part of the experiment, you will face a sequence of choice problems in which you will be asked to choose between two uncertain options. Specifically, on every line (see the example below), you will be asked to choose between:
-- Option A. A bet that pays $20 if the ball drawn from Box A is RED.
-- Option B. A bet that pays $20 if the ball drawn from Basket B is smaller than or equal to a certain number.
Example
Choice No.
Option A Win $20 if a RED ball is drawn from Box A Win $0 if a BLACK ball is drawn from Box A
A
B
Option B Win $20 if the number on the ball drawn from Basket B is smaller than or equal to the number indicated in each line. Win $0 otherwise
1 Bet on RED from Box A 10
2 Bet on RED from Box A 20
3 Bet on RED from Box A 30
Note that as you go down the list, the number in Option B increases, so the chances of being paid if you choose option B increase.
Option B
Play Lottery of Basket B
Basket B
The basket contains 100 balls numbered 1 to 100.
Each number appears in the basket exactly once.
All balls are equally likely to be drawn.
Back
Next


Instructions
Option A
Bet on RED from Box A
Box A
50% chance
OR
50% chance
The number of RED and BLACK balls in Box A is unknown. It contains either 9 BLACK and 1 RED balls OR 1 BLACK and 9 RED balls. Both compositions have equal chance (50%).
Switching Between Options
In the initial list of choice problems, the increase in number (in Option B) as you move from one line to the consecutive line is 10 (or 10 percent).
Note that the first (#0) and the last (#10) lines are pre-filled for you, and that once you switch from choosing Option A to Option B, it makes sense that you will continue to choose Option B in all consecutive choice problems (lower lines), since the chances of being paid according to Option B increase while Option A does not change. So it makes sense to switch only once on the list from Option A to Option B.
Once you finish making your choices in the initial list, you will face a "zoom-in" list in which the increase in the number in Option B is only 1 (1 percent).
If a choice problem (either from the initial list or the zoom-in list) is selected for payment, you will be paid according to your choice in that problem.
Option B
Play Lottery of Basket B
Basket B
The basket contains 100 balls numbered 1 to 100.
Each number appears in the basket exactly once.
All balls are equally likely to be drawn.
Back
Next


Please choose between A and B on every line
Choice No.
Option A
Win $20 if a RED ball is drawn from Box A
Win $0 if a BLACK ball is drawn from Box A
A
B
Option B
Win $20 if the number on the ball drawn from Basket B is smaller than or equal to the number indicated in each line.
Win $0 otherwise.
Box A
50% chance
OR
50% chance
The number of RED and BLACK balls in Box A is unknown. It contains either 9 BLACK and 1 RED balls OR 1 BLACK and 9 RED balls. Both compositions have equal chance (50%).
Bet on RED from Box A
Basket B
The basket contains 100 balls numbered 1 to 100. Each number appears in the basket exactly once. All balls are equally likely to be drawn.
You chose to bet on the number drawn from BASKET B in CHOICE NO. 7, and chose to bet that the colour of the ball drawn from BOX A is RED in CHOICE No. 8.
Note that the chance of winning if you bet on BASKET B is higher in CHOICE NO. 8 than in CHOICE NO. 7.
If this was not your intention, please go back to the relevant choice problem and update your choice, but if it was your intention - please confirm.
OK

