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# Shiftwork

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We analyze the organization of employment in nonsimultaneous shifts, considering the shift composition of manufacturing employment, both in the business cycle frequency and in the long run. With regard to the short run, we argue that shiftwork would be procyclical and that this, combined with the inherent lumpiness of shifts, may help resolve the puzzle of the procyclicality of labor productivity. With regard to the long run, we identify channels that may account for the increase in shiftwork in the past half-century and for the nonnegative cross-country correlation between shiftwork and the level of income.

Shiftwork, the operation of the same capital stock by different teams of workers on alternate hours, is a key facet of labor employment. The long-term decline in the workweek of individuals and the attendant increase in capital per worker have brought about an uncoupling of the working hours of individuals and businesses through a significant increase in the prevalence of shiftwork. As estimated by Foss (1984), the average workweek of manufacturing plants in the United States has increased by 24.7% between 1929 and 1976. Estimates in Mayshar and Solon (1993) reveal that the share of late-shift manufacturing production workers in U.S. metropolitan areas reached 30.1% in 1987–89, compared with only 23.7% in 1951–53. For 10 European Community (EC) countries in 1989, the

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average operating hours for industrial plants was 66, while the corresponding average weekly hours of full-time employees was 39, implying that the average EC firm operated  $66/39 = 1.69$  shifts per day (Commission of the European Communities 1991).

In spite of these magnitudes, the shift composition of employment has been oddly ignored by most labor economists.<sup>1</sup> It has been analyzed primarily by those concerned with the role of capital utilization in production or from the perspective of developing countries (see Betancourt and Clague 1981). This literature followed Marris (1964) in focusing almost exclusively on the phenomenon of long-run, planned capital utilization and in ignoring the role of short-run changes in shiftwork. Yet, as argued by Lucas (1970), the short-run responsiveness of shiftwork to fluctuations in demand may be highly significant for our understanding of business-cycle fluctuations in productivity.

In this article we present a unified treatment of shiftwork, both in the short run and in the long run. Our formulation of the firm's short-run behavior extends Lucas's argument by focusing on the critical role of the lumpiness of shifts. The ability to initiate or suspend shiftwork is a valuable option for firms. If much of the business-cycle variation in employment occurs along the discrete margin of introducing and curtailing lumpy shifts, then it becomes easier to account for the apparent flatness of marginal costs even in the short run and for the apparent procyclicality of labor productivity. Our formulation also seeks to improve the understanding of factors that can account for the long-run increase in the prevalence of shiftwork.

In accordance with this agenda, we formulate a model of a competitive firm whose short-run employment policy is conducted along three distinct dimensions: the total number of workers employed, the length of the workday of each employee, and the composition of employment across discrete nonsimultaneous shifts. In the long run, the firm also determines its capital stock and the level of normal employment for which that stock is designed.

Capital utilization can be altered either by operating equipment at varying levels of intensity (speed, heat) or by varying the time span in which structures and equipment are used. We confine ourselves here only to the second facet. Still, capital utilization is *not* an independent decision variable in our model; we suggest that the workday of capital is more realistically analyzed as but a by-product of the pattern of labor employment. Maintenance time aside, it is primarily for the purpose of conserving

<sup>1</sup> Thus, there is no reference to shiftwork in the *Handbook of Labor Economics* (other than the use of the shift premium as an illustration of a compensating differential, Ashenfelter and Layard [1986]).

on labor costs that capital is left idle. Alternatively put, if labor were homogeneous and indifferent to the daily timing of work there would be little reason for a competitive manufacturing firm that produces a storable output to leave its capital stock idle for part of the day.

Section I presents further evidence on key shiftwork practices in Western economies. Our model is presented in Section II. Demand behavior is then examined for the short run (Sec. III) and the long run (Sec. IV). All the formal proofs are relegated to the appendix.

### I. Shiftwork Practices in Western Economies

The observed patterns of shiftwork vary widely. We are concerned here only with the multiplicity of daily shifts rather than with their timing and possible rotation. With regard to daily timing, it is customary to distinguish between three shifts: the day shift, the evening shift (usually ending at about midnight), and the night shift.

Most of the available evidence on shiftwork is from cross-sectional surveys. In a unique recent survey (Commission of the European Communities 1991), over 25,000 industrial companies in 10 European Community countries were asked about their "average operating hours per week" (p. 55) and their "average contracted weekly working hours for a full-time employee" (p. 56). Altogether, 70% of the industrial firms in this survey reported engaging shifts, but only 37% of the workers employed by the firms that operated shifts actually worked late shifts. Key findings of the survey are presented in table 1, with countries arranged

**Table 1**  
**Shiftwork Patterns in the European Community, 1989**

	Average Weekly Hours		Shift Index		Percentage of Plants	
	Full-Time Employees	Plant Operation	All Companies	Companies with 1,000 + Employees	With Shifts	With Continuous Shifts
Portugal	44	54	1.23	...	19	4
Greece	40	64	1.60	2.66	52	...
Ireland	41	61	1.48	2.50	55	16
Spain	40	69	1.73	1.70	68	17
United Kingdom	37	76	2.05	2.24	73	14
Italy	39	73	1.87	2.15	83	9
Netherlands	39	74	1.90	2.46	63	11
Belgium	37	77	2.08	2.27	80	22
France	39	69	1.77	2.03	74	10
Germany	38	53	1.39	1.57	65	6
Europe	39	66	1.69	2.00	70	10

SOURCE.—Commission of the European Communities 1991.

NOTE.—Shift index = average plant hours divided by average employees' hours.

in ascending order of gross national product per capita. We observe that employee weekly hours in general decline with income while plant operating hours clearly do not. In fact, with the important exception of West Germany, there seems to be a distinct positive cross-country association between income and the prevalence of shiftwork.

The variation in plant operating hours, both across countries and across firms, is high. Much of this variation is explained by establishment size and by the industrial classification of firms: 94% of the EC plants with 1,000 or more employees reported operating shifts, in comparison with only 42% of the plants with fewer than 200 employees.<sup>2</sup> As table 1 shows, the shift index (the ratio of plant hours to employee hours) is significantly higher for plants with high employment, and the cross-country pattern noted above disappears after conditioning on plant size.

Shiftwork is more widespread in companies producing intermediate goods, and especially in continuous-process industries. Nevertheless, only 10% of the companies in the EC study reported continuous operation. Thus, most of EC companies could extend their operating hours but refrained from doing so, apparently for economic rather than technological considerations. In fact, when asked about their reasons for not expanding weekly operating hours, companies cited most often low-demand conditions on the one hand and adverse labor supply conditions (collective agreements and statutory provisions on working hours) on the other.

Figure 1 provides some information on the cyclicality of shiftwork. It is based on 1951–90 estimates of the percentage of U.S. manufacturing production workers in plants with more than 50 employees in major metropolitan areas who work on late shifts (Mayshar and Solon 1993) and Bureau of Labor Statistics data on employment of all manufacturing production workers (U.S. Department of Labor 1975, 1992). The figure shows that annual changes in the employment of late-shift production workers account for much of the annual changes in total manufacturing employment. Given the pronounced cyclical volatility of employment by manufacturing production workers, figure 1 demonstrates the significance of the procyclicality of employment by late-shift workers. As calculated by Mayshar and Solon (1993), about a half of manufacturing production workers who were laid off during cyclical downturns were late-shift workers, even though their average share in employment was only about a quarter.

On the basis of the evidence cited above and several other sources, the main stylized facts with regard to shiftwork are the following:

<sup>2</sup> It should be noted, however, that employment is not the right measure of size for this purpose since for a given plant the operation of an extra shift typically implies an increase in employment.

