Difficult Decisions*

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Abstract

We introduce an incentive-compatible mechanism that set-identifies behavior consistent with broad classes of complete preferences under uncertainty. Choices outside this set reveal an inability to rank alternatives, indicating incomplete preferences. Experimental findings show that choices incompatible with Subjective Expected Utility often cannot be rationalized by flexible models of complete preferences. These results reflect three sources of deliberate randomization – indifference, hedging (convexity), and incomplete preferences.

Keywords: Incomplete Preferences, Choice Under Uncertainty, Deliberate Randomization, Experiment

JEL: C91, D01, D81, D90

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Identifying preferences from choice data is one of the cornerstones of economic research and practice. Yet, the most basic aspect of preferences – the decision maker's (DM) mere ability to compare alternatives – has proven to be the most elusive.

The goal of the current investigation is to propose an incentive compatible mechanism through which a DM can reveal their inability to compare two alternatives and study their behavior in such situations. The approach pursued is indirect: we theoretically investigate the testable implications of completeness and the weakest ancillary assumptions on choice behavior in the proposed mechanism, and then implement the mechanism in a controlled experiment. Behavior that is inconsistent with such general models of complete preferences is then indicative of incomplete preferences.

In the experiment, the DM faces a binary choice between bets on which of two shapes is larger, and can explicitly randomize over the bets if they choose. The DM's limited ability to perceive differences between the shapes' areas induces the type of uncertainty that is characteristic of situations where DMs must choose under ambiguity, which is typical in economic decisions where objective probabilities are not known.[1] This allows us to abstract from uncertainty that arises from imprecise tastes – as the DM always prefers more money to less and thus always prefers placing a bet on the larger shape – and focus entirely on uncertainty that arises due to imprecise beliefs.

The mechanism also elicits the DM's "probability equivalent" (PE) for betting on each shape being larger using an incentivized generalization of the Becker, DeGroot, and Marschak (1964, BDM) mechanism. Crucially, we allow the DM to report an interval of PEs as opposed to merely a singleton.[2] The combination of these three tasks – the binary choice and the two PEs – is essential to the identification strategy we pursue theoretically and empirically.

First, note that a DM with probabilistic beliefs (including Subjective Expected Utility, SEU) reports a singleton PE ("matching probability") for each bet, and the two PEs must sum up to one. Moreover, this DM will never randomize (mix) between the bets, unless both PEs are equal to 0.5. An ambiguity averse DM with convex preferences could naturally prefer to mix (hedge) between the bets. However, if the DM's preferences are described

[1] A recent body of economic research also argues that "economists have much to learn from studies of imprecision in people's perception of sensory magnitudes" (Woodford, 2020, p. 580) as there is "reason to believe that reasoning about numerical information often involves imprecise mental representations of a kind directly analogous to those involved in sensory perception" (Khaw, Li, & Woodford, 2021, p. 1983). Indeed, de Clippel, Moscariello, Ortoleva, and Rozen (2025) document that the valuation of perceptual tasks is highly correlated with ambiguity aversion.

[2] The probability equivalent of an ambiguous bet is typically interpreted as the objective probability such that a DM is indifferent between the bet and a lottery that pays the same prize with the objective probability. We do not impose or require this interpretation, but we use the term as it is typically used to denote the response to a BDM mechanism.

by the Maxmin Expected Utility Model (MEU, Gilboa & Schmeidler, 1989) then they will always report singleton intervals for their PEs that sum up to less than or equal to one.[3]

On the other hand, reporting one or more non-singleton PE is indicative of imprecise beliefs but not necessarily of incomplete preferences. As noted by Cettolin and Riedl (2019) and Agranov and Ortoleva (in press), one cannot rule out more general models of complete and convex preferences that rationalize this behavior. In order to rule out such models, we show that very basic and uncontroversial properties of these preferences imply tight restrictions on behavior and argue that inconsistency with them is suggestive of incomplete preferences. To illustrate the mechanism, consider a DM who reports identical intervals of PEs equal to [0.45, 0.55] for both bets and chooses to mix between the two bets with probability 0.5. As noted above, choice of a non-singleton PE is inconsistent with SEU or MEU preferences. Furthermore, these choices are also inconsistent with much more general models of complete and convex preferences.[4]

Moving to the empirical evaluation, a striking feature of the experimental data is that many choices are consistent with SEU. However, when choices are inconsistent with SEU, subjects' behavior is almost always inconsistent with common multiple prior models, including Variational Preferences (Maccheroni, Marinacci, & Rustichini, 2006), and is often inconsistent with a very general class of complete and convex preferences. In other words, we show that observed behavior, both in the PE questions and choice between bets, cannot generally be rationalized by common multiple prior models and can only sometimes be rationalized by more general models of convex preferences. Moreover, at the individual level, we find that many subjects make choices that are inconsistent with convex preferences in some, but not all, of their decisions. This evidence suggests a conservative bound on behavior, including apparent deliberate randomization, that is associated with incomplete preferences.

Two general lessons can be learned from the current investigation. First, it is possible to identify incomplete preferences, even with very mild ancillary assumptions, by considering the behavior that all models of complete preferences should exhibit. Second, at least in our setup where uncertainty is over beliefs, the data suggest that deviations from the standard model of SEU cannot always be accommodated by much more general models of complete preferences but are instead due to the DM's difficulty in ranking alternatives and may be accounted for by a simpler model of incomplete preferences. In particular, an adaptation of the

[3]Note that once preferences are not SEU, PEs reported by a DM do not correspond to beliefs (though they are functions of them). For example, in MEU the PEs correspond to extreme points of the set of priors.

[4]Although Cerreia-Vioglio, Dillenberger, Ortoleva, and Riella (2019) consider only stochastic choice over objective lotteries, the general insight from their work is that convex preferences can lead to stochasticity of choice, as might be observed in the bet between the two shapes.

model of Bewley (2002) allows researchers to maintain many of the structural assumptions that are relaxed by more general models (when assuming completeness) while simultaneously accurately representing the data.

We show that our results are not due to confusion about the mechanism, trembles, ambiguity seeking, non-stable preferences, deriving utility from mere randomization, menu-dependent preferences, or violation of reduction of compound lotteries. Moreover, subjects tend to spend more time in rounds in which they choose non-singleton intervals, which indicates a deliberate and thoughtful choice process.

The remainder of the paper is organized as follows. Section 1 reviews the literature, Section 2 introduces the model and the main axioms, Section 3 contains the main theorems, Section 4 introduces the experiment design, Section 5 provides the main data results, Section 6 discusses some alternative models proposed to explain the data, and Section 7 concludes. Many additional theoretical and empirical results, in addition to a detailed description of the experimental design and implementation, are contained in a Supplemental Appendix.

1 Literature Review

The two experimental studies most closely related to the present investigation are Agranov and Ortoleva (in press) and Cettolin and Riedl (2019). In both studies, subjects are allowed to mix among alternatives in multiple price lists (which are analogous to our modified BDM mechanism for eliciting intervals of PEs).[5] Agranov and Ortoleva (in press) use this mechanism to elicit ranges of certainty and lottery equivalents for objective lotteries, and compare these ranges to choices in a standard-BDM mechanism, where subjects cannot explicitly mix. They find that many subjects report large ranges for each, and suggest that this could be due to complete and convex preferences, or to incomplete preferences.[6] The binary bets used in the current study focus attention on belief imprecision, and allow us to theoretically and empirically identify these competing explanations.

Cettolin and Riedl (2019) also study the problem of identifying behavior that is incompatible with models of complete preferences when beliefs are imprecise. They elicit a range of PEs of a chosen bet on the color of a randomly drawn ball from an urn that contains balls with an unknown color composition. Like us, the interval is defined when the subject

[5] Cubitt, Navarro-Martinez, and Starmer (2015) are, to the best of our knowledge, the first to employ this mechanism in order to elicit ranges of values or beliefs. In their study, these choices were not incentivized and the focus of their analysis was on the consequences of imprecise preferences rather than identifying incompleteness.

[6] Agranov and Ortoleva (in press) point out that the two sources may be linked, since some models of complete and convex preferences could be derived as the cautious-completion of incomplete preferences.

chooses to mix between the chosen ambiguous bet and the objective lottery for different probabilities of winning. A non-singleton interval is not consistent with SEU and MEU, but could be rationalized by more general models of ambiguity aversion or incomplete preferences. In the second part of their experiment they measure the subjects' willingness-to-pay to mix, relying on the identifying assumption that zero willingness-to-pay is incompatible with complete preferences and, therefore, indicates incompleteness. We build on Cettolin and Riedl (2019), but we use different stimuli, identification strategy, and relate incompleteness to deliberate randomization among the two ambiguous bets. The bets on shapes allow us to vary the difficulty of ranking the bets by varying the number of attributes (e.g., sides of different lengths), allowing the analyst to observe many within-subject choices. The PEs of the two complementary events allow us to distinguish between SEU and MEU – when the intervals are singletons, and identify behavior inconsistent with very weak assumptions on complete preferences – when the intervals are non-singletons. Furthermore, we are able to relate imprecise beliefs and behavior that is incompatible with complete preferences to deliberate randomization between the two ambiguous bets.

Other attempts to identify incompleteness experimentally involve choice deferral or choosing not to choose. Costa-Gomes, Cueva, Gerasimou, and Tejiščák (2022) and Gerasimou (2025) compare treatments in which subjects may defer their choices and where subjects are forced to choose. The greater inconsistency of choices in the latter (forced choice) setting is interpreted as evidence of incompleteness. Predating, but using a hybrid of Agranov and Ortoleva (in press) and Costa-Gomes et al. (2022), Danan and Ziegelmeyer (2006) elicit a range of certainty equivalents while allowing for choice deferral. The dynamic nature of these experiments leaves room for many alternative explanations, including preference instability, a preference for flexibility, a preference for randomization, or costly learning.[7] Nielsen and Rigotti (2024) avoid tying their hands to a specific assumption regarding how subjects with incomplete preferences will choose when forced to, by allowing them to delegate choice to an algorithm. However, in addition to not being incentive compatible, their identification is potentially hampered by the subjects' inability to train the algorithm to randomize, when randomization is strictly preferred to both options. Our incentive compatible mechanism allows us to elicit choices simultaneously, avoiding many of the pitfalls discussed above, while also not restricting the subject's ability to randomize, if desired.

Many other experimental studies seek to better understand the relationship between "imprecise" preferences or beliefs and, possibly stochastic, choice behavior. For example, Butler (2000) shows that preference reversals in binary lottery choice tasks are more likely

[7] Costa-Gomes et al. (2022) themselves report some evidence of introspective preference learning during the main phase of their experiment.

when preferences are "weak." Butler and Loomes (2007) elicit ranges of certainty and probability equivalents for two reference lotteries and compare this to repeated choice among the same two lotteries. Like us, beliefs (PEs in our case) are elicited for each choice object separately in addition to observing the (possibly stochastic) choice between the two objects. However, both papers use an unincentivized elicitation mechanism. Enke and Graeber (2023) investigate how "cognitive uncertainty" relates to the classic S-curve that often characterizes estimated probability weighting functions. In our context, their model of cognitive uncertainty, however, does not predict non-singleton intervals, but rather a singleton belief that is the weighted average of two scalars. In the second part of the experiment, the authors elicit a measure of confidence in this point estimate, but this unincentivized measure does not necessarily provide any information about the DM's preferences. On the other hand, our theory results carefully lay out implications of standard economic assumptions for the relationship between the PEs reported in our experiment and the subject's preferences.

Another burgeoning literature that seeks to understand how the difficulty of perceiving the differences between options may impact choice behavior[8] models the DM as rationally choosing which costly pieces of information to acquire (e.g., Matějka & McKay, 2015; Caplin, Dean, & Leahy, 2022, 2019). This literature also studies stochastic choice, but a crucial difference is that costly learning models generically imply a strict preference for one object once information gathering is complete. These models do not predict non-degenerate PEs. A similar contrast exists with the incomplete preference model of Karni (2024) (and Karni (2022)), as the behavior of the DM is stochastic to an outside observer, but the DM does not wish to randomize over options. When the DM cannot directly compare two alternatives, their choice is "triggered by impulses" that are inherently random, but result in the DM choosing a single option.

Our study is also related to the experimental literature on stochastic choice. Sopher and Narramore (2000), for example, allow subjects to select a mixture between two lotteries to evaluate various models of stochastic choice, including the Random Utility model and a model of deliberate randomization due to Machina (1985). Agranov and Ortoleva (2017) provide support for deliberate randomization using a repeated choice framework, where stochastic choice is more often observed for comparisons between lotteries deemed "hard."[9] Chew, Miao, Shen, and Zhong (2022) also find evidence in favor of deliberate randomization

[8]There is an older related literature that dates back at least to Tversky (1972) who writes: "Choice probabilities ... reflect not only the utilities of the alternatives in question, but also the difficulty of comparing them." This implies that any useful descriptive theory of choice must account for the potential difficulty of comparing objects.