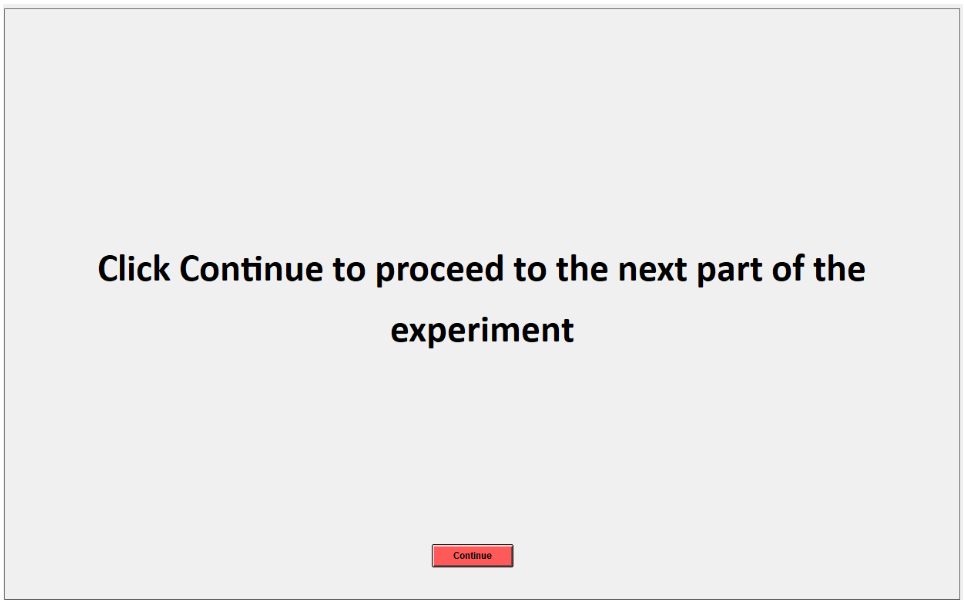

Please choose between A and B on every line
Choice No.
Option A
Win $20 if a RED ball is drawn from Box A
Win $0 if a BLACK ball is drawn from Box A
A
B
Option B
Win $20 if the number on the ball drawn from Basket B is smaller than or equal to the number indicated in each line.
Win $0 otherwise.
Box A
50% chance
OR
50% chance
The number of RED and BLACK balls in Box A is unknown. It contains either 9 BLACK and 1 RED balls OR 1 BLACK and 9 RED balls. Both compositions have equal chance (50%).
Bet on RED from Box A
Basket B
The basket contains 100 balls numbered 1 to 100. Each number appears in the basket exactly once. All balls are equally likely to be drawn.
Submit Choices

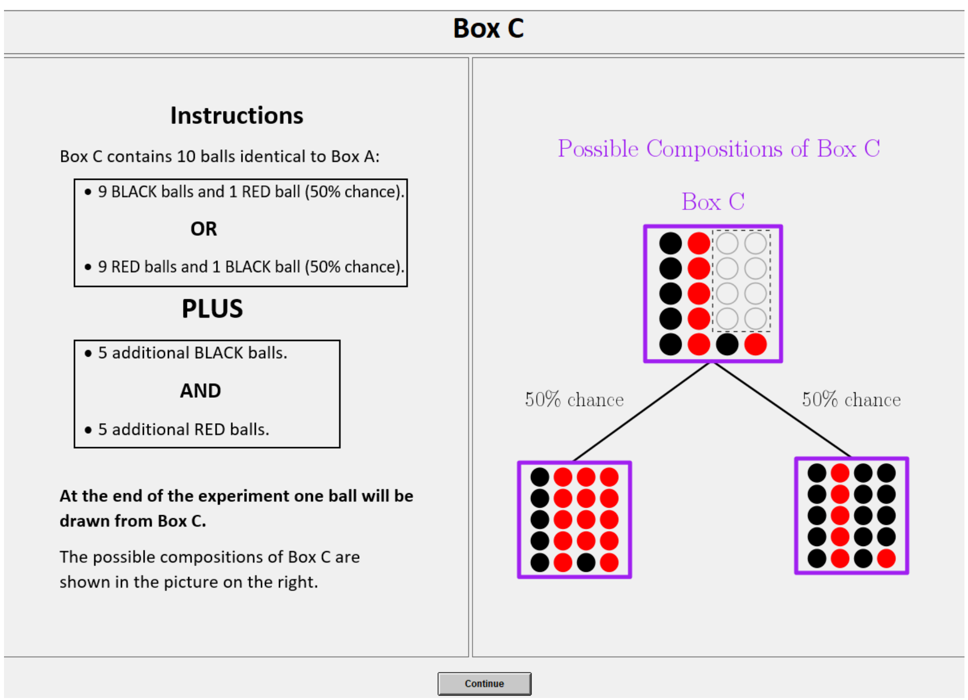




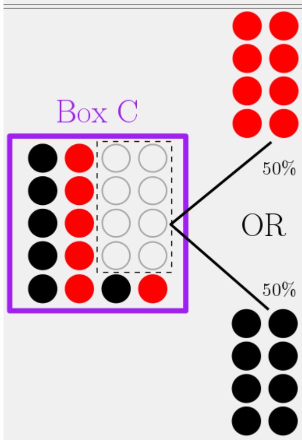
Please choose between A and B on every line

Choice No.	Option A: Win $20 if a RED ball is drawn from Box A. Win $0 if a BLACK ball is drawn from Box A	A	B	Option B: Win $20 if the number on the ball drawn from Basket B is smaller than or equal to the number indicated in each line. Win $0 otherwise.
0	Bet on RED from Box A	☑	☐	40
1	Bet on RED from Box A	☑	☐	41
2	Bet on RED from Box A	☑	☐	42
3	Bet on RED from Box A	☑	☐	43
4	Bet on RED from Box A	☑	☐	44
5	Bet on RED from Box A	☑	☐	45
6	Bet on RED from Box A	☑	☐	46
7	Bet on RED from Box A	☑	☐	47
8	Bet on RED from Box A	☐	☑	48
9	Bet on RED from Box A	☐	☑	49
10	Bet on RED from Box A	☐	☑	50


Click Continue to proceed to the next part of the experiment
Continue


Box C
Instructions
Box C contains 10 balls identical to Box A:
• 9 BLACK balls and 1 RED ball (50% chance).
OR
• 9 RED balls and 1 BLACK ball (50% chance).
PLUS
• 5 additional BLACK balls.
AND
• 5 additional RED balls.
At the end of the experiment one ball will be drawn from Box C.
The possible compositions of Box C are shown in the picture on the right.
Possible Compositions of Box C
Box C
50% chance
50% chance
Continue

Box C



Box C contains 10 balls identical to Box A (9 RED and 1 BLACK, or 1 RED and 9 BLACK) PLUS 5 additional RED balls AND 5 additional BLACK balls.