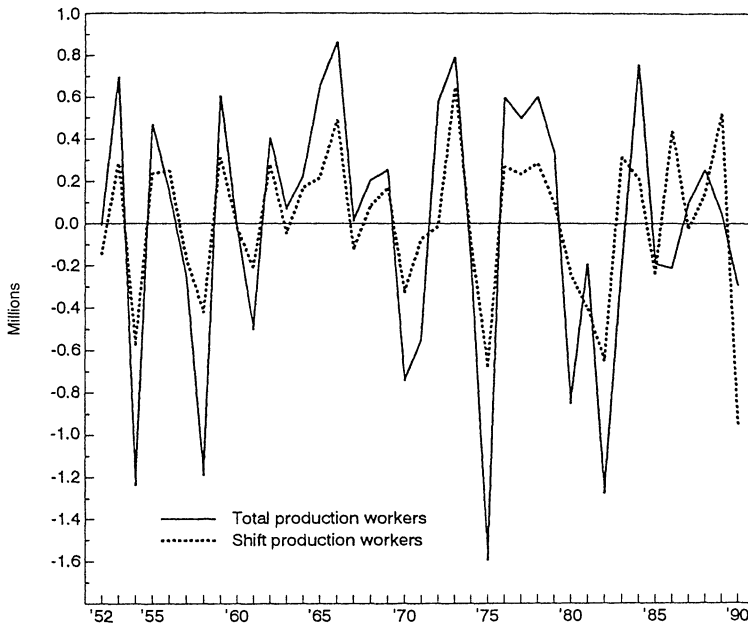


FIG. 1.—Annual changes in the employment of U.S. manufacturing production workers. Employment changes among shift workers account for much of the total change.

1. Shiftwork, not only in manufacturing but in other sectors as well, is widespread both in Europe and in the United States. There seems to be an upward trend in the prevalence of shiftwork.<sup>3</sup>
2. Shiftwork is conducted primarily by large firms (in terms of employment) and by capital intensive firms.<sup>4</sup>
3. Technologically mandated shiftwork in continuous-process industries accounts for a small fraction of shiftwork in manufacturing.<sup>5</sup>

<sup>3</sup> Based on surveys of individuals, the share of the total full-time wage and salary workers who worked on late shifts was 24% in the EC in 1989 (Commission of European Communities 1991) and 18% in the United States in 1991 (U.S. Department of Labor 1992). The share in manufacturing was only slightly higher in the United States, 19.5%, and still higher, 24.4%, among operators, fabricators, and laborers. Evidence on the upward trend of shiftwork in the United Kingdom and in Europe is reported by Bosworth and Dawkins (1981, tables 7.2, 7.12).

<sup>4</sup> Concerning employment size, see also Oi (1983) and Kostiuk (1990), and concerning capital intensity, see Bosworth and Dawkins (1981) and Betancourt and Clague (1981).

<sup>5</sup> Late shift employment among U.S. manufacturing production workers in continuous-process industries was estimated by Foss (1984, table 13) at only 11%. Shapiro (1995, table 1) reports that even among foundries and steel establishments, where minimizing temperature variation is an important technological

4. Among firms that engage shifts, the number of workers on the day shift is disproportionately larger than those on later shifts (see Shapiro 1995).
5. The shift premium in the United States seems to be surprisingly low; in Europe it is substantially higher. This may be partly related to the widespread regulation of shiftwork in Europe. There is some evidence that the shift premium has declined over time.<sup>6</sup>
6. Shiftwork is highly procyclical. In the United States the fraction of the variation in manufacturing production employment accounted for by shift workers is twice their share in employment and may be close to 50% (Mayshar and Solon 1993; and see also Shapiro 1993).
7. At least for the U.S. automobile and lumber industries, much of the cyclical variation in output and employment is related to the discrete initiation and abandonment of shifts.<sup>7</sup>

## II. The Model

There are two ingredients in our model: the technology and the employment-cost relation. In the long run, technology is assumed to be given by an instantaneous neoclassical production function,  $x = F^0(N, K)$ , where  $x$  is the flow rate of output (or linespeed),  $N$  is the number of simultaneous workers on the production line (a stock variable), and  $K$  is the capital stock. To allow for the possibility that the degree of

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consideration, 21% and 14% of plants, respectively, operate only one shift and few run more than two shifts. On the other hand, in several industries, such as textiles, companies often operate continuously though not necessarily for technological reasons (Foss 1984, table 21).

<sup>6</sup> Kostiuk (1990) and Shapiro (1995) report average shift premia for the second and third shifts in U.S. industry of about 5%. Kostiuk argues that this figure understates the actual premium because of a failure to correct for workers' heterogeneity. Shapiro argues that given the practices of rotation and long-term contracting, some of the premium for late-shift work may be reflected in a higher base wage. On the basis of the estimated premium for shiftworkers on rotation he estimates that the marginal premium for nightwork may in fact be 25%. A study by the U.K. National Board for Prices and Incomes (1970, sec. 181) reports that in 1969 the premium specified in collective labor agreements for nightwork ranged from 15% to 50%, with an average premium of 25% for permanent nightwork. Bosworth and Dawkins (1981, table 9.3) report a premium of 25% for permanent nightwork in the United Kingdom in 1977, with an upward time trend. Foss (1984, pp. 62–67) identifies a decline in the shift premium in the United States.

<sup>7</sup> Based on micro plant data for the lumber industry, Cardellicchio (1990) concludes that for each sawmill the key short-run decision variable is the yearly number of (8-hour) operative shifts. He demonstrates that the number of shifts varies significantly in response to changes in prices and costs. Aizcorbe (1990) and Bresnahan and Ramey (1994) demonstrate the significance of shift changes for automobile assembly plants.

substitutability between labor and capital may be lower *ex post*, the short run is characterized by an instantaneous production function,  $x = F(N; N_0, K_0)$ , where a zero subscript denotes variables determined *ex ante*. Short-run employment can differ from the level for which the plant was designed but at a cost:  $F(N; N_0, K_0) \leq F^0(N, K_0)$  for all  $N$ , with equality for  $N = N_0$ . For simplicity, the short-run production function is abbreviated to  $F(N)$ . Production is assumed to be additive over time, that is, if  $N(t)$  are employed at time  $t$ , the total output over a time interval is  $\int F(N(t))dt$ . Thus, if  $N(t) \equiv N$  in the  $H$  hours of a single shift, the shift output is  $HF(N)$ .

The short-run instantaneous production function  $F(N)$  is assumed to have the shape depicted in figure 2a, with an inverted U-shaped average product of labor whose maximum is attained at  $\underline{N} > 0$  (a function of  $K_0$  and  $N_0$ ). For employment levels beyond the threshold  $\underline{N}$  we assume decreasing marginal productivity and diminishing elasticity of product with respect to employment. Note that as formalized here, workers' productivity does not depend on whether they work by day or by night. This implicitly assumes that supervision, too, can be perfectly replicated; if not, we may have lower productivity for work on less-supervised shifts.