[9]Dwenger, Kübler, and Weizsäcker (2018) also find a preference for randomization that seems to arise from the difficulty of ranking objects of choice due to uncertainty regarding one's own preferences.

expressed as multiple-switching behavior in multiple price lists. Like Sopher and Narramore (2000), Feldman and Rehbeck (2022) allow subjects to choose mixtures between pairs of lotteries, and compare these to repeated choices among the same lotteries. They interpret a positive correlation as evidence of deliberate randomization and find more randomization among lotteries for which the odds-ratio is intermediate in value. Hence, even in these studies, which were focused primarily on stochastic choice, the results suggest an important, though not identifiable, connection between the difficulty of comparing objects and stochastic choice.

Finally, our paper is related to the theoretical literature on incomplete preferences. The main distinction is that instead of modeling behavior when preferences are incomplete (e.g., Bewley, 2002; Karni, 2024), or studying foundations and mathematical properties for incomplete preferences (e.g., Aumann, 1962; Ghirardato, Maccheroni, & Marinacci, 2004; Nau, 2006; Gilboa, Maccheroni, Marinacci, & Schmeidler, 2010; Ok, Ortoleva, & Riella, 2012; Galaabaatar & Karni, 2013; Faro, 2015; Riella, 2015), our paper focuses on exploring the testable implications for very general classes of complete preferences. A complementary theoretical approach to the problem of identifying incomplete preferences in other domains is provided by Eliaz and Ok (2006). In addition, in a paper related to our empirical findings, Ok and Tserenjigmid (2022) explicitly assume a relationship between stochasticity of choices and incompleteness of preferences.

2 Identification

Consider a set S of states, a convex set X of consequences, and the set $\mathcal{F}$ of all (simple) acts, which are measurable functions from S to X. For every $x \in X$, define $x \in \mathcal{F}$ to be the constant act such that $x(s) = x$ for all $s \in S$. Throughout the paper, we consider a binary state space $S = \{\lambda, \rho\}$ and acts that always result in the DM receiving one of two payments, either $m > 0$ or nothing. This feature of our environment allows us to focus solely on the consequences of imprecise beliefs. We thus, as a minor abuse of notation, let $X = [0, 1]$, which is to say we let $x \in [0, 1]$ denote the constant act that assigns a chance x of winning m as opposed to nothing in both states.

To facilitate the connection with the experimental implementation, we consider two special acts $L, R \in \mathcal{F}$ (short for "bet on left" and "bet on right"). The acts L and R are bets on the two possible states (λ and ρ): a correct bet wins the DM a payment of $m > 0$, and nothing otherwise.

As is typical, for every $f, g \in \mathcal{F}$ and $\alpha \in [0, 1]$, we define the act $\alpha f + (1 - \alpha)g \in \mathcal{F}$ as the act that yields the statewise-mixture $\alpha f(s) + (1 - \alpha)g(s) \in X$ for every $s \in S$. If

f, $g \in \mathcal{F}$ and $f(s) > g(s)$ for all $s \in S$, then we say f *statewise dominates* g. We model the DM's preference on $\mathcal{F}$ as the binary relation $\succeq$, and denote by $\succ$ and $\sim$ the asymmetric and symmetric parts of $\succeq$, respectively.

2.1 Elicitation Mechanism

The DM faces three questions. In the first two, which we call the *PE questions*, they are asked to choose between each of a sequence of lotteries (constant acts) and L or R, respectively. While the bet remains constant in each of these two questions, the probability of winning the lottery increases from 0 to 1. The DM may respond to each of the two questions using an interval. Let l_L, $u_L \in [0, 1]$ with $l_L \leq u_L$ denote the lower and upper bound reported by the DM for L, and let l_R, $u_R \in [0, 1]$ with $l_R \leq u_R$ denote the lower and upper bound reported by the DM for R. If $l_f = u_f$ for $f \in \{L, R\}$, then we say that the interval is *degenerate* and is thus equal to the DM's PE for the given bet. If an interval is not degenerate, we say it is *non-degenerate*. Importantly, the DM is not required to report a non-degenerate interval, and can report a PE if they desire.

If one of the two PE questions is selected to determine the DM's payment, a random lottery with probability of winning $r \in [0, 1]$ is drawn according to some full support distribution. When r is higher than the upper bound of the interval chosen by the DM for the relevant bet (either L or R), then they receive r – that is, they win m with probability r. If r is smaller than the lower bound chosen by the DM for the relevant bet, their payment is determined by the relevant bet, L or R, and they win m iff the relevant bet is correct about the state of the world. If r falls within the chosen interval, then they receive either the relevant bet or the lottery r, with equal chances.

In the third question facing the DM, which we call the *binary choice question*, we ask them to choose between L and R or to mix between the two bets. Mixing is achieved by choosing a number $\alpha \in [0, 1]$ to be their probability of selecting R, if not selecting L, and thus if this question is used for payment the DM receives the act $\alpha R + (1 - \alpha)L$.

The mechanism proposed for the first two questions generalizes the BDM mechanism (Becker et al., 1964) as applied to PEs (Grether, 1981; Karni, 2009) by allowing the DM to choose an interval of PEs. Moreover, betting on both an event and its complement (as in the work of Baillon, Huang, Selim, and Wakker (2018)), here L and R, allows the analyst to identify behavior that was not available before. Further, in the binary choice question the DM can mix between the two bets, and while convex (uncertainty averse) preferences could potentially rationalize randomization and intervals being chosen, the following sections demonstrate that even very general models of preferences impose refutable predictions on choices across the three questions.

2.2 Behavioral Axioms

The literature (e.g. Maccheroni et al. (2006)) has generally assumed preferences satisfy completeness, transitivity, some form of monotonicity, and then added some structure through additional axioms. The approach we take is to maintain completeness while using very weak forms of monotonicity and transitivity. The most common form of monotonicity assumed in the literature is statewise monotonicity, which requires $\succeq$ to be monotone with respect to statewise dominance. The Winning is Preferred Axiom is weaker in the sense that it is defined only on lotteries (constant acts) and assumes the DM prefers a higher chance of winning the prize.

Winning is Preferred (WP) For all x, $y \in X$, if $y > x$ then $y \succ x$.

The Completeness Axiom is the one to be evaluated in the current investigation, and assumes the DM can always rank any two acts.

Completeness (COM) For all f, $g \in \mathcal{F}$, either $f \succeq g$ or $g \succeq f$.

The standard notion of transitivity is provided below for ease of comparison, but weaker forms of it are used extensively in the identification.

Transitivity (TR) For all f, g, $h \in \mathcal{F}$, if $f \succeq g$ and $g \succeq h$, then $f \succeq h$.[10]

Behavior that rejects the assumption that preferences are monotone, complete, and transitive could be attributed to the failure of any of the three assumptions. Since our focus is on evaluating the empirical relevance of completeness, we want the other assumptions to be as weak as possible. To this end, we require the pairwise ranking to be extended by transitivity *only* when comparing to a third act that statewise dominates (dominated) the preferred (inferior) act in the pair.

Statewise Transitivity (STR) For all f, g, $h \in \mathcal{F}$ such that $f \succeq g$, if h statewise dominates f then $h \succ g$, and if g statewise dominates h then $f \succ h$.[11]

Weak Statewise Transitivity (WSTR) For all f, g, $h \in \mathcal{F}$ such that $f \succeq g$, if h statewise dominates f then $h \succeq g$, and if g statewise dominates h then $f \succeq h$.[12]

We now introduce structural assumptions that are commonly employed when modeling imprecise beliefs and ambiguity averse preferences. The most general property is convexity,

[10]Lemma 1 in Section B of the Supplemental Appendix shows that the transitivity of the strict preference relation is implied by TR.

[11]Note that COM and STR together imply WP.

[12]In the absence of statewise monotonicity (which is not essential in our set-up), TR does not imply STR, but STR does imply WSTR.

which is closely related to ambiguity aversion (Schmeidler, 1989; Gilboa & Schmeidler, 1989), and deliberate randomization (Cerreia-Vioglio et al., 2019).

Convexity (CON) For all $f,\, g,\, h \in \mathcal{F}$, if $f,\, h \succeq g$, then for all $\alpha \in (0,\, 1)$: $\alpha f + (1-\alpha)h \succeq g$.[13]

The following axioms assume how a DM evaluates hedging with constant acts, which is crucial in our setup, as the interval in the PE questions allows the subject to mix the bet (R or L) with a constant act (lottery). The Certainty Independence axiom assumes that mixing with a constant act does not change the DM's ranking of acts.

Certainty Independence (CI) (Gilboa & Schmeidler, 1989) For all $f,\, g, \in \mathcal{F}$, $x \in X$, and $\alpha \in (0,\, 1)$: $f \succ g \iff \alpha f + (1-\alpha)x \succ \alpha g + (1-\alpha)x$.

The Weak Certainty Independence axiom can accommodate a preference to hedge with a constant act.

Weak Certainty Independence (WCI) (Maccheroni et al., 2006) For all $f,\, g, \in \mathcal{F}$, if $x,\, y \in X$, then for all $\alpha \in (0,\, 1)$: $\alpha f + (1-\alpha)x \succeq \alpha g + (1-\alpha)x \implies \alpha f + (1-\alpha)y \succeq \alpha g + (1-\alpha)y$.[14]

3 Main Theoretical Results

We now turn to investigating the testable implications of two classes of complete preferences (based on the axioms presented in Section 2.2) utilizing the elicitation mechanism. A DM *isolates* if for each choice they make they ignore other decisions in front of them (narrow bracketing, Ellis & Freeman, 2024). This applies both to a single PE question (where the DM could consider their responses for all possible values of r)[15] and across the three

[13]Our definition of convexity differs from the definition given by some other papers. For a clarification of how this definition of convexity relates to the definition given by, for example, Cerreia-Vioglio, Maccheroni, Marinacci, and Montrucchio (2011), see Proposition 1 in Section B of the Supplemental Appendix and its proof, or Lemma 56 in their paper.

[14]WCI is implied by COM and CI.

[15]If a DM integrates their responses within a PE question by taking into account that the random number r is uniformly drawn (rather than responding to every value of r in isolation), calculates the probabilities of final outcomes according to the laws of probability (as they would if they satisfy Reduction of Compound Lotteries, ROCL), and has monotone preferences with respect to First Order Stochastic Dominance, then they will never choose a non-degenerate interval. Suppose the DM reports $l_f < u_f$ for a bet on $f \in \{L, R\}$, then the probability of winning m conditional $r \in [l_f, u_f]$ is $\frac{1}{u_f - l_f}\int_{l_f}^{u_f}\left[\frac{r}{2} + \frac{f}{2}\right]dr = \frac{1}{2}f + \frac{1}{2}\left(\frac{u_f + l_f}{2}\right)$. Whereas, if the DM reports a degenerate interval in the midpoint of the interval – $\frac{u_f + l_f}{2}$, the probability of winning m conditional on $r \in [l_f, u_f]$ is $\frac{1}{2}f + \frac{1}{u_f - l_f}\int_{(u_f+l_f)/2}^{u_f} r dr = \frac{1}{2}f + \frac{1}{2}\left(\frac{1}{2}u_f + \frac{1}{2}\frac{u_f + l_f}{2}\right) > \frac{1}{2}f + \frac{1}{2}\left(\frac{u_f + l_f}{2}\right)$. We are not aware of experimental studies that support this form of integration, with the exception of Freeman, Halevy, and Kneeland (2019), who use related reasoning. Since subjects who integrate r within a PE question will choose degenerate intervals, their choices would be indistinguishable from subjects with SEU/MEU preferences. In this sense, our main empirical findings should be thought of as a conservative

questions.[16] In the body of the paper we assume throughout that the DM isolates.

Since we study an environment where beliefs may be imprecise, a natural starting point is the celebrated Maxmin Expected Utility (MEU) model in which the DM has a set of priors and their utility of an act is their minimal expected utility over all priors in the set. It is straightforward to show that a DM whose preferences are represented by MEU will always report degenerate intervals (whose sum can be less than or equal to 1).[17] Intuitively, CI implies that a DM does not increase their utility by mixing an act with a constant act, which is exactly what choosing a non-degenerate interval in the PE question permits. Hence, a natural first step is to consider a class of preferences that relaxes CI, allowing DMs to benefit from mixing with a constant act. The class, which we call *Multiple Prior Preferences (MPP)*, is characterized by the set of axioms {COM, STR, CON, WCI} and includes common multiple prior models. Note as well that in addition to relaxing CI, this class imposes neither TR nor continuity.[18]

3.1 Multiple Prior Preferences

The testable implications of MPP in our elicitation mechanism are detailed in the following theorems that place quantitative restrictions on the DM's responses to the PE and binary choice questions.