Instructions

In the next part of the experiment, you will be asked to choose between bets on Basket B (as before) and bets on the colour of a ball drawn from Box A, if you observe that a RED ball is drawn from Box C. Similarly, you will be asked to choose between bets on Basket B and bets on the colour of a ball drawn from Box A, if you observe that a BLACK ball is drawn from Box C. You will be asked to make your choices before you know the actual colour of the ball drawn from Box C.

If a choice problem from this part of the experiment is selected for payment, a ball will be drawn from Box C and your choice for that case (BLACK from Box C or RED from Box C) will determine your payment.

To ensure you understand the instructions, please answer the following quiz. You must successfully complete the quiz before you may continue. You will get $0.25 for every question you answer correctly in the first attempt.

Back | Continue

Quiz

1. If you bet on RED from Box A and a BLACK ball is drawn from Box C. What would be your payment?

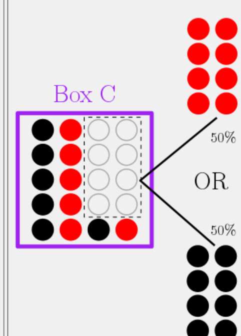
☐	$0
☐	$20
☑	It depends on the colour of the ball drawn from Box A.

Your answer is correct

2. If there are more BLACK balls than RED balls in Box A:

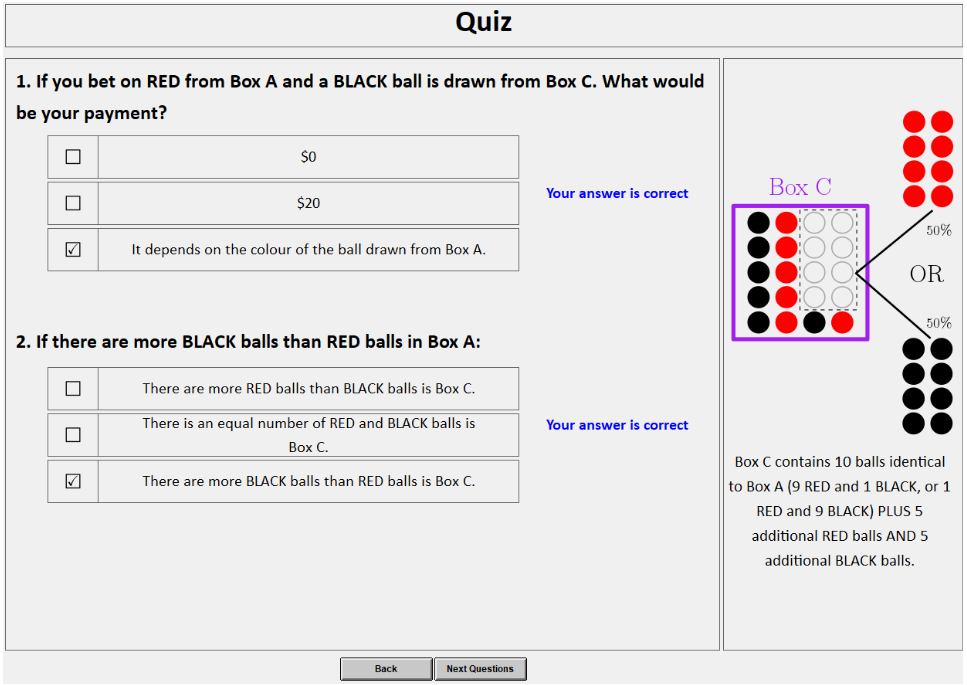
☐	There are more RED balls than BLACK balls is Box C.
☐	There is an equal number of RED and BLACK balls is Box C.
☑	There are more BLACK balls than RED balls is Box C.