While the shape of our instantaneous short-run production function  $F(N)$  appears in any microeconomics textbook in the guise of a U-shaped average variable cost curve, it is quite different from the one adopted in the related literature. In particular, this shape integrates two essential features: it admits a positive short-run marginal labor productivity but also stipulates an initial range of increasing average labor productivity and a minimum efficient scale of operations. The latter is designed to capture the idea that production is conducted by teams of workers in lumpy shifts. In contrast, the models of shiftwork by Winston and McCoy (1974) and Betancourt and Clague (1981) have lumpiness but assume a putty-clay technology, with no short-run substitutability. At the other pole, Lucas (1970) and most other researchers who are interested in the short run typically assume a constant elasticity of substitution type of short-run production function, with monotonically declining average labor productivity and no lumpiness. The former assumption implies that only hours can vary in the short run, in contrast with evidence of cyclical variations in linespeed and employment in operating shifts (see Aizcorbe 1990) and with differences in employment across operating shifts (see Shapiro 1995). The latter assumption clearly ignores the well-documented phenomenon of the discrete initiation and abandonment of shifts and the key feature emphasized by Marris (1964, p. 8): "factory work is team work."

Another special property of the above technology is in the incorporation of time. The characterization of the production function as instantana-



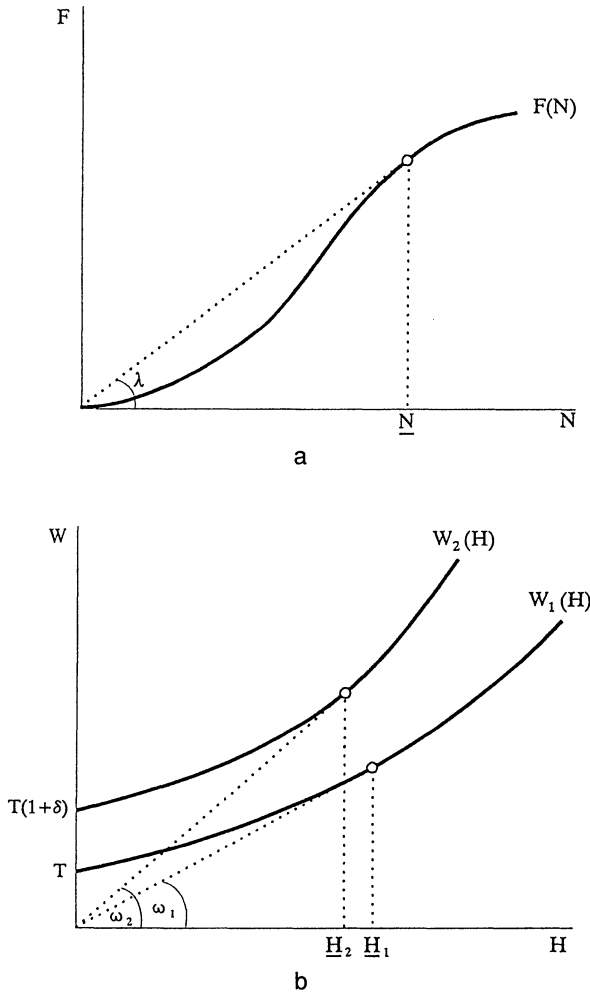


FIG. 2.—*a*, The instantaneous production function; *b*, the employment-cost functions

neous and time additive follows both Lucas (1970) and Winston and McCoy (1974). As noted by others, it is a natural idealization of manufacturing in an assembly line, where production is subdivided into multiple, simultaneously conducted tasks and where (unlike in continuous processes) operations conducted at any one instant can be independently replicated at any other. The ability to replicate production provides a powerful rationale for the existence of constant returns to scale for variations in workers' (and consequentially also in capital's) hours of employment. This time feature is also crucial for the central distinction that we

make here with regard to short-run changes in the shift composition of employment: the effect on output of an increase in employment should be rather different when employment is expanded *within* an operating shift or when a *new* shift is introduced. In the latter case, there is no presumption of diminished labor productivity since capital hours expand as well and there is no congestion effect leading to reduced labor productivity.

The second key ingredient of the model is the employment-cost relation. This relation reflects, in particular, workers' preferences concerning the timing and length of the workday. As depicted in figure 2*b*, we assume the existence of two employment-cost functions,  $W_1(H)$  and  $W_2(H)$ , representing the firm's total daily costs for employing one worker for  $H$  straight hours in each of the two shifts. The representative worker is taken to be indifferent between working any number of hours  $H$ , and working in any one of the two shifts, as long as the firm compensates him appropriately by incurring the corresponding employment costs  $W_i(H)$ . We assume that  $W_2(H) > W_1(H)$  for all  $H$ , reflecting the greater hardships of the second shift. We also assume that  $W_i(0) = 0$  but that the costs of employment are discontinuous at  $H = 0$ , involving a daily fixed cost,  $W_1(0+) = T > 0$  and  $W_2(0+) = (1 + \delta)T$ , with  $\delta > 0$ . The premium  $\delta T$  incorporates the amount required to compensate workers to overcome their initial dislike for working in the second shift. The two average hourly employment cost functions are assumed to be U-shaped, with minima attained at  $\underline{H}_1$  and  $\underline{H}_2$ , respectively, and with increasing marginal employment costs and increasing elasticity of employment costs with respect to hours beyond these threshold levels. No distinction is made here between regular hours of work and overtime. We also assume that there are no sunk fixed costs in hiring or training and that all workers are homogeneous.

Our formulation whereby workers in effect supply a reservation indifference curve from which the employer selects the hours-pay combination follows Leslie (1987) and Kinoshita (1987). This formulation of labor supply differs from the conventional one, where it is individuals who determine their hours of work. As others have noted, assigning the decision concerning job attributes (including hours of work) to employers seems to conform better with reality. The assumption of a U-shaped average hourly employment cost relation with a fixed cost component is also quite common. This is a natural way to explain why workers typically work uninterrupted daily hours. Workers clearly incur fixed (yet unsunk) daily costs of travel to and from work; in addition, when employment by one firm excludes employment by other firms, each employee incurs the fixed opportunity cost of alternative employment. The firm, too, may incur fixed yet unsunk daily carrying costs per employee that are independent of the daily hours worked. These costs include administrative

records keeping, use of supervisory capacity, and the provision of worker-specific equipment and space.<sup>8</sup>

The dual employment cost schedules,  $W_2(H) > W_1(H)$ , depict the compensating wage premium for shiftwork, reflecting the presumption that (unlike machines) “humans prefer to work in the day and sleep at night” (Marris 1964, p. 8).<sup>9</sup> In comparison, both Lucas (1970) and Winston and McCoy (1974) incorporate the effect of workers’ preferences for employment across hours of the day by introducing a rhythmic instantaneous wage rate function,  $w(t)$  with no fixed costs.<sup>10</sup> This formulation does not distinguish between work in discrete shifts and does not clarify whether one worker or more are employed in any given job during the day. Betancourt and Clague (1981), however, distinguish between discrete shifts but assume that the duration of employment in each shift is set exogenously.<sup>11</sup> Thus, like the formulations of Lucas and Winston and McCoy, their model, too, cannot address the issue of a firm’s trade-off between varying the number of workers employed or changing the time span of work per employee.

Given our formulation of the model, in the short run the competitive firm selects nonnegative  $N_1, N_2, H_1, H_2$  to maximize

$$p[H_1F(N_1) + H_2F(N_2)] - N_1W_1(H_1) - N_2W_2(H_2), \quad (1a)$$

where  $p$  is the price of output. This formulation ignores altogether the cost of maintenance and of depreciation due to use of the capital stock.<sup>12</sup>

<sup>8</sup> Note that we ignore fixed (and sunk) per-worker adjustment costs, related to initial hiring and training.

<sup>9</sup> The literature on the physiological and psychological ill-effects of work at night is very extensive (for references see Betancourt and Clague [1981, chap. 12.2]).