The first result utilizes only responses to the PE questions and does not assume any notion of transitivity.

Theorem 1. *If the DM's preferences satisfy WP, COM, CON, and WCI then* $u_L + u_R \leq 1$.

In spite of its very mild requirements, Theorem 1 still imposes tight restrictions on the permissible responses to the PE questions. Note that it naturally extends the result for MEU preferences that satisfy CI, by allowing non-degenerate intervals, but requiring the same inequality to hold for the sum of the upper bounds of the intervals. Figure 1 illustrates the testable implications of this theorem. The following theorem assumes STR and uses the responses to the binary choice question to impose further restrictions on behavior.

Theorem 2. *Let* $\alpha \in [0, 1]$ *be the weight assigned to* $f \in \{L, R\}$ *in the binary choice question. If the DM's preferences satisfy COM, STR, CON, and WCI then:*

lower bound on the frequency of subjects who exhibit incomplete preferences.

[16]Theoretical results assuming integration of the two PE questions along the lines of Baillon, Halevy, and Li (2022a) can be found in Section B.1 of the Supplemental Appendix.

[17]Theorem 6 in Section B.1 of the Supplemental Appendix proves this result directly for the class of preferences that includes α-MEU (Ghirardato et al., 2004), which requires CI but not CON.

[18]Continuity is defined in the Section B.3 of the Supplemental Appendix.

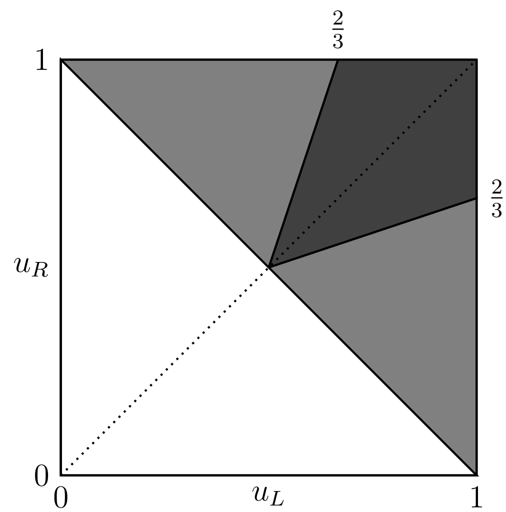


Figure 1: Testable implications - Theorems 1 and 4

Grey region violates Theorem 1 (MPP). Dark Grey region violates violates Theorems 1 and 4 (CP).

1. *if* $\alpha \geq \frac{3}{4}$*, then* $u_f \geq \frac{1}{2}$*, and*

2. *if* $\alpha \leq \frac{1}{2}$*, then* $u_f \leq \frac{1}{2}$*.*

Theorem 2 requires consistency between the upper bound chosen in the PE questions and the response to the binary choice question. For example, if a DM chooses the act R with a relatively high probability then they must also report a relatively high upper bound for the corresponding PE, and *vice versa*.

The following theorem maintains STR but drops WCI. We include it here, though it applies to a more general class of preferences (discussed below), as it applies to MPP as well.

Theorem 3. *Let* $\alpha \in [0, 1]$ *be the weight assigned to* $f \in \{L, R\}$ *by the DM in the binary choice question. If the DM's preferences satisfy COM, STR, and CON then:*

1. *if* $\alpha = 1$*, then* $l_f \geq \frac{1}{2}$*, and*

2. *if* $\alpha \leq \frac{3}{4}$*, then* $l_f \leq \frac{1}{2}$*.*

Theorem 3 requires consistency between the lower bound in the PE question and the response to the binary choice question. For example, choosing the act R with probability one in the binary choice question implies that the lower bound of the interval in the PE question for R should be greater than 0.5.

Table 1: Relationship Between Classes, Axioms, and Theorems Under Isolation

Class	Axioms	Theorems
MPP	COM, STR, CON, WCI	1, 2, 3
CP	COM, STR, CON	3, 4

3.2 Convex Preferences

An even more general class of preferences that drops WCI maintaining only {COM, STR, CON} is called *Convex Preferences (CP)*. This class is particularly important, as complete and convex preferences have been advanced as an explanation for observed randomization (notably by Agranov & Ortoleva, 2017; Cerreia-Vioglio et al., 2019; Agranov & Ortoleva, 2022). The following theorem, in addition to Theorem 3 above, demonstrates the testable implications of this class.

Theorem 4. *If the DM's preferences satisfy WP, COM, WSTR, and CON then*

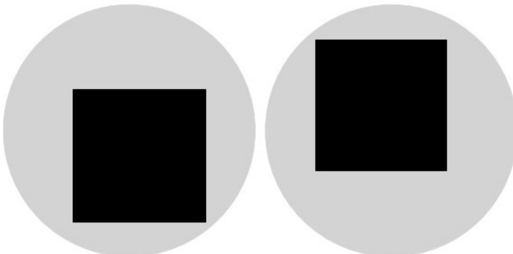
$$\frac{1}{2}\Big(\frac{u_L + u_R}{2} + \frac{1}{2}\Big) \geq \min\{u_L,\, u_R\}.$$

Theorem 4, like Theorem 1, does not rely on responses to the binary choice question and yet still imposes testable restrictions on responses to the PE questions. Note as well that PEs that do not satisfy Theorem 4 necessarily do not satisfy Theorem 1. This is illustrated in Figure 1.

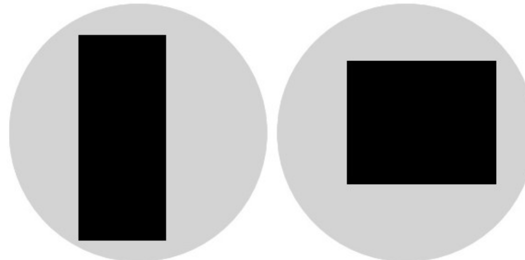
Table 1 summarizes the relation between the axioms, the class of preferences, and the corresponding theorems. We can also relate our classes to well-known models of complete and convex preferences. Yet, in all cases, our classes impose much less structure. For example, Multiple Prior Preferences assume less structure than Variational Preferences (Maccheroni et al., 2006) which itself is a generalization of MEU. Convex Preferences assume less structure than MPP and also Smooth Ambiguity Averse Preferences (Klibanoff, Marinacci, & Mukerji, 2005; Denti & Pomatto, 2022).[19] What is apparent is that our mechanism allows us to test a suite of behavioral axioms that imposes very little structure on preferences and corresponds to well-known models of complete and convex preferences that have been proposed as rationalization for observed ambiguity aversion and deliberate randomization.[20] Rejection of

[19]Note that, if one assumes that always winning is weakly preferred to always losing, i.e. $1 \succeq 0$, then Theorem 4 applies also to Uncertainty Averse Preferences (UAP, Cerreia-Vioglio et al. (2011), see Propositions 1 and 2 in Section B of the Supplemental Appendix). Moreover, Theorem 3 applies as well if one assumes strict monotonicity with respect to constant and non-constant acts, i.e. if $f(s) \succ g(s)$ for all s, then $f \succ g$ (Denti and Pomatto (2022), Axiom 2). Hence, UAP is included in CP when strict monotonicity is assumed.

[20]The relationship between these classes and models is formalized in Proposition 1 in Section B of the Supplemental Appendix.



(a) Image 2



(b) Image 8

Figure 2: Two Examples of Comparisons

these models then suggests a different explanation for observed behavior: incompleteness.

4 Experimental Design

Each round of the experiment involves making comparisons between two shapes of different sizes and judging which of the two has the larger area. Figure 2 includes two examples of such comparisons: in this specific case, each rectangle in Figure 2b has an area that equals one of the squares in Figure 2a, so the comparison between squares is based on the length of one side, while the comparison between rectangles relies on multiplying the lengths of two sides. Subjects must spend at least 45 seconds in each round before continuing to the next round, but we can record if they stopped interacting with the interface earlier.

We include 3 (of the 12) rounds in which the "correct" choice is obvious ("sanity checks"), in order to make sure that our results are not driven by subjects' confusion.[21] In the first round, subjects cannot see the image of the shapes, and subjects are informed during training that the larger shape is equally likely to appear on either side. In two other rounds it is relatively easy to notice the bigger shape.[22]

We have four main treatments, with subjects randomly distributed among them in a two-by-two factorial design. The two dimensions we vary are which part of the experiment subjects interact with first,[23] and which of two different orderings of the questions in each

[21] An important feature of our binary induced value design is that the correct answer is objective and observable to the researcher. This feature thus facilitates these sanity checks.

[22] See Section C.3 of the Supplemental Appendix for additional details on the shapes.

[23] The experiment includes a second part, which we call the "lottery" rounds, where subjects choose between binary lotteries in which the probability of winning is represented by the area of the shape. This type of problem is much closer to choice among lotteries that has been used in the literature, but does not facilitate exact identification results as the main part of the experiment. Subjects face the two parts in random order (each consists of 12 rounds), and are trained and quizzed on each part separately. Additional details regarding this part can be found in Section C.1 of the Supplemental Appendix.

round they see. These treatments are meant to control for order effects among the different parts and among the different questions, respectively. Additionally, we vary several other factors to control for possible confounds resulting in 64 possible configurations. The construction and rationale for these configurations are described in detail in Section C of the Supplemental Appendix.

Subjects are trained on the interface before completing a quiz (for which they are rewarded based on performance) that includes five questions, in order to verify their understanding of the interface and incentive system. After completing the training and quiz, the subjects complete 12 rounds of decision problems (each round includes two PE questions and the binary choice question).

At the end of the experiment, one of the three questions in one of the rounds is used to determine if the subject has won a \$30 CAD prize.[24] Their payment depends on their choices, a random lottery r, and the resolution of any remaining uncertainty. If a subject ends up winning the prize they receive it in addition to their earnings from the training and quizzes (between an additional \$5 and \$10 CAD depending on their performance on the quizzes). Payments to subjects are made using an electronic funds transfer within 24 hours of the deadline for each session.

The subjects were undergraduate students at the University of Toronto from a variety of programs who registered in the Toronto Experimental Economics Lab subject pool and were recruited using ORSEE (Greiner, 2015). The experiment was coded with oTree (Chen, Schonger, & Wickens, 2016) and all sessions were conducted online.

4.1 Description of Interface

In each round, subjects respond to the two PE questions and the binary choice question as described in Section 2.1. All three questions are presented to the subject on the same page together with an image of the shapes, and subjects enter their responses using a graphical user interface. An example of the interface viewed by subjects in one of the rounds can be found by clicking here.[25]

Subjects enter their responses to the two PE questions using a double-slider, with which they determine the lower bound - l_f, and the upper bound - u_f, for shape $f \in \{L, R\}$. As discussed in Section 2.1, subjects may report a degenerate or non-degenerate interval, which is incentivized using a discrete implementation of the mechanism: $r, l_f, u_f \in \{0, 0.01, \ldots, 1\}$. Figure 4 in Section C of the Supplemental Appendix presents an example of one of the

[24] The question that is used for payment is randomly determined before subjects begin the experiment rounds, see the work of Baillon et al. (2022a).

[25] https://teel.economics.utoronto.ca/wp-content/uploads/2023/05/incomplete_interface.htm.

PE questions, including the dynamically updated text that describes the payoff relevant consequences of the subject's selections.

To respond to the binary choice question as described in Section 2.1, subjects use a single slider to set the color composition of an urn that contains 100 balls that are each red or blue (marked with R for 'right' and L for 'left'). Let $x \in \{0, 1, \ldots, 100\}$ be the number of red balls, then $\alpha = x/100$. An example of this particular elicitation device, including the dynamically updated text that describes the relevant payoff consequences of the subject's selections, is depicted in Figure 5 in Section C of the Supplemental Appendix.

5 Experimental Results

The analysis in this section is based on the choices made by 218 subjects who completed the experiment.[26]

5.1 Main Results: Violations of Convex Models

Table 2 summarizes the aggregate results at the round level, which consists of two PE questions and a binary choice question. In 59% of the rounds, the subjects choose two degenerate intervals. In most of these rounds (53% overall) the observed behavior is consistent with Subjective Expected Utility (SEU, which requires the two degenerate PEs to sum up to 1, and if they are different from 0.5 then they bet with probability 1 on the shape with the higher PE in the binary choice question).[27] SEU choices essentially fall into two categories: in 31% of rounds overall subjects are confident which shape is bigger (report PEs that equal 1 and 0), while in 21% of rounds overall both (degenerate) PEs are equal to 0.5 – expressing indifference. Moreover, if we exclude the 3 "sanity checks" rounds and restrict the sample to only subjects who made the "correct" choices in them,[28] the share of rounds in which choices are consistent with SEU is similar.