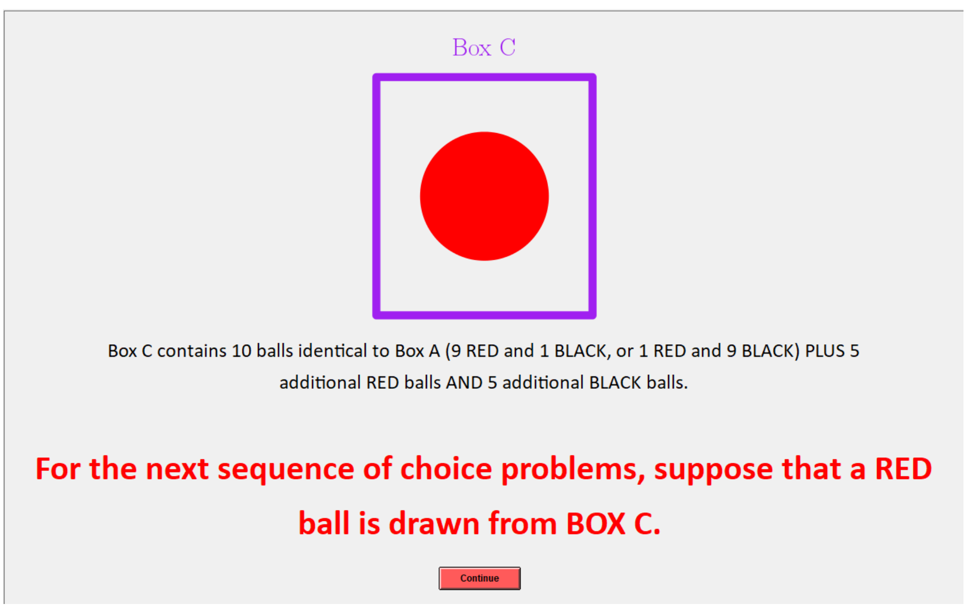
Your answer is correct



Box C contains 10 balls identical to Box A (9 RED and 1 BLACK, or 1 RED and 9 BLACK) PLUS 5 additional RED balls AND 5 additional BLACK balls.

Back | Next Questions


Quiz
1. If you bet on RED from Box A and a BLACK ball is drawn from Box C. What would be your payment?
☐ $0
☐ $20
☑ It depends on the colour of the ball drawn from Box A.
Your answer is correct
2. If there are more BLACK balls than RED balls in Box A:
☐ There are more RED balls than BLACK balls is Box C.
☐ There is an equal number of RED and BLACK balls is Box C.
☑ There are more BLACK balls than RED balls is Box C.
Your answer is correct
Box C
50%
OR
50%
Box C contains 10 balls identical to Box A (9 RED and 1 BLACK, or 1 RED and 9 BLACK) PLUS 5 additional RED balls AND 5 additional BLACK balls.
Back
Next Questions


Box C
Box C contains 10 balls identical to Box A (9 RED and 1 BLACK, or 1 RED and 9 BLACK) PLUS 5 additional RED balls AND 5 additional BLACK balls.
For the next sequence of choice problems, suppose that a RED ball is drawn from BOX C.
Continue

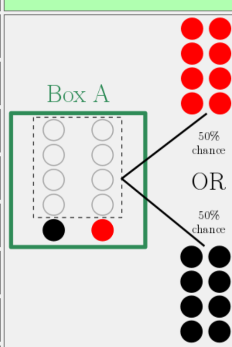
Remember that in part I (before Box C was introduced) you switched from betting on a RED ball being drawn from Box A to betting on the number drawn from Basket B at 48.

That is, for smaller numbers - you chose to bet on Box A, and for 48 and higher numbers - you chose to bet on the number drawn from Basket B.

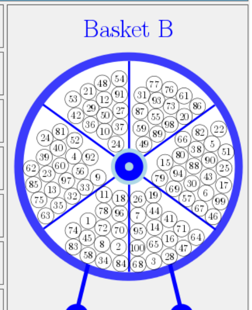
Continue

Please choose between A and B on every line IF A RED BALL IS DRAWN FROM BOX C

Choice No.	Option A Win \$20 if a RED ball is drawn from Box A Win \$0 if a BLACK ball is drawn from Box A	A	B	Option B Win \$20 if the number on the ball drawn from Basket B is smaller than or equal to the number indicated in each line. Win \$0 otherwise.
0	Bet on RED from Box A	☑	☐	0
1	Bet on RED from Box A	☑	☐	10
2	Bet on RED from Box A	☑	☐	20
3	Bet on RED from Box A	☑	☐	30
4	Bet on RED from Box A	☑	☐	40
5	Bet on RED from Box A	☑	☐	50
6	Bet on RED from Box A	☑	☐	60
7	Bet on RED from Box A	☐	☑	70
8	Bet on RED from Box A	☐	☑	80
9	Bet on RED from Box A	☐	☑	90
10	Bet on RED from Box A	☐	☑	100



The number of RED and BLACK balls in Box A is unknown. It contains either 9 BLACK and 1 RED balls OR 1 BLACK and 9 RED balls. Both compositions have equal chance (50%).



The basket contains 100 balls numbered 1 to 100. Each number appears in the basket exactly once. All balls are equally likely to be drawn.

Box C Description

Submit Choices

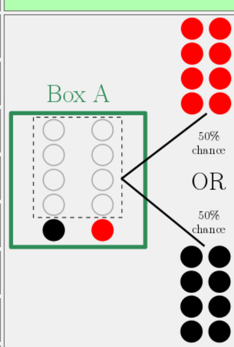
Remember that in part I (before Box C was introduced) you switched from betting on a RED ball being drawn from Box A to betting on the number drawn from Basket B at 48.