<sup>10</sup> Lucas (1970) allows employment  $N(t)$  to vary within the day. In his formulation the firm selects an employment function  $N(t)$  to maximize  $\int p[F^0(N(t), K_0) - w(t)N(t)]dt$ . Winston and McCoy (1974) derive from the instantaneous wage function  $w(t)$  an increasing convex cumulative wage relation  $W^*(H)$  for the daily wage bill needed to employ one worker over the  $H$  least expensive hours. Taking output  $X_0$  as given, they then assume that the firm selects  $H, N$ , and  $K$  to minimize the cost  $NW^*(H) + rK$ , given  $HF^0(N, K) = X_0$ , where  $F^0(N, K)$  exhibits constant returns to scale. Due to their putty-clay assumption, both  $N$  and  $K$  are only long-term decision variables.

<sup>11</sup> Betancourt and Clague (1981) postulate a wage cost  $W_i$  for employing a worker in shift  $i$  for the fixed  $H_0$  hours, with  $W_2 > W_1$ . They then compare the least-cost option of producing given output  $X_0$  in either one or two shifts. With one shift, the least cost input combination  $N, K$  minimizes  $W_1N + rK$ , given  $H_0F^0(N, K) = X_0$ ; with two shifts it minimizes  $(W_1 + W_2)N + rK$ , given  $2H_0F^0(N, K) = X_0$ .

<sup>12</sup> The existence of such costs, in the linear form  $(H_1 + H_2)vK_0$ , does not affect the qualitative conclusions below, though their introduction increases the threshold employment level. As noted by a referee, convexity of these costs may provide another reason for the firm to limit its daily hours of operation.

Note that whereas each laborer works either  $H_1$  or  $H_2$  hours per day, capital operates  $H_1 + H_2$  hours. We also ignore the 24 hours constraint on the maximum daily duration of operation.

### III. The Model in the Short Run

#### A. The Short-Run Optimum

As is evident from the formulation of the firm's profits in (1a) above, in the short run there is no interdependence between shifts. For  $i = 1, 2$ , each pair,  $N_i, H_i$ , can be considered as independently selected to maximize

$$\Pi_i(N_i, H_i) = pN_iF(N_i) - N_iW_i(H_i). \quad (1b)$$

Given this short-run independence of shifts, we omit in this section the subscripts identifying shifts and analyze the selection of the optimal employment  $N$  and shift duration  $H$  to maximize the shift operation profit,  $\Pi(N, H)$ . Particular attention will be paid to the corner solution of no shiftwork by checking that at the local optimum for  $H$  and  $N$  the shift operation profit is nonnegative.

The two first-order conditions for optimization with respect to  $H$  and  $N$  are

$$pF(N)/N = W'(H) \quad (2)$$

and

$$pF'(N) = W(H)/H. \quad (3)$$

Condition (2) states that if the firm considers asking all workers to work longer hours, the value of the additional product,  $pF(N)$ , should be matched by the extra cost  $NW'(H)$ . Condition (3) requires that the value of the product of a marginal worker,  $pHF'(N)$ , should equal that worker's wage,  $W(H)$ .

Conditions (2) and (3) can be combined to imply

$$\eta_F(N)\eta_W(H) = 1, \quad (4)$$

where  $\eta_F(N) \equiv NF'(N)/F(N)$  denotes the elasticity of the production rate with respect to employment, and  $\eta_W(H) \equiv HW'(H)/W(H)$  denotes the elasticity of the employment cost with respect to hours of work. Condition (4), which is also the first-order condition for the problem of minimizing the shift cost of producing a given quantity, presents the

expansion path along which employment and hours may vary in the short run.

Given (2) and (3), the condition for nonnegative shift operation profit can be stated in either one of two forms:

$$W'(H) \geq W(H)/H \quad \text{or} \quad F(N)/N \geq F'(N). \quad (5)$$

The first condition implies that hours have to exceed the threshold  $\underline{H}$ , where the elasticity  $\eta_w(\underline{H})$  is greater than one and increasing. The second condition implies that employment has to exceed the threshold  $\underline{N}$ , where the short-run elasticity  $\eta_F(N)$  is smaller than one and decreasing.

The local (sufficient) second-order conditions for the optimization of (1b) require

$$W''(H) > 0, F''(N) < 0 \quad (6a)$$

and

$$pHNW''(H)F''(N) + [pF'(N) - W'(H)]^2 < 0. \quad (6b)$$

Given the absence of decreasing marginal productivity to hours of work, the extent of hours worked must be checked by rising marginal costs,  $W''(H) > 0$ . Conversely, since the firm can obtain any number of workers for the same wage, the number of workers has to be checked by the standard requirement of diminishing marginal product of labor  $F''(N) < 0$ . Our assumptions that  $\eta_w$  is increasing in  $H$  and  $\eta_F$  is decreasing in  $N$  are shown in the appendix, Section I to guarantee condition (6b). Since the second-order conditions are satisfied, the equilibrium  $N$  and  $H$  are jointly determined by (2), (3), and (5).

### B. Comparative Statics in the Short Run

By condition (5), the firm will operate only in the ranges of employment and hours beyond the threshold levels  $\underline{N}$  and  $\underline{H}$ . We thus obtain (see appendix, Sec. II) the following.

PROPOSITION 1.—Work in a shift is conducted if and only if the maximal instantaneous average product per worker,  $\lambda = F(\underline{N})/\underline{N}$ , exceeds workers' minimal average (hourly) real employment cost,  $\omega/p = W(\underline{H})/p\underline{H}$ . As a result, both an increase in product price  $p$  and a decrease in the employment cost schedule  $W(H)$  tend to encourage initiating shiftwork.

Since  $W_2(H) > W_1(H)$ , proposition 1 provides a simple explanation of why a firm might operate the first shift and not the second. The higher minimum real employment costs of the second shift,  $\omega_2/p$ , may exceed

the maximum labor productivity,  $\lambda$ , rendering the initiation of that shift unprofitable. Proposition 1 in fact identifies three price ranges for the short run, identified by two threshold prices,

$$\begin{aligned} p_1^* &= [W_1(\underline{H}_1)/\underline{H}_1]/[F(\underline{N})/\underline{N}] = \omega_1/\lambda & \text{and} \\ p_2^* &= [W_2(\underline{H}_2)/\underline{H}_2]/[F(\underline{N})/\underline{N}] = \omega_2/\lambda. \end{aligned} \quad (7)$$

When the price exceeds  $p_2^*$  both shifts will operate, when the price falls short of  $p_1^*$ , no production will take place, and in between, only the first shift will operate. If productivity on the late shift were lower than in the day shift, we will have to distinguish between the maximum labor productivity in each of the two shifts:  $\lambda_1$  and the lower  $\lambda_2$ . Such productivity differentials will increase the *effective* shift premium, as represented by the gap between the threshold prices  $p_2^* = \omega_2/\lambda_2$  and  $p_1^* = \omega_1/\lambda_1$ .

Corresponding to the above results on the initiation of shiftwork, the model implies simple comparative static results concerning variations in employment in an operating shift (see appendix, Sec. III).

PROPOSITION 2.—In the short run (i) as a result of a higher product price, both the number of workers and the working hours per worker in an operating shift will increase and (ii) as a result of a higher fixed costs of employment, ( $T$ ), the number of workers in an operating shift will decrease but their hours of work will increase.