[26]Some results for the "lottery" rounds can be found in Section D.10 of the Supplemental Appendix. They resemble the findings in the main part of the experiment reported in the following, though (as noted in footnote 23) they do not yield exact identification results as derived in Section 2.

[27]An additional 3% of rounds (56% overall) are consistent with CP (which, of course, includes SEU and MEU) and have two degenerate intervals. In only 0.5% of rounds subjects report two degenerate intervals that sum to less than 1 and make choices that are consistent with MEU. In 0.3% of all the rounds, subjects chose two degenerate intervals for PEs that sum to more than 1 and additionally do not randomize in the binary choice question (which may be rationalized by ambiguity-seeking behavior but not necessarily CP). The remaining 2.5% of rounds do not fit nicely into any of the categories described above. For instance, in some rounds two different degenerate intervals that sum to 1 are reported, yet in the binary choice question the subject assigns a 100% chance to betting on the shape with the lower PE.

[28]These are choices in the remaining 9 rounds made by 114 subjects.

Table 2: Aggregate Distribution of Behavior Consistent with Models of Complete Preferences

	Total	SEU	CP	MPP
Two Degenerate PE Intervals	59.3 (61.3)	52.9 (54.2)	56.1 (57.4)	55.5 (57.1)
At Least One Non-degenerate PE Interval	40.7 (38.7)	0 (0)	23.2 (24.2)	3.9 (3.4)

Percentage of all rounds, all subjects. The numbers in the parentheses report the percent of rounds that satisfy the given theorems in non-sanity check rounds for subjects who passed all three sanity checks.
Note: MPP are a subset of CP

Recall that the leading rationale for deliberate randomization when preferences are complete is convexity (Agranov & Ortoleva, 2017; Cerreia-Vioglio et al., 2019). Therefore, evaluating CP when the subject chooses to randomize, whether by selecting a non-degenerate interval in the PE questions or in the binary choice question, boils down to a test of Completeness (COM) and Statewise Transitivity (STR). In 41% of all rounds subjects choose at least one non-degenerate interval and in 91% of these rounds subjects randomize in the binary choice question. Choices in 23% of all rounds involve a non-degenerate PE and are consistent with Convex Preferences (CP), and in only 4% of all rounds the choices involve a non-degenerate PE and are consistent with MPP.[29] These results imply that, conditional on reporting a non-degenerate interval, subjects' choices are inconsistent with MPP and CP in 90% and 43% of rounds, respectively. In other words, when subjects report at least one non-degenerate PE their behavior is almost always inconsistent with common multiple prior models and often inconsistent with even more general models of convex preferences. Moreover, the magnitudes of these violations are also quite large, making them robust to small perturbations. For example, in rounds where subjects report at least one non-degenerate interval, the average violation of Theorem 4, which pertains to CP, is 0.14 (see Figure 3 in Section 5.4 for more details on MPP). Choices in about one sixth of all rounds include randomization in the PE questions but are inconsistent with completeness, statewise transitivity, and convexity – suggesting that incompleteness may be an important rationale for randomizing.[30] As above, excluding the sanity check rounds and those subjects who do not make correct choices in at least one of them, has little effect on the fraction of remaining rounds that are consistent with MPP and CP.

Recall that consistency with MPP and CP requires that the choices in all three ques-

[29]This section employs the discrete versions of our theoretical results which can be found in Section B.2 of the Supplemental Appendix. Note that, if a behavior is consistent with continuous data then it is also consistent with discrete data. As a result, it is more demanding to identify a behavior that is inconsistent with a class of preferences based on discrete data.

[30]It is crucial to recall that a subject with complete preferences can make choices consistent with SEU in some rounds, while in other rounds their choices will be inconsistent with SEU but consistent with MPP or CP.

Table 3: Percentage of Consistent Choices* Conditional on Non-Degenerate PEs

		PEs Satisfy		
		Theorem 1	Theorem 4	
Choices Satisfy	Theorems 2 and 3	9.5 (8.8)	-	49.7 (58.9)
	Theorem 3	-	56.9 (62.5)	70.6 (66.5)
		12.9 (11.1)	82.3 (92.4)	

*Does not necessarily require mixing in the binary choice question. Rounds where subjects report at least one non-degenerate interval. At least one non-degenerate interval is reported in 1066 (397) rounds. The numbers in the parentheses report the percent of choices that satisfy the given theorems in non-sanity check rounds for subjects that passed all three sanity checks.

tions to satisfy Theorems 1, 2, and 3, and Theorems 3 and 4, respectively. Recall also that Theorems 1 and 4 place restrictions on behavior in the PE questions alone (Figure 1) whereas Theorems 2 and 3 place restrictions on behavior in the PE questions conditional on responses in the binary choice question. Table 3 summarizes the empirical importance of these two types of restrictions focusing, for clarity, only on those rounds where subjects report at least one non-degenerate PE interval. Inconsistency of choices with MPP originates mainly from the PE questions: only 13% of choices in these questions are consistent with MPP, and in 74% of them (10% overall) the choices are also consistent with the binary choice question (recall that MPP imposes restrictions both on the lower and upper bounds). On the other hand, for CP the required consistency of the choices in the PE questions with the binary choice question potentially plays a more substantial role: while 82% of choices in the PE questions are consistent with CP, 69% of them (57% overall) are consistent with their responses to the binary choice question (particularly, consistency of the lower bounds). This is a natural result, since while MPP have some linear structure (satisfy WCI), CP have no such structure and are consistent with randomization. Hence, most rejections of CP rely on responses to the PE questions that are inconsistent with the binary choice question.[31]

At the individual level, most subjects make choices that are inconsistent with general models of complete and convex preferences. Table 4 describes the proportion of subjects that are inconsistent with our classes in one or more rounds. This indicates that the results are not driven entirely by "pure" types, which are either always or never consistent with MPP or CP. However, there are 25 subjects (or 12% all subjects) that are consistent with SEU in all rounds (21 of them, or 18% of 114, passed all sanity checks). More than half of the subjects are inconsistent with MPP in 5 or more rounds (40% when controlling for

[31] Theorem 3 requires STR while Theorem 4 only requires WSTR, hence some of the impact of Theorem 3 may be due to this strengthening of transitivity.

Table 4: Individual analysis: number of subjects inconsistent with alternative models

Percentage of Subjects	All	Quiz ≥ 4
Inconsistent with MPP	83.9 (76.3)	77.8
Inconsistent with MPP 3 or more rounds	66.1 (54.4)	61.1
Inconsistent with MPP 5 or more rounds	54.6 (39.5)	50.0
Inconsistent with CP	72.9 (62.3)	63.3
Inconsistent with CP 3 or more rounds	42.2 (29.8)	35.6
Inconsistent with CP 5 or more rounds	18.8 (7.9)	12.2
	N=218	N=90

The numbers in the parentheses report the percent of choices that satisfy the given theorems in non-sanity check rounds for subjects that passed all three sanity checks.

the 3 sanity checks), and 42% are inconsistent with the more permissive model of CP in 3 or more rounds (30% after controlling for sanity checks). These results are remarkable since there are only 9 rounds with "nontrivial" choice problems, and suggest that sometimes subjects choose like SEU (for example in the 3 sanity checks, but in other problems too), and sometimes make choices that are inconsistent with general models of complete preferences when beliefs are imprecise.

It is not, for instance, lack of understanding on the part of subjects that is driving rejection of these models of complete preferences. The last column of Table 4 demonstrates that the inconsistency rate for subjects who answered at least four of the five quiz questions correctly on their first attempt is similar to the sample that includes all subjects. We draw two conclusions from this fact. First, the inconsistency rate of subjects who performed well on the quiz is a meaningful measure of inconsistency with general models of complete preferences. Second, subjects who did not perform well on the quiz behaved as if they had a good understanding of the interface. We believe that the latter is explained by two features of our experimental design and interface. First, during the quiz, subjects are not able to move on from a question until they answer it correctly, so they, in a sense, forced to learn the correct answer. Second, in each round, they are provided with dynamic text that explains the implications of their choices (see Figures 4 and 5 in Section C of the Supplemental Appendix.).

Analysis of response time data also supports the conclusion that our results are not driven by subjects who are rushing through the experiment. Subjects who made choices inconsistent with MPP spent more time adjusting their sliders. For example, when subjects made choices inconsistent with MPP in a round, it took them a median time of 42.5 seconds to finish interacting with the sliders; while when subjects made choices consistent with MPP in these questions, it took a median time of 31.0 seconds to reach a final decision using the

Table 5: Percentage of rounds explained by various models, conditional on randomizing in the binary choice question

Consistent with SEU (Indifference)	32.4 (36.0)
Consistent with CP (Convexity)	37.5 (38.3)
Inconsistent with CP (Incompleteness)	30.1 (25.7)

Subjects randomize in 1643 (686) out 2616 (1026) rounds or 62.8% (66.9%). The numbers in the parentheses report the percent of choices that satisfy the given theorems in non-sanity check rounds for subjects that passed all three sanity checks.

sliders. For CP, the median times are 39.6 seconds and 34.6 seconds, respectively.[32]

These results are also robust to several other explanations, including integration or mistakes/fatigue. A discussion of these issues and other robustness results are presented in Section D of the Supplemental Appendix.

5.2 Sources of Randomization

Subjects choose to mix in the binary choice question in 63% of all rounds and subjects choose to mix in 91% of the rounds in which at least one non-degenerate interval is chosen in the PE questions. Hence, there is a high correlation between imprecise beliefs – as indicated by non-singleton PEs – and the propensity to randomize among bets on which shape has the greater area.

As a result of our identification in Section 2, we are able to attribute mixing in the binary choice question to different sources. Table 5 indicates the extent to which choices can be rationalized by various models and places each round into one of three mutually exclusive and exhaustive categories. Conditional on mixing in the binary choice question, 32% of choices can be explained by indifference, i.e. reporting two degenerate PE intervals equal to 0.5, whereas an additional 38% can be explained by CP. Only 10% of these rounds can be rationalized by MPP. The remaining 30% of observations then fall into two categories: rounds with at least one non-degenerate interval where choices are inconsistent with CP (approximately 26% of these rounds) and, to a lesser extent, rounds where two degenerate intervals are reported and choices are not consistent with SEU or CP. Recall that this is a lower bound on the incidence of incomplete preferences, since even if choices are consistent with complete preferences, they do not necessarily validate this assumption.

[32]Detailed reaction time data that records the exact time at which they made any adjustment of any slider is available for 215 of our 218 subjects. Reaction time data for two subjects was lost due to a server error, and there are errors in one of the subject's reaction time data that seem to be due to a local error on their device.

5.3 Comparing Sets of Images

There are two sets of images that are particularly helpful for demonstrating the circumstances in which behavior is more likely to reject flexible models of complete preferences, and the qualitative differences between incompleteness and indifference. Each subject faces a round with two rectangles (Image 8 in Figure 2b), which we call the *rectangles of interest*, and a round with two squares (either Image 2 in Figure 2a or the nearly identical Image 2B in Figure 6 in Section C.3 of the Supplemental Appendix), which we call the *squares of interest*. Importantly, the areas of the two squares and rectangles of interest are essentially identical.[33]

The goal of these images is to investigate how subjects' behavior varies when they confront problems that are similar in terms of how challenging it is to identify the larger shape, but differ in their number of attributes – side-length in this case. The ability of subjects to identify the larger shape in the two pairs is roughly the same.[34] However, choices involving rectangles of interest exhibit a much higher rate of inconsistency with complete preferences than squares of interest: 51% versus 30% of rounds are inconsistent with MPP, and the rate of inconsistency with CP is 28% and 19%, respectively. This finding suggests a fundamental difference between the two sets of images. Moreover, the uncertainty regarding the difference between the squares of interest is more frequently attributed to indifference (subjects choosing two PEs of 0.5) as opposed to imprecise beliefs than for the rectangles of interest. For squares of interest, subjects report 0.5 for their two PEs 50% of the time versus only 18% of the time for rectangles of interest, whereas subjects reported at least one non-degenerate interval 32% of the time versus 54% of the time for squares and rectangles, respectively. Finally, median response time for rectangles of interest is 48.4 seconds while for squares it is 38.7 seconds. Our mechanism allows us to uncover through choices these two sources of uncertainty that would otherwise be indistinguishable from analysis of the success rate alone.[35]

Additional insights can also be derived from the comparison of behavior for other pairs of images as well. For example, while otherwise similar, the relative difference in the size of the right and left shapes increases from Image 3 to Image 9. As expected, so does the subjects' accuracy in betting correctly, increasing from 43.7% to 60.3%, respectively.