That is, for smaller numbers - you chose to bet on Box A, and for 48 and higher numbers - you chose to bet on the number drawn from Basket B.

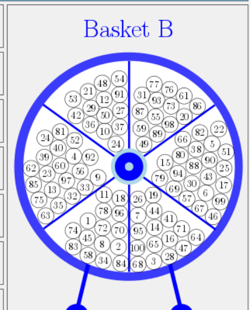
Continue

Please choose between A and B on every line IF A RED BALL IS DRAWN FROM BOX C

Choice No.	Option A Win $20 if a RED ball is drawn from Box A Win $0 if a BLACK ball is drawn from Box A	A	B	Option B Win $20 if the number on the ball drawn from Basket B is smaller than or equal to the number indicated in each line. Win $0 otherwise.
0	Bet on RED from Box A	☑	☐	0
1	Bet on RED from Box A	☑	☐	10
2	Bet on RED from Box A	☑	☐	20
3	Bet on RED from Box A	☑	☐	30
4	Bet on RED from Box A	☑	☐	40
5	Bet on RED from Box A	☑	☐	50
6	Bet on RED from Box A	☑	☐	60
7	Bet on RED from Box A	☐	☑	70
8	Bet on RED from Box A	☐	☑	80
9	Bet on RED from Box A	☐	☑	90
10	Bet on RED from Box A	☐	☑	100



The number of RED and BLACK balls in Box A is unknown. It contains either 9 BLACK and 1 RED balls OR 1 BLACK and 9 RED balls. Both compositions have equal chance (50%).



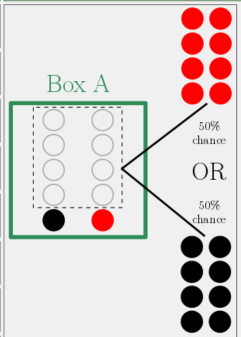
The basket contains 100 balls numbered 1 to 100. Each number appears in the basket exactly once. All balls are equally likely to be drawn.

Box C Description

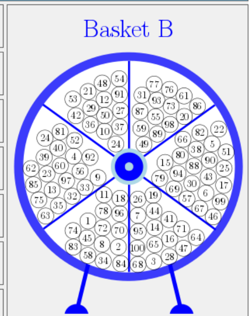
Submit Choices

Please choose between A and B on every line IF A RED BALL IS DRAWN FROM BOX C

Choice No.	Option A Win $20 if a RED ball is drawn from Box A Win $0 if a BLACK ball is drawn from Box A	A	B	Option B Win $20 if the number on the ball drawn from Basket B is smaller than or equal to the number indicated in each line. Win $0 otherwise.
0	Bet on RED from Box A	☑	☐	60
1	Bet on RED from Box A	☑	☐	61
2	Bet on RED from Box A	☑	☐	62
3	Bet on RED from Box A	☑	☐	63
4	Bet on RED from Box A	☑	☐	64
5	Bet on RED from Box A	☐	☑	65
6	Bet on RED from Box A	☐	☑	66
7	Bet on RED from Box A	☐	☑	67
8	Bet on RED from Box A	☐	☑	68
9	Bet on RED from Box A	☐	☑	69
10	Bet on RED from Box A	☐	☑	70



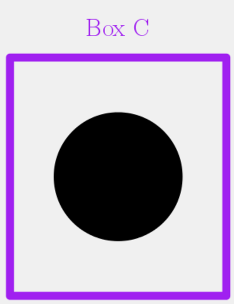
The number of RED and BLACK balls in Box A is unknown. It contains either 9 BLACK and 1 RED balls OR 1 BLACK and 9 RED balls. Both compositions have equal chance (50%).



The basket contains 100 balls numbered 1 to 100. Each number appears in the basket exactly once. All balls are equally likely to be drawn.

Box C Description

Submit Choices



Box C contains 10 balls identical to Box A (9 RED and 1 BLACK, or 1 RED and 9 BLACK) PLUS 5 additional RED balls AND 5 additional BLACK balls.

For the next sequence of choice problems, suppose that a BLACK ball is drawn from BOX C.

Continue

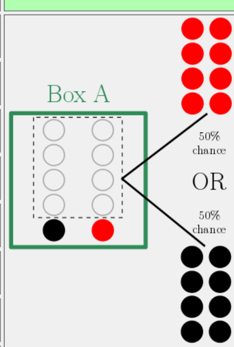
Remember that in part I (before Box C was introduced) you switched from betting on a RED ball being drawn from Box A to betting on the number drawn from Basket B at 48.

That is, for smaller numbers - you chose to bet on Box A, and for 48 and higher numbers - you chose to bet on the number drawn from Basket B.