When the price changes, employment and hours in an operating shift will in fact vary along the expansion path given by condition (4). Our assumptions guarantee that the slope of that path in  $H, N$  be positive. If the difference between the cost relations of the first and second shifts is primarily in the intercept, and not in the curvature, result (ii) in proposition 2 could explain why firms tend to employ fewer workers in a second shift. If the employment-cost relation for the late shift is not only higher than for the first but steeper as well then the prediction of longer hours on the late shift is no longer valid.<sup>13</sup>

### C. Short-Run Implications

The literature on shiftwork has, on the one hand, not concerned itself with the short run. On the other hand, the literature concerned with short-run variations in employment has, by and large, ignored the issue of employing *alternative* teams of workers in discrete, multiple shifts.<sup>14</sup>

<sup>13</sup> The 1970 study by the U.K. National Board for Prices and Incomes (1970, sec. 176) reports that shifts often vary in duration and that the average length of work on permanent night shifts is 10 hours rather than 8 hours.

<sup>14</sup> Hansen and Sargent (1988) present a dynamic model of the operation of overtime by only a fraction of the workers who work regular hours.

Our model fills this gap and suggests a rich pattern of short-run variations in firms' employment along three dimensions. According to propositions 1 and 2, a cyclical increase in demand (price) provides an incentive for the firm to hire additional workers in an already operating shift (which does not increase the average employment costs  $W(H)/H$  but suffers from diminishing marginal productivity) and to increase hours of work (with the reverse consequences). In addition, when the price reaches the threshold levels  $p_1^*$  and  $p_2^*$ , the firm responds by initiating inoperative shifts, which results in a discrete increase in employment without incurring diminishing labor productivity.

The foregoing arguments are illustrated by figure 3, which depicts the average and marginal variable costs of producing a daily output  $X$ . This figure shows that the firm's ability to engage a second shift is tantamount to an option to engage a second, more costly plant. When the price reaches (from below) the bottom threshold  $p_1^*$ , the firm engages  $\underline{N}$  workers for  $\underline{H}_1$  hours, producing output  $\underline{X}_1$  in a single shift. When the price reaches  $p_2^*$ , the firm engages the second shift, discretely increasing output from  $\hat{X}$  to  $\hat{X} + \underline{X}_2$ , while employment increases from  $\hat{N}$  to  $\hat{N} + \underline{N}$ .

The possibility of initiating or curtailing shiftwork identifies a major source of short-run flexibility for the firm at the business-cycle frequency. This option serves as a buffer, enabling the firm to expand

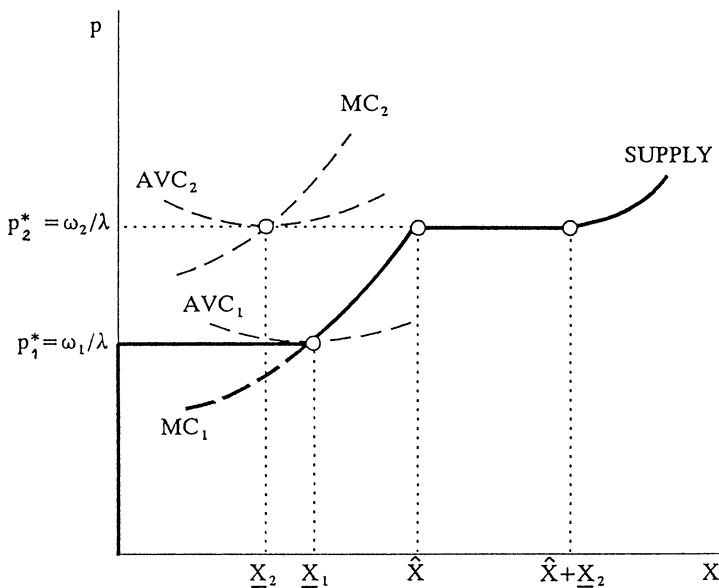


FIG. 3.—Short-run supply with two shifts. The late shift is like a second, more costly, plant.

output discretely in response to a surge in demand by increasing employment and capital utilization, without having to augment its capital stock or incurring a drop in labor productivity. In fact, since the second shift is operated with fewer workers ( $N_2 < \hat{N}$ ), average labor productivity should *increase* when the second shift is initiated.<sup>15</sup> Thus, at the threshold price  $p_2^*$  (and in fact also at  $p_1^*$ ) the marginal cost curve may be considered to be flat, and the firm may *seem* to operate in a local nonconvex range of increasing short-run labor productivity.<sup>16</sup>

Our argument that changes in the shift composition of employment may explain the procyclicality of measured labor productivity at the plant level differs from other explanations of this puzzle in the literature. The argument does not rely on fixed adjustment costs of employment or on noncompetitive behavior. Unlike Lucas's (1970) formulation of a similar argument, our model relies crucially on two *infra-marginal* sources of lumpiness (nonconvexity), in employment costs and in production, as depicted in figure 2. Our explanation also does not rely on an exogenous procyclical change in an omitted factor of production. While the endogenous change in capital utilization plays an integral role in this explanation, it should be reemphasized that in our formulation there is no direct cost to increased utilization of capital; it is determined simply as a by-product of the shift composition of employment. While the required time series microevidence on the shift composition of employment is still scarce, our explanation has the advantage of being directly testable once such data become available.

The threshold prices at which different firms in an industry will initiate shiftwork are likely to be firm specific. Moreover, as discussed below, firms that operate multiple shifts are also likely to be those in which *average* labor productivity is relatively high to begin with. As the cited

<sup>15</sup> As suggested in Mayshar and Solon (1993), this conclusion will be reinforced if the initiation of the second shift does not entail a proportional increase in the number of overhead production workers who provide maintenance and other ancillary services.

<sup>16</sup> That is, suppose that one were to try and depict daily product,  $X = H_1F(N_1) + H_2F(N_2)$ , as a function of daily labor,  $L = H_1N_1 + H_2N_2$ , ignoring the compositional distinction between day and night workers. At the threshold  $p_2^*$ , when the labor input increases from  $\hat{L} = H\hat{N}$  to  $\hat{L} + L_2 = H\hat{N} + H_2N_2$ , the average product  $X/L$  would increase from  $\hat{\lambda} = \hat{X}/\hat{L} = F(\hat{N})/\hat{N}$  to  $(\hat{X} + X_2)/(\hat{L} + L_2)$ , which is higher since it is a weighted average of  $\hat{\lambda}$  and the *maximal* average productivity  $\lambda$ . The reference to nonconvexity is deceptive, however, since employment in this range is inherently nonhomogeneous in that late-shift workers command a compensating wage premium. Thus, the cost function is *not* concave at this range. A price-taking firm would not be able to gain anything from attempting to "convexify" its production function by alternating between running one and two shifts (as was suggested to us by Robert Hall).



empirical evidence indicates, these are also likely to be larger firms. Thus, if a significant fraction of the business-cycle variation in manufacturing employment is in fact related to temporary layoffs and worker recalls due to the discrete curtailment or resumption of shifts, we may be able to explain at least some of the puzzling aggregate procyclicality of productivity both by a firm-specific effect and by a composition effect. The above-cited evidence in Mayshar and Solon (1993) and in Shapiro (1993) provides some support for this argument. It demonstrates the high volatility and procyclicality of aggregate variations in late-shift employment and in capital utilization in manufacturing. The evidence by Aizcorbe (1990), Cardellicchio (1990), and Bresnahan and Ramey (1994) suggests that at least for some industries much of this cyclical variation is indeed along the discrete margin of initiating and curtailing shifts.