[33]The shapes in both Image 2 and 8 take up 35.45% and 34.63% of their circles, respectively, while in Image 2B they take up 34.95% and 34.38% of their circles, respectively.

[34]The average frequency of betting correctly on the squares and rectangles of interest are 58.1% and 55.3%, respectively, and are not statistically significantly different at the 10% level for either group based on paired t-test.

[35]These results are robust to differences in the sequence of images that precede the squares and rectangles of interest and to the order in which the squares and rectangles are seen. See Section C.3 in the Supplemental Appendix for more details.

Additionally, the relative difference in areas is the same in Image 6 to Image 12, yet the absolute difference is smaller for Image 12. Weber's Law predicts that the error rate should be higher for Image 12, but this is not confirmed in our data. Finally, a change in the size of the gray circle in which the shapes are framed appears to have no effect on the likelihood of rejection nor the betting accuracy. A detailed breakdown of the results by image can be found in Section D.7 of the Supplemental Appendix.

5.4 Tradeoff Between Completeness and Other Axioms

Thus far we have shown that if choices in a round are inconsistent with SEU then they often cannot be rationalized by the most general models of Convex Preferences (CP). More specifically, if randomization in the binary choice problem cannot be explained by SEU (i.e, through indifference), then it is almost as likely as not that the behavior is inconsistent with convex preferences. In this subsection, we try to provide an indication of the magnitude of these violations and evaluate whether complete preferences are the best tool for modeling behavior that is inconsistent with SEU. On the one hand, if complete and convex preferences are correct on average, or close to correct in general, then perhaps they are still the most convenient tool for analysis. On the other hand, if complete and convex models provide a poor fit, then perhaps a model of incomplete preferences could provide a better fit while, at the same time, maintaining some of the structural assumptions relaxed by the general models of complete preferences. It is thus natural to explore the empirical tradeoff between Completeness (COM) and the other axioms, and see what additional structure can be imposed without sacrificing accuracy when completeness is relaxed.

The Bewley (2002) model is the leading theory of incomplete preferences that focuses on imprecise beliefs, and has been successfully applied to study important economic environments (Rigotti & Shannon, 2005; Lopomo, Rigotti, & Shannon, 2011, 2022). In our context, the model assumes that the DM has a convex set of beliefs regarding the likelihood that one shape is larger than the other. The DM then prefers to bet on one shape over another only if it is larger according to *all* beliefs in the set. Otherwise, in its original form, the model assumes that the DM will select the "status quo option". As there is no "status quo" in our setting, we assume that the DM will instead randomize when they cannot rank the bets. The amended Bewley model predicts that if the DM reports an interval $[a, b] \subseteq [0, 1]$ for the bet on one shape, then they report an interval of $[1-b, 1-a]$ for the bet on other shape (ignoring discreteness issues).

The Bewley (2002) model provides an excellent comparison for both MPP and CP because its characterization (Gilboa et al., 2010) can be used to show that (if WP is assumed)

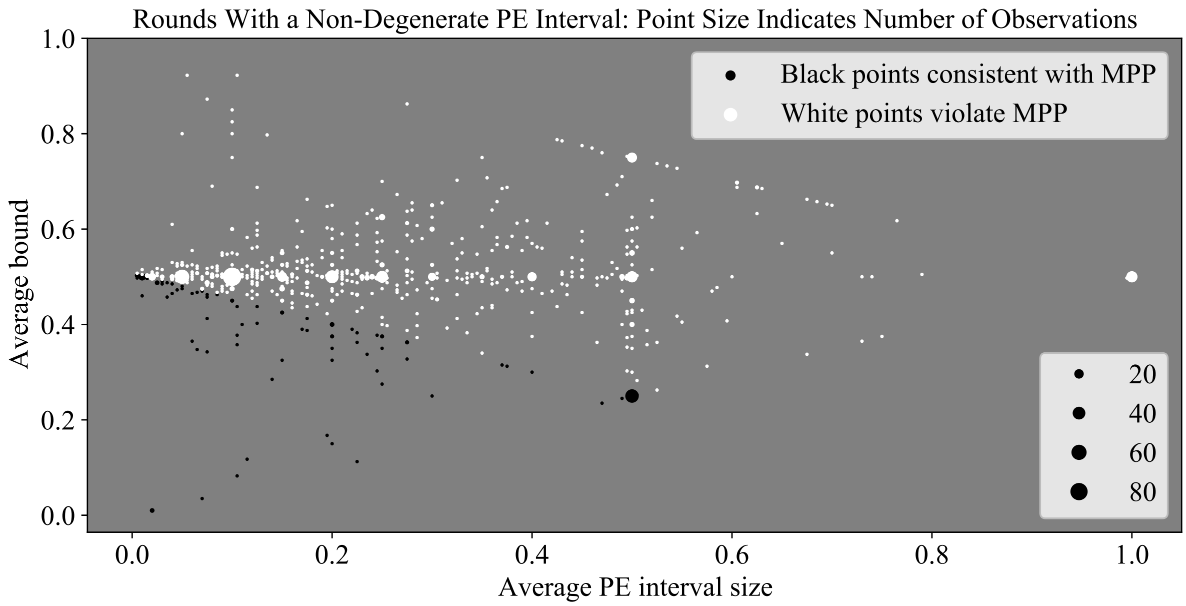


Figure 3: Empirical distribution of average bounds and interval sizes

PE intervals that are inconsistent with MPP are depicted in white, while those that are consistent are depicted in black.

it imposes strictly more structure than MPP and CP (other than that COM is relaxed). In particular, the Bewley model imposes Transitivity (TR), Independence, and Continuity. We thus employ the Bewley model to demonstrate the tradeoff between Completeness and other standard axioms.

To begin with, we have shown that the intervals of PEs we observe are much more likely to be inconsistent with MPP than with CP. This difference may be interpreted as evidence that the WCI axiom is responsible for the high rejection rate of MPP. On the other hand, if the Bewley model, which imposes an abundance of structure (including Independence), performs well relative to MPP in our data, then it would suggest that the Completeness assumption may be the culprit. In other words, the additional structural assumptions may only be problematic in conjunction with Completeness.

The MPP model is so frequently rejected in our data because in rounds with a non-degenerate PE interval, the average sum of the upper bounds for the two shapes is 1.26, well above the constraint of 1 imposed by Theorem 1. The MPP model is therefore incorrect "on average." As shown by Corollary 1 in Section B.1 of the Supplemental Appendix, Theorem 1 requires that a larger PE interval requires a lower average of the four bounds of the PE intervals (l_L, u_L, l_R, and u_R). This prediction is in stark contrast with that of the Bewley model, which predicts that the average of the four bounds should always be 0.5, no matter how wide the intervals are.

Figure 3 depicts the distribution of average bounds and interval widths that we observe in our data in rounds with a non-degenerate PE. Only about 10 percent of the rounds with non-degenerate PEs (depicted in black) are consistent with the MPP restrictions, and the observations that violate MPP are typically far from the range of values consistent with the constraint imposed by MPP. The Bewley model, in contrast, performs extremely well on average, as can be surmised from Figure 3. The horizontal line at 0.5 is highly populated, with the majority (51%) of average bounds between 0.49 and 0.51. Thus, by dropping COM a significant amount of additional structure, including Independence, can be maintained while, at the same time, improving the goodness-of-fit in our data relative to MPP. Hence, in addition to the Bewley model providing a more tractable and accurate alternative to MPP, this suggests that COM may be more responsible for the observed violations of MPP than the additional behavioral assumptions required by this class, WCI in particular.

The above insights should be taken with caution. Our experimental setup was not designed to test a particular theory of incomplete preferences. Although the amended Bewley (2002) model is superficially promising, all we intend to highlight here is that it maintains a lot of structure relative to the general classes of complete preferences we have identified, but appears to fit choices better.

6 Alternative Hypotheses

As discussed up to this point, our experiment was designed to evaluate the empirical validity of the completeness axiom. The current section presents and evaluates two models that could, in principle, generate randomization.

6.1 Additive Perturbed Utility

Fudenberg, Iijima, and Strzalecki (2015) propose a model of stochastic choice in which the DM chooses a distribution over the options in a menu in order to maximize their expected utility minus a convex (perturbation) function that depends only on the chosen probability of choosing each option (and is independent of the menu considered). An implication of the model is that the DM may find it more costly to implement a deterministic rather than a stochastic choice rule, and hence it can rationalize deliberate randomization. According to this model, a DM will choose to randomize when the expected difference in value between the options is small relative to the cost of implementing them deterministically, or alternatively

– the benefit of randomizing, implied by the perturbation function.[36]

Recall that in our mechanism subjects are able to randomize not only over the two shapes in the binary choice question (as in the deliberate randomization literature), but also over a bet on a given shape - f, and ranges of objective probabilities of winning the prize - r, by including them in the PE interval. To understand the implications of this model in our setting, suppose that in a particular PE question the DM reports a degenerate interval equal to some objective lottery $r \in (0, 1)$, and let $r' := r - 0.01$.[37] Since, r' is not included in the PE interval, it follows that betting on f with probability 1 is preferred to randomizing over f and r' with equal probabilities. These choices bound the cost of deterministic choice (i.e., the benefit of randomizing) from above by the utility difference between the objective lotteries of winning the prize with probabilities r and r', which could not be very large. Moreover, since the cost depends only on the probability of choosing the options (in our case: $0, \frac{1}{2}, 1$), and not on the menu itself, it is identical for all f and r and for all decision problems. Therefore, this model is incompatible with the prevalence of both degenerate and non-degenerate intervals observed in our data at the individual level.

6.2 Minimax Regret Model

The Minimax-Regret model (Savage, 1951) is also a candidate for rationalizing our data. Suppose the DM has an interval $[\underline{p}, \overline{p}] \subseteq [0, 1]$ of possible probabilities for the state $s = \lambda$, i.e. that the shape on the left is larger, and chooses an act f from the set of available acts F, which includes the options r (the objective lottery), L, or randomizing between the two, so as to minimize their regret:

$$V(f) = \max_{p \in [\underline{p}, \overline{p}],\, g \in F} \left[\Big(pg(\lambda) + (1-p)g(\rho) \Big) - \Big(pf(\lambda) + (1-p)f(\rho) \Big) \right].$$

This structure implies that subjects in our experiment would report intervals whenever their interval of possible beliefs $[\underline{p}, \overline{p}]$ is non-degenerate, but it also imposes structure onto what the intervals can look like, since: $\overline{p} = 2u_L - l_L \leq 1$ and $\underline{p} = 2l_L - u_L \geq 0$. In our data, when a non-degenerate interval of PEs is provided for a shape, it is inconsistent with the conditions imposed by the Minimax-Regret model 73% of the time. One of the main reasons we observe so many rejections of the Minimax-Regret model in rounds with non-degenerate intervals is because it implies that intervals are relatively small, in particular $\overline{p} - \underline{p} = 3(u_L - l_L)$, so $u_L - l_L < 0.34$, but our intervals in rounds with non-degenerate

[36] Due to the convex nature of the perturbation function, the cost of randomizing is necessarily smaller than choosing any option with probability 1.

[37] The boundary cases of choosing 0 or 1 follow a similar logic with the obvious adjustments.

intervals are quite large, about 27 percentage points on average.

7 Conclusion

The assumption that decision makers can always rank all alternatives seems to many scholars normatively and descriptively unappealing. However, the revealed preference approach that has been the workhorse of economic research has made this assumption especially challenging to evaluate empirically. The current work proposes a methodology to evaluate this assumption using an incentivized mechanism in which choices of decision makers with complete preferences would be set-identified. Choices outside of this set are indicative of incomplete preference.

We apply this methodology to the domain of choice under uncertainty, where beliefs may be imprecise, in which very general models of complete preference have been developed to accommodate ambiguity-sensitive behavior. We let subjects choose between bets and elicit their PEs, allowing them to express an interval of such equivalents. We also investigate subjects' desire to randomize and if and how it is related to incompleteness. We find that although in most decision problems subjects' choices are consistent with Subjective Expected Utility, in many other cases – in which subjects choose intervals of PEs – their choices are incompatible with very general classes of complete preferences.