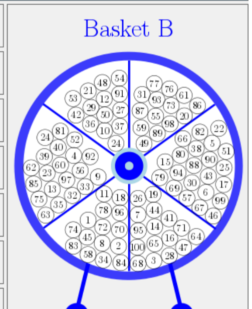
Continue

Please choose between A and B on every line IF A BLACK BALL IS DRAWN FROM BOX C

Choice No.	Option A Win \$20 if a RED ball is drawn from Box A Win \$0 if a BLACK ball is drawn from Box A	A	B	Option B Win \$20 if the number on the ball drawn from Basket B is smaller than or equal to the number indicated in each line. Win \$0 otherwise.
0	Bet on RED from Box A	☑	☐	0
1	Bet on RED from Box A	☑	☐	10
2	Bet on RED from Box A	☑	☐	20
3	Bet on RED from Box A	☑	☐	30
4	Bet on RED from Box A	☐	☑	40
5	Bet on RED from Box A	☐	☑	50
6	Bet on RED from Box A	☐	☑	60
7	Bet on RED from Box A	☐	☑	70
8	Bet on RED from Box A	☐	☑	80
9	Bet on RED from Box A	☐	☑	90
10	Bet on RED from Box A	☐	☑	100



The number of RED and BLACK balls in Box A is unknown. It contains either 9 BLACK and 1 RED balls OR 1 BLACK and 9 RED balls. Both compositions have equal chance (50%).



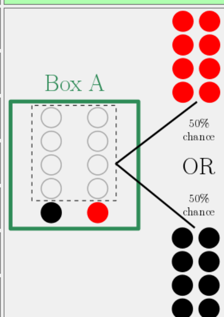
The basket contains 100 balls numbered 1 to 100. Each number appears in the basket exactly once. All balls are equally likely to be drawn.

Box C Description

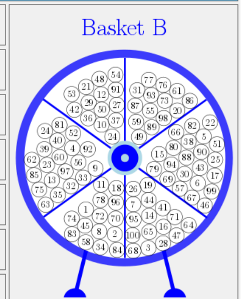
Submit Choices

Please choose between A and B on every line IF A BLACK BALL IS DRAWN FROM BOX C

Choice No.	Option A Win $20 if a RED ball is drawn from Box A Win $0 if a BLACK ball is drawn from Box A	A	B	Option B Win $20 if the number on the ball drawn from Basket B is smaller than or equal to the number indicated in each line. Win $0 otherwise.
0	Bet on RED from Box A	☑	☐	30
1	Bet on RED from Box A	☑	☐	31
2	Bet on RED from Box A	☑	☐	32
3	Bet on RED from Box A	☑	☐	33
4	Bet on RED from Box A	☐	☑	34
5	Bet on RED from Box A	☐	☑	35
6	Bet on RED from Box A	☐	☑	36
7	Bet on RED from Box A	☐	☑	37
8	Bet on RED from Box A	☐	☑	38
9	Bet on RED from Box A	☐	☑	39
10	Bet on RED from Box A	☐	☑	40



The number of RED and BLACK balls in Box A is unknown. It contains either 9 BLACK and 1 RED balls OR 1 BLACK and 9 RED balls. Both compositions have equal chance (50%).



The basket contains 100 balls numbered 1 to 100. Each number appears in the basket exactly once. All balls are equally likely to be drawn.

Box C Description

Submit Choices

End of the Experiment

The experiment is over. Take a look at your performance.

Part 2 was randomly selected for payment. The ball drawn from Box C is RED.

The table below displays the choice problem that was selected for payment.

Please choose between A and B IF A RED BALL IS DRAWN FROM BOX C

Choice No.	Option A Win $20 if a RED ball is drawn from Box A Win $0 if a BLACK ball is drawn from Box A	A	B	Option B Win $20 if the number on the ball drawn from Basket B is smaller than or equal to the number indicated in each line. Win $0 otherwise.
4	Bet on RED from Box A	☑	☐	40

The ball drawn from Box A is RED.

The breakdown of your payment is:

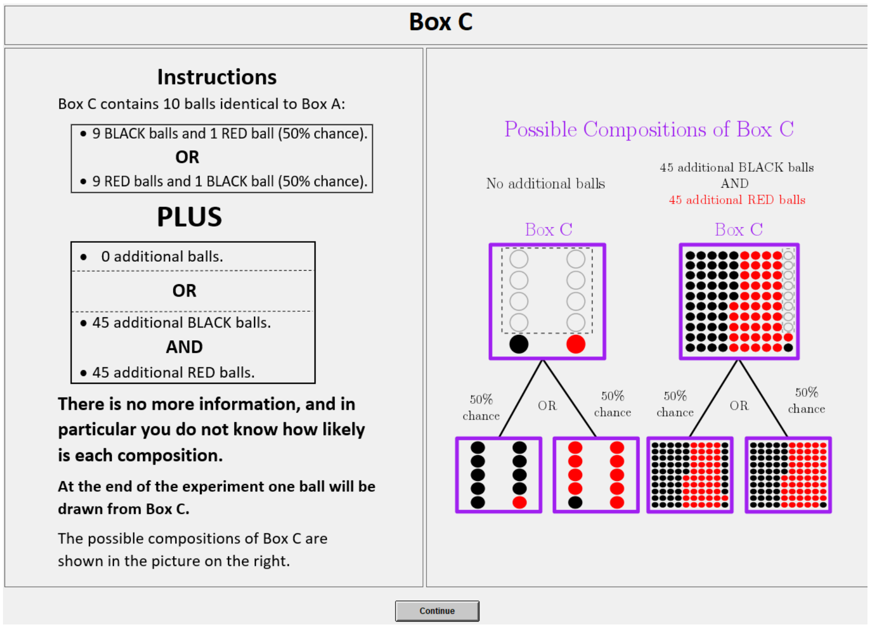
Show-up fee: $7.00
Quiz (12 X $0.25): $3.00
Payment from the Experiment: $20.00
Your Total Payment is: $30.00

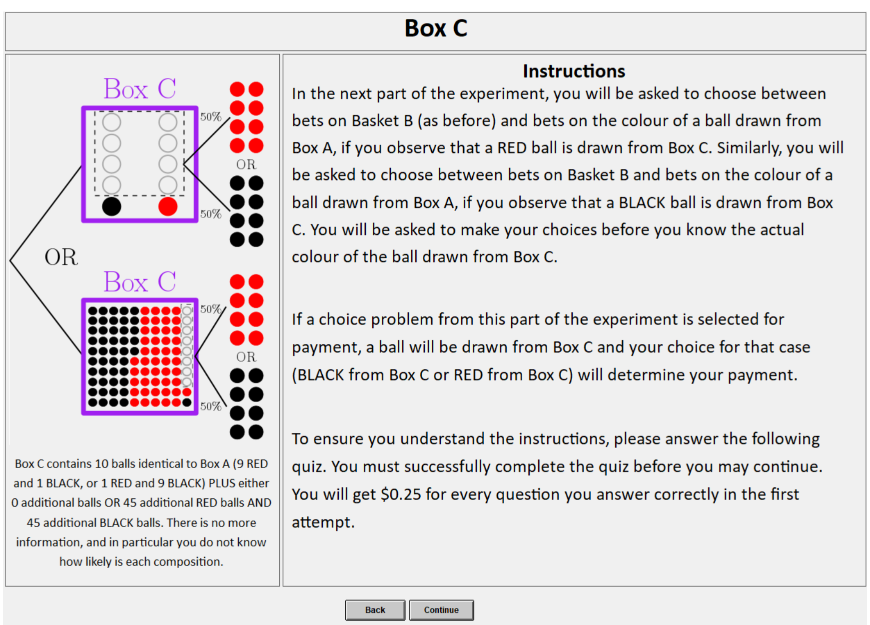
Please record your payment in your receipt.

OK

B.4.2 Ambiguous-signals treatment

Below are only the screens that are different in the ambiguous signal treatment





Quiz

1. If you bet on RED from Box A and a BLACK ball is drawn from Box C. What would be your payment?

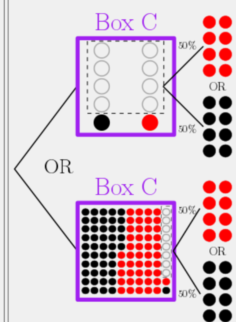
☐	$0
☐	$20
☑	It depends on the colour of the ball drawn from Box A.

Your answer is correct

2. If there are more BLACK balls than RED balls in Box A:

☐	There are more RED balls than BLACK balls is Box C.
☐	There is an equal number of RED and BLACK balls is Box C.
☑	There are more BLACK balls than RED balls is Box C.

Your answer is correct



Box C contains 10 balls identical to Box A (9 RED and 1 BLACK, or 1 RED and 9 BLACK) PLUS either 0 additional balls OR 45 additional RED balls AND 45 additional BLACK balls. There is no more information, and in particular you do not know how likely is each composition.

Back | Next Questions

Quiz

1. If you bet on RED from Box A and a BLACK ball is drawn from Box C. What would be your payment?

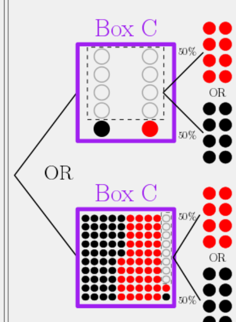
☐	$0
☐	$20
☑	It depends on the colour of the ball drawn from Box A.