#### IV. The Model in the Long Run

##### A. The Long-Run Optimum and Comparative Statics

In the long run, the firm selects the scale and design of its capital stock (represented here by  $K_0$  and  $N_0$ ). It is clearly that type of choice that accounts for the cross-sectional and secular patterns of shiftwork that were presented in Section II above. More specifically, we are interested here in how shiftwork responds, through such long-term factors, to changes in the cost of capital and in the cost of employment. We are also interested in how the possibility of shiftwork can affect the firm's long-term decisions.

To simplify things, we perceive the firm's long-run selection of its capital stock and capital design as a once and for all decision. In particular, we will assume that the capital stock does not have any alternative use and does not depreciate after its installation. We further simplify by assuming that only the output price varies over time, and that it follows a stationary, time-independent stochastic process,  $\tilde{p}$ .

Conditional on its expectations of future prices and its given capital and employment costs, the firm maximizes its expected discounted profits,

$$E \int e^{-rt} \{ \Pi^*[K_0, N_0, p_t, W_1(\cdot)] + \Pi^*[K_0, N_0, p_t, W_2(\cdot)] \} dt - P_k K_0,$$

where  $\Pi^*[K_0, N_0, p, W_i(\cdot)] = \text{Max}_{N,H} [pHF(N, N_0, K_0) - NW_i(H)]$  denotes the short-run maximum shift operation profit.<sup>17</sup> Given the as-

<sup>17</sup> Note that  $E\Pi^*[K_0, N_0, p, W_i(\cdot)] = \int_{\tilde{p}_i}^{\infty} \Pi^*[K_0, N_0, p, W_i(\cdot)] f(p) dp$ , where  $f(p)$  is the density function associated with  $\tilde{p}$  and where shift  $i$ 's price threshold  $\tilde{p}_i^* = \omega_i/\lambda_i$ , depends through  $\lambda_i$  on  $(K_0, N_0)$ . Note further that by applying the envelope argument to the short-run optimization it follows that for an operating shift,  $\Pi_K^*[K_0, N_0, p, W_i(\cdot)] \equiv pH_i F_3(N_i; N_0, K_0)$  and  $\Pi_N^*[K_0, N_0, p, W_i(\cdot)] \equiv pH_i F_2(N_i; N_0, K_0)$ , where  $H_i$  and  $N_i$  are at the ex post short-run optimum levels.

sumed time independence of the price process, this is equivalent to maximizing

$$E\{\Pi^*[K_0, N_0, \tilde{p}, W_1(\cdot)] + \Pi^*[K_0, N_0, \tilde{p}, W_2(\cdot)]\} - C(K_0, \rho).$$

Here the capital rental costs,  $rP_k K_0$ , is expressed more generally as  $C(K_0, \rho)$ , where we identify the rental rate of capital  $rP_k$  by the parameter  $\rho$  and assume that  $C_\rho > 0$ ,  $C_K > 0$ ,  $C_{K\rho} > 0$ . This generalization is needed in order to facilitate a rising capital cost schedule, where  $C_{KK}$  is strictly positive, which is needed in some cases to guarantee the second-order conditions.

The first-order conditions to determine the optimal capital stock  $K_0$  and the design  $N_0$  are simply

$$E\{\Sigma_i \Pi_k^*[K_0, N_0, \tilde{p}, W_i(\cdot)]\} = C_K(K_0, \rho) \quad (8a)$$

and

$$E\{\Sigma_i \Pi_N^*[K_0, N_0, \tilde{p}, W_i(\cdot)]\} = 0. \quad (8b)$$

These conditions determine  $K_0$  and  $N_0$  as functions of the long-term rental cost of capital,  $\rho$ , the shift employment costs,  $W_1(H)$  and  $W_2(H)$ , and the technology. If the second-order conditions are met, it immediately follows that a decrease in the capital costs  $\rho$  will lead to an increase in the capital stock  $K_0$ .

In order to examine the indirect effects of such changes on shiftwork, however, it is necessary to impose more structure on the technological relation between the ex ante and ex post production functions:  $F^0(N_0, K_0)$  and  $F(N; N_0, K_0)$ . We will avoid doing so by considering only two polar cases that are spanned by our formulation and that are common in the literature.

At one pole is the case in which substitutability is undiminished in the short run (putty-putty). In this formulation  $F(N; N_0, K_0) \equiv F^0(N, K_0)$  for all  $N$ , and as a result,  $N_0$  is not a long-run decision variable and condition (8b) drops. At the other pole is the case of no short-run substitutability (putty-clay) assumed by Winston and McCoy (1974) and by Betancourt and Clague (1981), where  $F(N; N_0, K_0) = F^0(N_0, K_0)$  for  $N \geq N_0$  and  $F(N; N_0, K_0) \leq NF^0(N_0, K_0)/N_0$  for  $N \leq N_0$ . Here, employment in an operating shift is in effect only a long-term decision. In both these polar formulations, the essential production function is  $F^0(N, K)$ , and the two cases can be distinguished according to whether employment is determined before or after the stochastic price is realized.

In both formulations, hours of work and the number of operating shifts

are determined in the short run as described above. That is, given the ex ante choice of  $K_0$  and  $N_0$ , the firm will operate either two shifts, one shift, or none at all, depending on the relation of the realized price  $p$  to the two threshold prices,  $p_1^*$  and  $p_2^*$ . These trigger prices are functions of the short-run average labor productivity at threshold employment, which in turn depends on the choice of  $N_0$  and  $K_0$ :  $\lambda(N_0, K_0) = \text{Max}_N[F(N; N_0, K_0)/N]$ .

The link between the long-term decisions  $N_0, K_0$  and the short-term operation of shiftwork is thus through the maximal average labor productivity  $\lambda$ . In the case of undiminished substitutability, where the only long-run decision variable is the capital stock  $K_0$ , the focus is on the dependence of  $\lambda = \text{Max}_N[F^0(N, K_0)/N]$  on  $K_0$ . In the case of no short-run substitutability, by equations (2) and (5) it becomes evident that not only the initiation of shiftwork but also hours of work in any operating shift are an increasing function of the product  $p\lambda(N_0, K_0)$ . It is thus convenient to view the average labor productivity,  $\lambda = F^0(N_0, K_0)/N_0$ , as the firm's long-term decision variable instead of  $K_0$ . In this amended formulation the firm selects  $\lambda$  and  $N_0$ , and this choice dictates the capital stock  $K_0 = g(\lambda, N_0)$ , as implied by inverting the above relation.

We obtain (see appendix, Sec. IV) the following.

PROPOSITION 3.—(a) In both polar formulations, (i) the key long-run technological determinant of shiftwork is the maximal average labor productivity and (ii) under constant returns to scale, the incentives to initiate shiftwork or to extend hours in an operating shift are unaffected by a change in capital costs;<sup>18</sup> (b) with undiminished substitutability, a decrease in capital costs enhances the incentive to initiate shiftwork and to expand hours, provided that in the relevant range there is increasing returns to scale;<sup>19</sup> and (c) with no short-run substitutability and a constant elasticity of substitution (CES) long-run production function, the incentive to initiate shiftwork and to expand hours are enhanced by a decrease in capital costs, provided that  $(\sigma - 1)(v - 1)$  is negative, where  $v$  and  $\sigma$  are the scale and substitution elasticities.

We will not formally pursue the effects of changes in labor costs. One observation though deserves to be made concerning the effect of a lower

<sup>18</sup> This conclusion conflicts with Winston and McCoy's (1974) main result, that under constant returns to scale a reduction in capital cost will increase hours of work if the elasticity of substitution is larger than one.