Much of behavioral economics has been devoted to studying environments in which decisions are difficult and choices are inconsistent with a "standard" model of preferences. The response in face of such evidence (ever since the St. Petersburg paradox and Daniel Bernoulli's resolution of it using logarithmic utility) has been to propose more general models of complete preference. We hypothesize that much of the experimental evidence emerges in situations where decisions are difficult, and agents find it challenging to rank the alternatives they face. Exactly in this twilight zone, decision makers may rely on procedures, algorithms, and heuristics to make choices. For example, in the present investigation, a decision maker who could not rank a bet and a lottery chose to randomize between the two. Once we view randomization as a procedure the decision maker employs to make a choice, it becomes natural to expect that in other environments they may use the same or other similar procedures. The behavioral (complete) preferences that have been proposed to rationalize these difficult decisions may have captured some of the procedures' properties (just like Convex Preferences in the current investigation), but ignore the context in which they are applied by universally applying them. Naturally, more work is needed to extend the current paper's methodology to other domains, such as choice over time, interpersonal, and interactive decisions. We believe that it promises to provide a new perspective on behavioral economics by connecting to models of bounded and procedural rationality on which decision makers rely in making difficult decisions.

A Proofs

Proof of Theorem 1: Assume by negation that $u_L + u_R > 1$, and notice that this implies $\min\{u_R, u_L\} > 0$. As a first step we show that for $f \in \{L, R\}$ for all small $\epsilon > 0$ there exists $z_f \in [u_f - \epsilon, u_f]$ such that $\frac{1}{2}z_f + \frac{1}{2}f \succeq z_f$. Consider two cases:
If $l_f < u_f$ for $f \in \{L, R\}$, and such z_f does not exist then COM implies that for all random lotteries $r \in [u_f - \epsilon, u_f]$ we have $r \succ \frac{1}{2}r + \frac{1}{2}f$, and thus the DM could do strictly better by reducing their upper bound for f by ϵ since then if a random lottery $r \in (u_f - \epsilon, u_f]$ is drawn, which has a strictly positive probability of happening, the DM would then get r instead of $\frac{1}{2}r + \frac{1}{2}f$, and be strictly better off since $r \succ \frac{1}{2}r + \frac{1}{2}f$.
If $u_f = l_f$ for $f \in \{L, R\}$, and such z_f does not exist then COM implies that for all random lotteries $r \in [u_f - \epsilon, u_f]$ we have $r \succ \frac{1}{2}r + \frac{1}{2}f$, which combined with COM and CON implies that $r \succ f$ (otherwise COM says $f \succeq r$ and $r \succeq r$ so CON says $\frac{1}{2}r + \frac{1}{2}f \succeq r$, contradicting the assumption that such z_f does not exist) and thus $\frac{1}{2}r + \frac{1}{2}f \succeq f$ by CON (since $f \succeq f$ by COM). It follows that the DM could do strictly better off by reducing both u_f and l_f by ϵ as then: they get $\frac{1}{2}r + \frac{1}{2}f$ instead of f, which is weakly better – if the random lottery drawn is $r = u_f - \epsilon$, r instead of f, which is strictly better – if the random lottery is $r \in (u_f - \epsilon, u_f)$ a set that has a strictly positive chance of happening, and r instead of $\frac{1}{2}r + \frac{1}{2}f$, which is strictly better – if the random lottery is $r = u_f$. This concludes the first step. Hence, COM and CON imply that for all small $\epsilon > 0$ there exist $z_L \in [u_L - \epsilon, u_L]$ and $z_R \in [u_R - \epsilon, u_R]$ such that:

$$\frac{1}{2}z_R + \frac{1}{2}R \succeq z_R \text{ and } \frac{1}{2}z_L + \frac{1}{2}L \succeq z_L.$$

WCI then implies that:

$$\frac{1}{2}z_L + \frac{1}{2}R \succeq \frac{1}{2}z_R + \frac{1}{2}z_L \text{ and } \frac{1}{2}z_R + \frac{1}{2}L \succeq \frac{1}{2}z_L + \frac{1}{2}z_R.$$

and using CON:

$$\frac{1}{2}\Big(\frac{1}{2}z_L + \frac{1}{2}R\Big) + \frac{1}{2}\Big(\frac{1}{2}z_R + \frac{1}{2}L\Big) = \frac{1}{4}z_R + \frac{1}{4}z_L + \frac{1}{4}L + \frac{1}{4}R \succeq \frac{1}{2}z_R + \frac{1}{2}z_L.$$

Note that $\frac{1}{4}L(s) + \frac{1}{4}R(s) = \frac{1}{4}$ for all $s \in S$, so $\frac{1}{4}L + \frac{1}{4}R = \frac{1}{4} \in X$. Thus, if we pick ϵ small enough so that $z_L + z_R > 1$ (which is possible since we assumed that $u_L + u_R > 1$), we contradict WP since:

$$\frac{1}{4}z_R + \frac{1}{4}z_L + \frac{1}{4}L + \frac{1}{4}R = \frac{1}{2}\Big(\frac{1}{2}z_R + \frac{1}{2}z_L\Big) + \frac{1}{2}\Big(\frac{1}{2}\Big) < \frac{1}{2}z_R + \frac{1}{2}z_L.$$

□

Proof of Theorem 2: Without loss of generality assume $f = R$. COM, CON, and the DM's choice of how to randomize over L and R tells us $\frac{3}{4}R + \frac{1}{4}L = \frac{1}{2}R + \frac{1}{2}\frac{1}{2} \succeq \frac{1}{2}R + \frac{1}{2}L = \frac{1}{2}$. Then, STR and WCI tell us $\frac{1}{2}R + \frac{1}{2}z \succ z$ for $z \in [0, \frac{1}{2})$, and thus the DM can strictly benefit by increasing u_R to $\frac{1}{2}$ if $u_R < \frac{1}{2}$.

COM, CON, and the DM's choice of how to randomize over L and R tells us $\frac{1}{2}R + \frac{1}{2}L = \frac{1}{2} \succeq \frac{3}{4}R + \frac{1}{4}L = \frac{1}{2}R + \frac{1}{2}\frac{1}{2}$, $\frac{1}{2}R + \frac{1}{2}L = \frac{1}{2} \succeq R$, and $\frac{3}{4}R + \frac{1}{4}L = \frac{1}{2}R + \frac{1}{2}\frac{1}{2} \succeq R$. Then, STR and WCI tell us $z \succ \frac{1}{2}R + \frac{1}{2}z$, $z \succ R$, and $\frac{1}{2}R + \frac{1}{2}z \succ R$, for $z \in (\frac{1}{2}, 1]$, and thus the DM can strictly benefit by decreasing u_R (and l_R if $l_R > \frac{1}{2}$) to $\frac{1}{2}$ if $u_R > \frac{1}{2}$. □

Proof of Theorem 3: Without loss of generality assume $f = R$. Assume $l_R < \frac{1}{2}$ and we will reach a contradiction. The DM's selection of R tells us $R \succeq \frac{1}{2}L + \frac{1}{2}R = \frac{1}{2}$ by COM, thus $\frac{1}{2}\frac{1}{2} + \frac{1}{2}R \succeq \frac{1}{2}$ by COM and CON, $R \succ x$, for $x < \frac{1}{2}$ by STR, and the DM's selection of R also tells us $R \succeq \frac{1}{4}L + \frac{3}{4}R = \frac{1}{2}\frac{1}{2} + \frac{1}{2}R$ by COM, so, $R \succ \frac{1}{2}x + \frac{1}{2}R$, for $x < \frac{1}{2}$ by STR. But, then the DM can do strictly better by increasing l_R (as well as u_R if $u_R < \frac{1}{2}$) to $\frac{1}{2}$.

Assume $l_R > \frac{1}{2}$ and we will reach a contradiction. The DM's selection of $\alpha R + (1-\alpha)L$ tells us $\alpha R + (1-\alpha)L \succeq R$ by COM, which means $\frac{3}{4}R + \frac{1}{4}L = \frac{1}{2}R + \frac{1}{2}\frac{1}{2} \succeq R$ by COM and CON, because letting $\beta = \frac{1}{4(1-\alpha)}$ it is evident $\beta(\alpha R + (1-\alpha)L) + (1-\beta)R = \frac{3}{4}R + \frac{1}{4}L = \frac{1}{2}R + \frac{1}{2}\frac{1}{2}$. So, $\frac{1}{2}R + \frac{1}{2}x \succ R$ for $x > \frac{1}{2}$ by STR, and the DM strictly benefits from reducing l_R to $\frac{1}{2}$. □

Proof of Theorem 4: Assume by negation that the inequality does not hold, and notice that this implies $\min\{u_L, u_R\} > 0$, and we will reach a contradiction. Since we assumed by negation that:

$$\frac{u_L + u_R}{4} + \frac{1}{4} < \min\{u_L, u_R\},$$

there exists $\epsilon > 0$ such that $\frac{1}{4}(u_L + u_R) + \frac{1}{4} < \min\{u_L, u_R\} - \epsilon$. Pick z_L and z_R as in the proof of Theorem 1 given ϵ, note that $\frac{1}{4}L(s) + \frac{1}{4}R(s) = \frac{1}{4}$ for all $s \in S$, and that WSTR then implies:

$$\frac{1}{2}z_R + \frac{1}{2}R \succeq \min\{z_L, z_R\} \text{ and } \frac{1}{2}z_L + \frac{1}{2}L \succeq \min\{z_L, z_R\}.$$

It follows from CON that

$$\frac{1}{2}\Big(\frac{1}{2}z_R + \frac{1}{2}R\Big) + \frac{1}{2}\Big(\frac{1}{2}z_L + \frac{1}{2}L\Big) = \frac{z_L + z_R}{4} + \frac{1}{4} \succeq \min\{z_L, z_R\},$$

But then WSTR implies that:

$$\frac{u_L + u_R}{4} + \frac{1}{4} \succeq \min\{z_L, z_R\} \text{ and } \frac{u_L + u_R}{4} + \frac{1}{4} \succeq \min\{u_L, u_R\} - \epsilon,$$

which contradicts WP and the fact that $\frac{1}{4}(u_L + u_R) + \frac{1}{4} < \min\{u_L, u_R\} - \epsilon$. □

References

Agranov, M., & Ortoleva, P. (2017). Stochastic choice and preferences for randomization. *Journal of Political Economy*, *125*(1), 40–68.

Agranov, M., & Ortoleva, P. (2022). Revealed preferences for randomization: An overview. In *Aea papers and proceedings* (Vol. 112, pp. 426–430).

Agranov, M., & Ortoleva, P. (in press). Ranges of randomization. *Review of Economics and Statistics*.

Aumann, R. J. (1962). Utility theory without the completeness axiom. *Econometrica*, *30*(3), 445–462.

Baillon, A., Halevy, Y., & Li, C. (2022a). Experimental elicitation of ambiguity attitude using the random incentive system. *Experimental Economics*, *25*(3), 1002–1023.

Baillon, A., Halevy, Y., & Li, C. (2022b). Randomize at your own risk: on the observability of ambiguity aversion. *Econometrica*, *90*(3), 1085–1107.

Baillon, A., Huang, Z., Selim, A., & Wakker, P. P. (2018). Measuring ambiguity attitudes for all (natural) events. *Econometrica*, *86*(5), 1839–1858.

Becker, G. M., DeGroot, M. H., & Marschak, J. (1964). Measuring utility by a single-response sequential method. *Behavioral science*, *9*(3), 226–232.

Bewley, T. F. (2002). Knightian decision theory. part i. *Decisions in Economics and Finance*, *25*(2), 79–110.

Butler, D. J. (2000). Do non-expected utility choice patterns spring from hazy preferences? an experimental study of choice errors. *Journal of Economic Behavior & Organization*, *41*(3), 277–297.

Butler, D. J., & Loomes, G. C. (2007). Imprecision as an account of the preference reversal phenomenon. *American Economic Review*, *97*(1), 277–297.

Caplin, A., Dean, M., & Leahy, J. (2019). Rational inattention, optimal consideration sets, and stochastic choice. *The Review of Economic Studies*, *86*(3), 1061–1094.

Caplin, A., Dean, M., & Leahy, J. (2022). Rationally inattentive behavior: characterizing and generalizing shannon entropy. *Journal of Political Economy*, *130*(6), 1676–1715. doi: https://doi.org/10.1086/719276

Cerreia-Vioglio, S., Dillenberger, D., Ortoleva, P., & Riella, G. (2019). Deliberately stochastic. *American Economic Review*, *109*(7), 2425–45.

Cerreia-Vioglio, S., Maccheroni, F., Marinacci, M., & Montrucchio, L. (2011). Uncertainty averse preferences. *Journal of Economic Theory*, *146*(4), 1275–1330.

Cettolin, E., & Riedl, A. (2019). Revealed preferences under uncertainty: Incomplete preferences and preferences for randomization. *Journal of Economic Theory*, *181*, 547–585.

Chen, D. L., Schonger, M., & Wickens, C. (2016). otree—an open-source platform for laboratory, online, and field experiments. *Journal of Behavioral and Experimental Finance*, *9*, 88–97.