Your answer is correct

2. If there are more BLACK balls than RED balls in Box A:

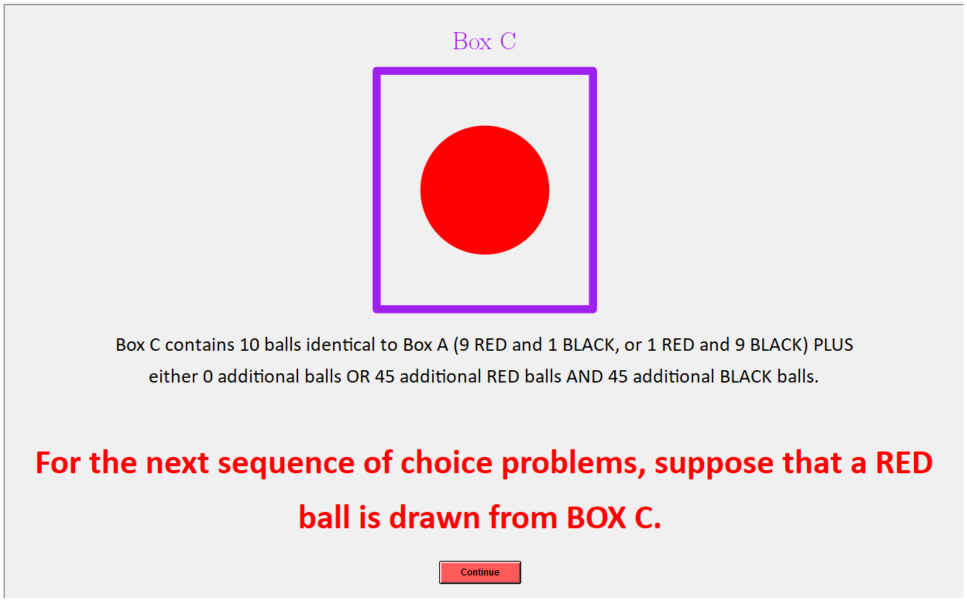
☐	There are more RED balls than BLACK balls is Box C.
☐	There is an equal number of RED and BLACK balls is Box C.
☑	There are more BLACK balls than RED balls is Box C.

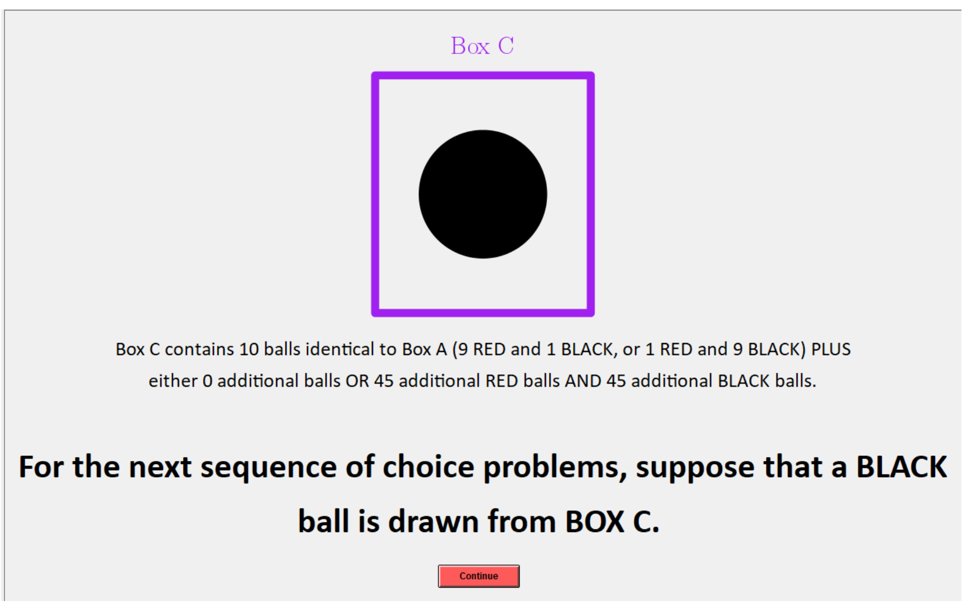
Your answer is correct



Box C contains 10 balls identical to Box A (9 RED and 1 BLACK, or 1 RED and 9 BLACK) PLUS either 0 additional balls OR 45 additional RED balls AND 45 additional BLACK balls. There is no more information, and in particular you do not know how likely is each composition.

Back | Next Questions


Box C
Box C contains 10 balls identical to Box A (9 RED and 1 BLACK, or 1 RED and 9 BLACK) PLUS
either 0 additional balls OR 45 additional RED balls AND 45 additional BLACK balls.
For the next sequence of choice problems, suppose that a RED ball is drawn from BOX C.
Continue


Box C
Box C contains 10 balls identical to Box A (9 RED and 1 BLACK, or 1 RED and 9 BLACK) PLUS
either 0 additional balls OR 45 additional RED balls AND 45 additional BLACK balls.
For the next sequence of choice problems, suppose that a BLACK ball is drawn from BOX C.
Continue