<sup>19</sup> Note that in the case of increasing returns (and price taking in the product market) we rely on the rising cost of capital to determine long-term scale. Betancourt and Clague's (1981) second proposition states that economies of scale are detrimental to capital utilization since it is then cheaper to produce any given output in one large shift rather than in two. This consideration is inapplicable here since the expansion of the capital stock is checked by increasing capital costs.

effective shift premium. Lower wages (or higher productivity) on the late shift clearly increase the shift operation profits and are likely to enhance the profitability of marginal capital for those price realizations when the second shift is operating. In addition, a lower effective shift premium lowers the trigger price  $p_2^*$  for initiating shiftwork. For both of these reasons, a lower shift premium is likely to encourage the expansion of the capital stock. Under the conditions specified in proposition 3 above, this induced expansion of the capital stock may then further promote the extent of shiftwork.

### B. Long-Run Implications

The main long-run phenomena concerning shiftwork that we want to explain are the secular increase in shiftwork and the puzzling observation that capital is not more fully utilized through shiftwork in countries where labor is relatively more abundant. Our attempt to address these issues reveals that the key technological determinant of shiftwork is the threshold, maximal *average* labor productivity.

Our focus on the role of average labor productivity has important implications. Proposition 3 suggests that if there is a link between reduced capital costs and increased shiftwork, via increased labor productivity, it is likely to rely on the existence of either economies of scale or, otherwise, on a high ex ante elasticity of substitution. The case of undiminished ex post substitutability directs attention to the role of economies of scale. According to this explanation, lower relative capital costs encourage firms to expand their capital stock. Economies of scale guarantee that such increased capital intensity of plants increases workers' productivity and thus diminishes the profitability of idling late shifts. The alternative polar case of no ex post substitutability directs attention (when there are decreasing returns to scale) to the role of a high elasticity of substitution. Given such a high ex ante elasticity, a decrease in capital costs raises the capital stock sufficiently to increase labor's average productivity and to increase the incentive for shiftwork.

These two alternative explanations are not mutually exclusive. In a more general framework, with diminished yet positive ex post substitutability, both explanations for a link between capital costs and shiftwork may be valid. Nevertheless, given the inconclusiveness of the empirical evidence on the magnitudes of the elasticities of scale and of substitution, we interpret proposition 3 as implying only a weak link between capital costs and shiftwork.<sup>20</sup> Indeed, also in the related literature there is dis-

<sup>20</sup> One may recall that even under conventional neoclassical assumptions, a lower relative capital cost does not necessarily increase average labor productivity. Thus with a constant return Cobb-Douglas production function, average labor productivity in equilibrium is proportional to the real wage rate and is *independent* of capital costs.

agreement concerning that link. According to conventional wisdom, cheaper capital cost is likely to result in diminished incentives to economize on capital and in lower utilization. Oi (1981), however, has informally claimed that the opposite may be true, and that a decline in the relative cost of capital may have been a principal reason for the increase in capital utilization in the past half-century.

What else can then account for the long-term stylized facts on shiftwork? The above argument that higher capital costs need not promote shiftwork suggests also that higher real wages need not be detrimental to shiftwork. In the long run, increased labor costs may result in more capital per worker and thus in a higher average labor productivity that may at least partially compensate for the wage increase. Also, if the income effect of increased real wages is dominant, this may be reflected in a reduction of workers' hours of employment. This is likely to encourage firms to expand the number of operating shifts as a substitute for the reduced hours per shift.

The most important effect of higher labor costs on shiftwork, however, may be indirect, through the impetus for labor-saving technological and organizational changes. As emphasized by Oi (1983), particularly relevant in this context (and related to *ex ante* substitutability) may be the adoption of assembly line operation that enables firms to switch, through a more capital-intensive and less flexible mode of operation, from low-scale customized production to market-oriented, large-scale production. That change has increased average labor productivity and, via standardizing workers' tasks, may have also resolved to a major extent the constraints on firms' limited supervisory capacity on late shifts. This organizational change can thus be interpreted as having contributed to lowering both threshold prices  $p_2^*$  and also to narrowing the effective shift premium  $p_2^* - p_1^*$ . In accordance with proposition 1, this should indeed encourage shiftwork. As discussed above, a lowering of the effective late-shift premium is also likely to have a positive feedback of increasing the profitability of the capital stock, reinforcing the incentives to expand the capital stock and thus possibly further enhancing workers' average productivity and promoting a more balanced shift composition of employment.

## V. Conclusion

Our aim was to explore the factors that account for shiftwork in the short run and in the long run. In particular, our model is concerned with three empirical phenomena: the short-run procyclicality of shiftwork and of labor productivity, the long-run increase in shiftwork in the past half-century, and the absence of more intense capital utilization in countries where capital is relatively scarce.

With regard to the short run, our model provides a unified simple framework for analyzing three alternative dimensions of the firm's de-

mand for labor. We consider not only variations in the number of workers and in hours of work by each worker but also variations in the shift composition of employment. Our analysis suggests that shiftwork will be procyclical, in conformity with the evidence. We have argued that such procyclical changes in the shift composition of employment, both within and across firms, may provide an important clue in resolving the puzzling observation of the procyclicality of average labor productivity. This argument is based on the inherent lumpiness involved when the number of operating shifts is changed, on the likelihood that average workers' productivity may in fact increase along that discrete margin and on the fact that shiftwork is mostly conducted by large firms with relatively high average productivity.

With regard to the long run and to cross-sectional evidence, we find that the two key factors accounting for shiftwork are a high average labor productivity and a low effective premium for late-shift employment. We argued that changes in capital costs may have an ambiguous effect on shiftwork. On the other hand, we noted that higher relative labor costs need not be detrimental to shiftwork. The effects of labor-saving technological changes, especially the introduction of standardized assembly line operations, may in fact provide the key link between high real wages, large-scale, capital-intensive modes of production and more prevalent shiftwork.

## Appendix

### Conditions and Proofs

#### I. The Second-Order Condition (6b)

By use of (2) and (3), one obtains  $(pF' - W')^2 = p(W' - W/H)(F/N - F') = pW'F'(\eta_w - 1)(1 - \eta_f) \geq 0$ . Dividing (6b) by this expression (in the range  $H > \underline{H}$  and  $N > \underline{N}$ ), (6b) becomes

$$[(NF''/F')/(1 - \eta_f)][(HW''/W')/(\eta_w - 1)] + 1 < 0. \quad (A1)$$

By differentiating the elasticities we get

$$\eta'_f \equiv \partial \eta_f / \partial N = (\eta_f/N)(1 - \eta_f + NF''/F') \quad (A2)$$

and

$$\eta'_w \equiv d\eta_w/dH = (\eta_w/H)(1 - \eta_w + HW''/W'). \quad (A3)$$

Substituting from (A2) and (A3) into (A1), the latter is equivalent to

$$[(-N\eta'_f/\eta_f)/(1 - \eta_f) + 1][(H\eta'_w/\eta_w)/(\eta_w - 1) + 1] > 1. \quad (A4)$$

Given that  $\eta_w > 1 > \eta_f$ , our assumption that  $\eta'_w > 0 > \eta'_f$  indeed guarantees (A4).