Chew, S. H., Miao, B., Shen, Q., & Zhong, S. (2022). Multiple-switching behavior in choice-list elicitation of risk preference. *Journal of Economic Theory*, *204*, 105510.

Costa-Gomes, M. A., Cueva, C., Gerasimou, G., & Tejiščák, M. (2022). Choice, deferral, and consistency. *Quantitative Economics*, *13*(3), 1297–1318.

Cubitt, R. P., Navarro-Martinez, D., & Starmer, C. (2015). On preference imprecision. *Journal of Risk and Uncertainty*, *50*(1), 1–34.

Danan, E., & Ziegelmeyer, A. (2006). *Are preferences complete? an experimental measurement of indecisiveness under risk* (resreport No. 2006-01). Max Planck Institue of Economics, Strategic Interaction Group. Retrieved from https://ideas.repec.org/p/esi/discus/2006-01.html

de Clippel, G., Moscariello, P., Ortoleva, P., & Rozen, K. (2025). *Caution in the face of complexity* (Tech. Rep.). Mimeo.

Denti, T., & Pomatto, L. (2022). Model and predictive uncertainty: A foundation for smooth ambiguity preferences. *Econometrica*, *90*(2), 551–584.

Dwenger, N., Kübler, D., & Weizsäcker, G. (2018). Flipping a coin: Evidence from university applications. *Journal of Public Economics*, *167*, 240–250.

Eliaz, K., & Ok, E. A. (2006). Indifference or indecisiveness? choice-theoretic foundations of incomplete preferences. *Games and Economic Behavior*, *56*(1), 61–86.

Ellis, A., & Freeman, D. J. (2024). Revealing choice bracketing. *American Economic Review*, *114*(9), 2668–2700.

Enke, B., & Graeber, T. (2023). Cognitive uncertainty. *The Quarterly Journal of Economics*, *138*(4), 2021–2067.

Faro, J. H. (2015). Variational bewley preferences. *Journal of Economic Theory*, *157*, 699–729.

Feldman, P., & Rehbeck, J. (2022). Revealing a preference for mixtures: An experimental study of risk. *Quantitative Economics*, *13*(2), 761–786.

Freeman, D. J., Halevy, Y., & Kneeland, T. (2019). Eliciting risk preferences using choice lists. *Quantitative Economics*, *10*(1), 217–237.

Fudenberg, D., Iijima, R., & Strzalecki, T. (2015). Stochastic choice and revealed perturbed utility. *Econometrica*, *83*(6), 2371–2409.

Galaabaatar, T., & Karni, E. (2013). Subjective expected utility with incomplete preferences. *Econometrica*, *81*(1), 255–284.

Gerasimou, G. (2025). *Eliciting and distinguishing between weak and incomplete preferences: Theory, experiment and computation* (resreport). University of Glasgow.

Ghirardato, P., Maccheroni, F., & Marinacci, M. (2004). Differentiating ambiguity and ambiguity attitude. *Journal of Economic Theory*, *118*(2), 133–173.

Gilboa, I., Maccheroni, F., Marinacci, M., & Schmeidler, D. (2010). Objective and subjective rationality in a multiple prior model. *Econometrica*, *78*(2), 755–770.

Gilboa, I., & Schmeidler, D. (1989). Maxmin expected utility with non-unique prior. *Journal of Mathematical Economics*, *18*(2), 141–153.

Greiner, B. (2015). Subject pool recruitment procedures: organizing experiments with orsee. *Journal of the Economic Science Association*, *1*(1), 114–125.

Grether, D. M. (1981). *Financial incentive effects and individual decisionmaking* (Tech. Rep.). California Institution of Technology.

Karni, E. (2009). A mechanism for eliciting probabilities. *Econometrica*, *77*(2), 603–606.

Karni, E. (2022). Incomplete risk attitudes and random choice behavior: an elicitation mechanism. *Theory and Decision*, *92*(3), 677–687.

Karni, E. (2024). Irresolute choice behavior. *International Journal of Economic Theory*, *20*(1), 70–87.

Khaw, M. W., Li, Z., & Woodford, M. (2021). Cognitive imprecision and small-stakes risk aversion. *The Review of Economic Studies*, *88*(4), 1979–2013.

Klibanoff, P., Marinacci, M., & Mukerji, S. (2005). A smooth model of decision making under ambiguity. *Econometrica*, *73*(6), 1849–1892.

Lopomo, G., Rigotti, L., & Shannon, C. (2011). Knightian uncertainty and moral hazard. *Journal of Economic Theory*, *146*(3), 1148–1172.

Lopomo, G., Rigotti, L., & Shannon, C. (2022). Uncertainty and robustness of surplus extraction. *Journal of Economic Theory*, *199*, 105088.

Maccheroni, F., Marinacci, M., & Rustichini, A. (2006). Ambiguity aversion, robustness, and the variational representation of preferences. *Econometrica*, *74*(6), 1447–1498.

Machina, M. J. (1985). Stochastic choice functions generated from deterministic preferences over lotteries. *The Economic Journal*, *95*(379), 575–594.

Matějka, F., & McKay, A. (2015). Rational inattention to discrete choices: A new foundation for the multinomial logit model. *American Economic Review*, *105*(1), 272–98.

Nau, R. (2006). The shape of incomplete preferences. *The Annals of Statistics*, *34*(5), 2430–2448.

Nielsen, K., & Rigotti, L. (2024). *Revealed incomplete preferences* (Tech. Rep.).

Ok, E. A., Ortoleva, P., & Riella, G. (2012). Incomplete preferences under uncertainty: Indecisiveness in beliefs versus tastes. *Econometrica*, *80*(4), 1791–1808.

Ok, E. A., & Tserenjigmid, G. (2022). Indifference, indecisiveness, experimentation, and stochastic choice. *Theoretical Economics*, *17*(2), 651–686.

Riella, G. (2015). On the representation of incomplete preferences under uncertainty with indecisiveness in tastes and beliefs. *Economic Theory*, *58*(3), 571–600.

Rigotti, L., & Shannon, C. (2005). Uncertainty and risk in financial markets. *Econometrica*, *73*(1), 203–243.

Savage, L. J. (1951). The theory of statistical decision. *Journal of the American Statistical association*, *46*(253), 55–67.

Schmeidler, D. (1989). Subjective probability and expected utility without additivity. *Econometrica: Journal of the Econometric Society*, 571–587.

Sopher, B., & Narramore, J. M. (2000). Stochastic choice and consistency in decision making under risk: An experimental study. *Theory and Decision*, *48*(4), 323–350.

Tversky, A. (1972). Elimination by aspects: a theory of choice. *Psychological Review*, *79*(4), 281–299.

Woodford, M. (2020). Modeling imprecision in perception, valuation, and choice. *Annual Review of Economics*, *12*, 579–601.

Supplemental Appendix

B More Theory Results

The results below use the axioms from Section 2, which are provided here for the reader's convenience.

Winning is Preferred (WP) For all $x, y \in X$, if $y > x$ then $y \succ x$.

Completeness (COM) For all $f, g \in \mathcal{F}$, either $f \succeq g$ or $g \succeq f$.

Transitivity (TR) For all $f, g, h \in \mathcal{F}$, if $f \succeq g$ and $g \succeq h$, then $f \succeq h$.

Statewise Transitivity (STR) For all $f, g, h \in \mathcal{F}$ such that $f \succeq g$, if h statewise dominates f then $h \succ g$, and if g statewise dominates h then $f \succ h$.

Weak Statewise Transitivity (WSTR) For all $f, g, h \in \mathcal{F}$ such that $f \succeq g$, if h statewise dominates f then $h \succeq g$, and if g statewise dominates h then $f \succeq h$.

Convexity (CON) For all $f, g, h \in \mathcal{F}$, if $f, h \succeq g$, then for all $\alpha \in (0, 1)$: $\alpha f + (1-\alpha)h \succeq g$.[38]

Certainty Independence (CI) (Gilboa & Schmeidler, 1989) For all $f, g, \in \mathcal{F}$, $x \in X$, and $\alpha \in (0, 1)$: $f \succ g \iff \alpha f + (1-\alpha)x \succ \alpha g + (1-\alpha)x$.

Weak Certainty Independence (WCI) (Maccheroni et al., 2006) For all $f, g, \in \mathcal{F}$, if $x, y \in X$, then for all $\alpha \in (0, 1)$: $\alpha f + (1-\alpha)x \succeq \alpha g + (1-\alpha)x \implies \alpha f + (1-\alpha)y \succeq \alpha g + (1-\alpha)y$.

We begin with three simple lemmas that we use without citation throughout the paper due to their simple and intuitive nature. These lemmas show that the strict preference versions of TR and WCI are implied by TR and WCI respectively, and that the strict preference version of CON is implied by COM, CON, and TR.

Lemma 1. If the preferences of the DM satisfy TR, then for all $f, g, h \in \mathcal{F}$, if $f \succ g$ and $g \succeq h$, or $f \succeq g$ and $g \succ h$, then $f \succ h$.

Proof. First, assume $f \succ g$ and $g \succeq h$. TR tells us $f \succeq h$. If it is not the case that $f \succ h$ then $f \sim h$ and we can reach a contradiction since $g \succeq h$ and then TR tells us $g \succeq f$ which contradicts $f \succ g$.

[38] Our definition of convexity differs from the definition given by some other papers. For a clarification of how this definition of convexity relates to the definition given by, for instance, Cerreia-Vioglio et al. (2011), see Proposition 1 in Section B of the Supplemental Appendix and its proof, or Lemma 56 in their paper.

Second, assume $f \succeq g$ and $g \succ h$. TR tells us $f \succeq h$. If it is not the case that $f \succ h$ then $f \sim h$ and we can reach a contradiction since $f \succeq g$ and then TR tells us $h \succeq g$, which contradicts $g \succ h$. □

Lemma 2. If the preferences of the DM satisfy WCI, then for all $f,\ g, \in \mathcal{F},\ x,\ y \in X$, and $\alpha \in (0,\ 1)$:

$$\alpha f + (1-\alpha)x \succ \alpha g + (1-\alpha)x \implies \alpha f + (1-\alpha)y \succ \alpha g + (1-\alpha)y.$$

Proof. Assume not, so $\alpha f+(1-\alpha)x \succ \alpha g+(1-\alpha)x$, and thus $\alpha g+(1-\alpha)y \sim \alpha f+(1-\alpha)y$ (because $\alpha f+(1-\alpha)x \succ \alpha g+(1-\alpha)x$, WCI tells us that $\alpha f+(1-\alpha)y \succeq \alpha g+(1-\alpha)y$, so, if it is not the case that $\alpha f+(1-\alpha)y \succ \alpha g+(1-\alpha)y$, then it must be that $\alpha g+(1-\alpha)y \sim \alpha f+(1-\alpha)y$), but then WCI tells us $\alpha g+(1-\alpha)x \succeq \alpha f+(1-\alpha)x$, and we immediately have a contradiction with $\alpha f+(1-\alpha)x \succ \alpha g+(1-\alpha)x$. □

Lemma 3. If the preferences of the DM satisfy COM, CON, and TR, then for all $f,\ g,\ h \in \mathcal{F}$, if $f,\ h \succ g$, then for all $\alpha \in (0,\ 1)$: $\alpha f + (1-\alpha)h \succ g$.

Proof. Suppose $f,\ h \succ g$. COM tells us either $f \succeq h$ or $h \succeq f$. Without loss of generality assume $f \succeq h$. Then CON tell us that for all $\alpha \in (0,\ 1)$: $\alpha f + (1-\alpha)h \succeq h$ and thus TR (and Lemma 1) tells us for all $\alpha \in (0,\ 1)$: $\alpha f + (1-\alpha)h \succ g$. □

The following proposition relates our axioms to those of some of the major models of choice under uncertainty (with complete preferences).

Proposition 1. *Given X and S as defined above, and WP:*

(i) COM, WSTR, CON, and TR, are implied by the assumptions of the Uncertainty Averse Preferences (UAP) model (axioms A.1 through A.5 from the work of Cerreia-Vioglio et al. (2011)),

(ii) STR and WCI, and the assumptions of the UAP model, are implied by the Variational Preferences (VP) model (axioms A.1 through A.6 from the work of Maccheroni et al. (2006)),

(iii) COM, STR, and TR, are implied by the Smooth Ambiguity Preferences model (Klibanoff et al., 2005; Denti & Pomatto, 2022),

(iv) *CON is implied by the Smooth Ambiguity Averse Preferences (SAAP) model (Klibanoff et al., 2005; Denti & Pomatto, 2022).*

Proof. Define X and S as in Section 2, and assume WP. COM and TR are together strictly weaker than axiom A.1 (Weak Order) in the work of Cerreia-Vioglio et al. (2011) and are together strictly weaker than Axiom 1 in the work of Denti and Pomatto (2022). WSTR is implied by axioms A.1 (Weak Order) and A.2 (Monotonicity) in the work of Cerreia-Vioglio et al. (2011). CON is implied by A.1 (Weak Order), A.3 (Convexity), and A.5 (Continuity), in the work of Cerreia-Vioglio et al. (2011), as is shown by Lemma 56 in their work.