## II. Proof of Proposition 1

The threshold levels  $H$  and  $N$  are defined by the conditions,  $H\overline{W}'(H) - \overline{W}(H) = 0$  and  $N\overline{F}'(N) - \overline{F}(N) = 0$ . If  $\lambda = F(N)/N \geq \overline{W}(H)/pH = \omega/p$ , then  $\text{Max}_{N,H}\{\overline{\Pi}(H, N)\} \geq \overline{\Pi}(H, N) = p\overline{HN}(\lambda - \omega/p) \geq 0$ . Conversely, if  $\lambda < \omega/p$ , then for any  $H, N$ ,  $\overline{\Pi}(H, N) \equiv p\overline{HN}[F(N)/N - \overline{W}(H)/H] \leq p\overline{HN} \cdot \text{Max}_{N,H}\{F(N)/N - \overline{W}(H)/H\} = p\overline{HN}(\lambda - \omega/p) < 0$ . Incentives to initiate shiftwork in the short run are thus increased if either  $\lambda$  is increased or  $\omega/p$  is decreased. Clearly, if for all  $H$ ,  $W^0(H) \geq \overline{W}(H)$ , and  $W^0(H)/pH$  is minimized at  $H^0$ , then  $W^0(H^0)/pH^0 \geq \overline{W}(H^0)/pH^0 \geq \overline{W}(H)/pH$ . An increase in price, leaving  $W(H)$  unchanged, also simply diminishes  $\omega/p$ .

## III. Proof of Proposition 2

Forming the total differentials of the first-order conditions (2) and (3) [and making substitutions from (2) and (3)] yields

$$-NW''dH - [p(F/N - F')]dN = -Fdp + 0dT$$

and

$$-(W' - W/H)dH + pHF''dN = -HF'dp + dT.$$

If  $J_1$  denotes the determinant of the Jacobian matrix with respect to  $H$  and  $N$ , the second-order conditions imply  $J_1 > 0$ . We then obtain

$$(i) \partial H/\partial T = (1/J_1)p(F/N - F') = (1/J_1)(pF/N)(1 - \eta_f) > 0;$$

$$\partial N/\partial T = -(1/J_1)NW'' < 0.$$

and

$$(ii) \partial H/\partial p = (1/J_1)(-pHFF'/N)(NF''/F' + 1 - \eta_f)$$

$$= (1/J_1)(pHF^2/N)(-\eta'_f) > 0;$$

$$\partial N/\partial p = (1/J_1)F[HW''NF'/F - (W' - W/H)]$$

$$= (1/J_1)(FW/H)(HW''/W' - \eta_w + 1) = (1/J_1)(FW/\eta_w)\eta'_w > 0.$$

## IV. Proof of Proposition 3

(i) *The case of undiminished substitutability.*—Here only  $K_0$  is determined in the long run. Second-order conditions guarantee that  $K_0$  will

increase when  $\rho$  decreases. Thus, all we have to check is the comparative static effect of a change in capital on the short-run threshold prices  $p_1^*$  and  $p_2^*$  and on hours. By the envelope theorem,  $\partial\{\text{Max}_N[F^0(N, K_0)/N]\}/\partial K_0 = F_K^0(N, K_0)/N$ . The elasticity of scale is defined by  $v(N, K) = (NF_N^0 + KF_K^0)/F^0$ . Thus,  $\partial\{\text{Max}_N[F^0(N, K_0)/N]\}/\partial K_0 = (v - 1)F^0(N, K_0)/NK_0$ , and it is positive if and only if  $v(N, K_0) > 1$ . A higher-than-one elasticity of scale at threshold employment is thus needed to ensure that lower capital costs for a competitive firm will lower its threshold prices.

Using the framework in the proof of proposition 2 above, in the short run,  $\partial H/\partial K_0 = (1/J_1)(-p^2 H)[F_K^0 F_{NN}^0 + F_{NK}^0(F^0/N - F_N^0)]$ . When  $F^0(N, K)$  exhibits (locally) a constant elasticity of scale  $v$ ,  $F_{NK}^0(N, K)$  is homogeneous of degree  $(v - 1)$ , so that by Euler's theorem,  $F_{NK}^0 = [(v - 1)F_N^0 - NF_{NN}^0]/K$ . Substituting, and using (4) and the definitions of  $\eta_F$  and  $\eta_W$ , one obtains after manipulation,  $\partial H/\partial K_0 = (v - 1)(1/J_1)[p^2(F^0)^2 H/KN](-\eta_F)$ . All the bracketed expressions other than  $(v - 1)$  are positive. Thus, the sign of  $\partial H/\partial K_0$  is positive if and only if there are (locally) increasing returns to scale.

(ii) *The case of no short-run substitutability.*—From equation (2) and (5), hours in shift  $i$  can be expressed as a function of the product  $p\lambda$ :

$$H_i^*(p\lambda) = 0 \text{ if } p\lambda \leq \omega_i; \quad H_i^*(p\lambda) = W_i'^{-1}(p\lambda) \text{ if } p\lambda \geq \omega_i, \quad (\text{A5})$$

where  $W_i'^{-1}(\cdot)$  is the inverse of the marginal employment cost  $W_i'(H_i)$ . Define by  $E(\lambda)$  the expected daily operating profit per worker

$$E(\lambda) = E\{\tilde{p}\lambda[H_1^*(\tilde{p}\lambda) + H_2^*(\tilde{p}\lambda)] - W_1[H_1^*(\tilde{p}\lambda)] - W_2[H_2^*(\tilde{p}\lambda)]\}. \quad (\text{A6})$$

In the case of no ex post substitutability, the firm's long-run problem is then to select average productivity  $\lambda$  and shift employment  $N_0$  so as to maximize  $N_0 E(\lambda) - C[g(\lambda, N_0), \rho]$ , where the price thresholds depend on  $\lambda$ . The two first-order conditions for the joint determination of  $\lambda$  and  $N_0$  are

$$NE'(\lambda) - C_K[g(\lambda, N), \rho]g_\lambda(\lambda, N) = 0 \quad (\text{A7})$$

and

$$E(\lambda) - C_K[g(\lambda, N), \rho]g_N(\lambda, N) = 0. \quad (\text{A8})$$

If the long-term production function  $F^0(N, K)$  has constant returns to scale, the function  $g(\lambda, N)$ , the  $K$ -inverse of  $F(N, K)/N$ , assumes the separable shape  $Ng^0(\lambda)$ . In this case, by factoring out  $C_K$  from (A7) and (A8) one obtains the condition:  $E(\lambda)/E'(\lambda) = Ng_N(\lambda, N)/g_\lambda(\lambda, N)$



$= g^0(\lambda)/g^0(\lambda)$ . This condition determines  $\lambda$ , and thus also the capital-labor ratio and the short-run hours of production, independently of the capital costs  $\rho$ .

More generally, total differentiation of the system of two equations (A7)–(A8) yields

$$\partial\lambda/\partial\rho = (C_{K\lambda}g_{\lambda}/J_2)(g_{\lambda N}g_N/g_{\lambda} - g_N/N - g_{NN}), \quad (\text{A9})$$

where  $J_2$  is the determinant of the Jacobian matrix. When the ex ante production function is CES,  $F^0(N, K) = (aN^{1-1/\sigma} + bK^{1-1/\sigma})^{\sigma/(\sigma-1)}$ . After tedious substitutions, (A9) becomes

$$\begin{aligned} \partial\lambda/\partial\rho = \{ & (C_{K\lambda}/J_2)[a/(b^2v^2)](K/N)^{1/\sigma} / [(F/K)^{1/\sigma} F^{(\sigma-1)(v-1)/\sigma}] \} \\ & \times [(\sigma-1)(v-1)/\sigma]. \end{aligned} \quad (\text{A10})$$

Since the second-order conditions require that  $J_2$  be positive, the sign of the whole expression is equal to the sign of  $(\sigma-1)(v-1)$ .

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