Axioms A.1 through A.6 in the work of Maccheroni et al. (2006) imply axioms A.1 through A.5 in the work of Cerreia-Vioglio et al. (2011), this implication is trivially true for each of the mentioned axioms from the work of Cerreia-Vioglio et al. (2011) except for A.4 (Risk Independence), whose implication is trivial given WP and our particular X. Further, WCI is axiom A.2 (Weak Certainty Independence) in the work of Maccheroni et al. (2006). The implication of STR via axioms A.1 through A.6 in the work of Maccheroni et al. (2006) is trivial given Theorem 3 from their work and WP.

STR is implied by Axiom 1 and Axiom 2 in the work of Denti and Pomatto (2022). Given Definition 1 in the work of Denti and Pomatto (2022), which provides a representation of Smooth Ambiguity Preferences, it is trivial to show that SAAP, which uses this representation and further imposes that the function ϕ is concave, satisfies CON.

□

Proposition 1 establishes that, given WP,[39] axioms COM, WSTR, and CON, are together weaker than (i.e., implied by) the UAP model and axioms COM, STR, CON, and WCI, are together weaker than (i.e., implied by) the VP model as both UAP and VP additionally assume TR (and Continuity) from the supplementary materials), and axioms COM, STR, and CON, are together weaker than (i.e., implied by) SAAP as SAAP additionally assumes TR and indirectly results in continuity being imposed.[40] Further, given WP, all of the axioms in this paper together (COM, STR, CON, TR, CI, and continuity from the supplementary materials) thus constitute the model of Maxmin Expected Utility (Gilboa & Schmeidler, 1989), as is discussed by Maccheroni et al. (2006).

[39] Proposition 2 in Section B.1 establishes that, in the context of our experiment, WP is a weak assumption to make relative to the quite general UAP model.

[40] Implication of continuity from the supplementary materials by the Smooth Ambiguity Preference model is established by Lemma 6 from the work of Denti and Pomatto (2022).

Next, we introduce a proposition that relates WP to the work of Cerreia-Vioglio et al. (2011). The proposition demonstrates that WP is a quite weak assumption about preferences.

Proposition 2. *Given X and S as defined above, if always winning is preferred to always losing, namely $y = 1 \succeq x = 0$, then WP is implied by axioms A.1, A.2, A.4, and A.5, from the work of Cerreia-Vioglio et al. (2011).*

Proof. Define X and S as in Section 2. Assume $y = 1 \succeq x = 0$ and that axioms A.1, A.2, A.4, and A.5, from the work of Cerreia-Vioglio et al. (2011) are satisfied. It must then be that $y = 1 \succ x = 0$, because if $y \sim x$ then Axiom A.1 (Weak Order, transitivity part) and Axiom A.4 (Risk Independence) can be used to show that for all $z \in [0, 1]$ we have $z \sim x \sim y$, because Axiom A.4 (Risk Independence) tells us $0 \sim 1 \Rightarrow 0 \sim \alpha$ for all $\alpha \in (0, 1)$. But, $z \sim x \sim y$ is not possible since Axiom A.1 (Weak Order, transitivity part) and Axiom A.2 (Monotonicity) would then tell us the DM is indifferent between all acts and that contradicts Axiom A.1 (Weak Order, nontrivial part), so $y = 1 \succ x = 0$.

Now assume that $\exists z, w \in X$ such that $z < w$ and $z \succeq w$, and we shall reach a contradiction. By Axiom A.1 (Weak Order, completeness part) either $x \succeq z$ or $z \succeq x$ and $x \succeq w$ or $w \succeq x$. If $x \succ z$ then there is $v > z$ such that $v \sim x$ by Axiom A.5 (Continuity) since $y \succ x \succ z$. If, instead, $z \succeq x$ and $x \succeq w$ then there is $v \geq z$ such that $v \sim x$ by Axiom A.5 (Continuity). If, instead, $z \succeq x$ and $w \succeq x$ then there is $v \leq z$ such that $v \sim w$ by Axiom A.5 (Continuity). In any of these cases we have $v, u \in X$ such that $v \neq u$ and yet $v \sim u$. This is problematic since then Axiom A.1 (Weak Order, transitivity part) and Axiom A.4 (Risk Independence) can be used to show if $t, s \in (0, 1)$ then $t \sim s$, since for $t \in (v, u)$ we have $t \sim v$ since $\alpha u + (1-\alpha)v \sim \alpha v + (1-\alpha)v = v$ for all $\alpha \in (0, 1)$, and then the indifference region $[v, u]$ (supposing without loss that $u > v$) can be iteratively expanded towards $(0, 1)$ since $\alpha v + (1-\alpha)0 \sim \alpha u + (1-\alpha)0$ and $\alpha v + (1-\alpha)1 \sim \alpha u + (1-\alpha)1$ for all $\alpha \in (0, 1)$. However, $t \sim s$ for all $t, s \in (0, 1)$ combined with the fact that $y = 1 \succ x = 0$ creates a contradiction with Axiom A.1 (Weak order, transitivity and completeness parts) and Axiom A.5 (Continuity). □

The rest of this appendix presents results in the continuous slider setting introduced in Sections 2 and 3.

B.1 Continuous Interval Results

Here, we introduce a corollary to Theorem 1. Let the **average bound** b_{av} be defined as $b_{av} = \frac{1}{4}(l_L + u_L + l_R + u_R)$. One can think of it (very loosely speaking) as the average of the two PEs, allowing for non-degenerate intervals. Similarly, the **average interval size** s_{av} is defined as $s_{av} = \frac{1}{2}(u_L - l_L + u_R - l_R)$, which is simply the average length of the PE intervals. The following describes the permissible relationship between the two, for a DM whose preferences belong to the Multiple Prior Preferences class.

Corollary 1. *If the DM's preferences satisfy WP, COM, CON, and WCI, then:*

$$b_{av} \leq \frac{1}{2} - \frac{1}{2}s_{av}.$$

Proof. If the DM's preferences satisfy WP, COM, CON, and WCI, then Theorem 1 tell us $u_L + u_R \leq 1$, so:

$$b_{av} = \frac{1}{2}(u_L + u_R) - \frac{1}{2}s_{av} \leq \frac{1}{2} - \frac{1}{2}s_{av}.$$

□

Corollary 1 establishes a monotonic bound on the relation between the two averaged variables for MPP. For example, if preferences are MEU (and hence satisfy CI) then $s_{av} = 0$ and therefore $b_{av} \leq \frac{1}{2}$. For more general multiple prior preferences, where the PE intervals might not be degenerate - the bound is even lower.

Next, we introduce some additional theorems not referred to in our results that impose restrictions onto DM behavior when their preferences satisfy different subsets of our axioms.

First, we define a class, called Weak Convex Preferences (WCP), which is characterized the set of axioms {WP, COM, WSTR, CON, TR}. The following theorem, in addition to Theorem 4 in Section 3 of the main text, demonstrates the testable implications of this class.

Theorem 5. *If the DM selects $f \in \{L, R\}$ when asked how they would like to randomize over L and R, their preferences satisfy WP, COM, WSTR, CON, and TR, and the DM isolates the three choice problems, then $u_f \geq \frac{1}{2}$.*

Proof. Without loss of generality assume $f = R$. Assume $u_R < \frac{1}{2}$ and we will reach a contradiction. The DM's selection of R tells us $R \succeq \frac{1}{4}L + \frac{3}{4}R = \frac{1}{2}\frac{1}{2} + \frac{1}{2}R$ by COM. So, $R \succeq \frac{1}{2}x + \frac{1}{2}R$, for $x < \frac{1}{2}$ by WSTR. The DM's selection of R also tells us $R \succeq \frac{1}{2}L + \frac{1}{2}R = \frac{1}{2}$ by COM, thus $\frac{1}{2}\frac{1}{2} + \frac{1}{2}R \succeq \frac{1}{2}$ by COM and CON, and $R \succ x$ for $x < \frac{1}{2}$ by WP and TR. But, then we have the desired contradiction since the DM can do strictly better by increasing both u_R and l_R to $\frac{1}{2}$. □

Note that, if one assumes that always winning is weakly preferred to always losing, i.e. $1 \succeq 0$, then WCP, characterized by Theorems 4 and 5, assumes less structure than Uncertainty Averse Preferences (UAP) (Cerreia-Vioglio et al., 2011). Additionally, UAP is included in CP so long as one assumes a stronger version of monotonicity: if $f(s) \succ g(s)$ for all s, then $f \succ g$; Axiom 2 from Denti and Pomatto (2022).

Theorem 6. *If the preferences of the DM satisfy WP, COM, WSTR, TR, and CI, and the DM isolates the three choice problems, then $l_L = u_L$ and $l_R = u_R$.*

Proof. Suppose the DM responds to the PE question about $f \in \{L, R\}$ with $u_f > l_f$. WP, COM, TR, CI, and the fact the the DM does not wish to decrease u_f tells us that for small $\epsilon > 0$ such that $u_f - \epsilon > l_f$ there is $z \in [u_f - \epsilon, u_f]$ such that $\frac{1}{2}f + \frac{1}{2}z \succ z = \frac{1}{2}z + \frac{1}{2}z \Rightarrow f \succ z$ because if not $u_f - \epsilon \succeq \frac{1}{2}f + \frac{1}{2}(u_f - \epsilon) \Rightarrow \frac{1}{2}(u_f - \epsilon) \succeq \frac{1}{2}f$ and thus for all $y \in (u_f - \epsilon, u_f]$ we have $\frac{1}{2}y \succ \frac{1}{2}f \Rightarrow y \succ \frac{1}{2}f + \frac{1}{2}y$ and the DM could do strictly better by lowering u_f. COM and the fact the the DM does not wish to increase l_f tells us that for all $\epsilon > 0$ there is $x \in [l_f, l_f + \epsilon]$ such that $\frac{1}{2}x + \frac{1}{2}f \succeq f$, so TR tells us $\frac{1}{2}x + \frac{1}{2}f \succ z$, and taking ϵ to zero COM and WSTR thus tell us $u_f < 1$. Thus, COM and the fact the the DM does not wish to increase u_f tells us that for all $\epsilon > 0$ there is $q \in [u_f, u_f + \epsilon]$ such that $q \succeq \frac{1}{2}f + \frac{1}{2}q$ and thus TR and CI tell us $\frac{1}{2}q \succeq \frac{1}{2}f \Rightarrow \frac{1}{2}q + \frac{1}{2}x \succeq \frac{1}{2}f + \frac{1}{2}x \succeq f \succ z \Rightarrow \frac{1}{2}q + \frac{1}{2}x \succ z$, which contradicts WP as ϵ goes to zero. □

Theorem 6 implies that a DM with α-MEU preferences (Ghirardato et al., 2004) will not report a non-degenerate PE interval. This is, of course, also true for MEU preferences as they are included in α-MEU.

B.1.1 Integration

We define the notion of integrating *across* questions. For each random lottery $r \in [0, 1]$ and each of the three choice questions indexed by $j \in \{1, 2, 3\}$, let $Q_j(r)$ denote the act assigned to the DM when the random lottery drawn is r and question j is used to determine their payment. We then say the DM is **integrating** if they answer a subset of more than one question $\mathcal{Q} \subseteq \{1, 2, 3\}$ in **conjunction**, which means that when the DM answers the subset of questions they have **weights** $\beta_j \geq 0$ for each $j \in \mathcal{Q}$ such that $\sum_{j \in \mathcal{Q}} \beta_j = 1$, and there exists no alternative way of answering the questions that would result in alternate acts $\tilde{Q}_j(r) \in \mathcal{F}$ for each $r \in [0, 1]$ and $j \in \mathcal{Q}$, such that there is an open interval of r with:

$$\sum_{j \in \mathcal{Q}} \beta_j \tilde{Q}_j(r) \succ \sum_{j \in \mathcal{Q}} \beta_j Q_j(r),$$

and $\forall\, r \in [0, 1]$:

$$\sum_{j \in \mathcal{Q}} \beta_j \tilde{Q}_j(r) \succeq \sum_{j \in \mathcal{Q}} \beta_j Q_j(r).$$

Theorem 7. *If the DM's preferences satisfy COM, STR, CON, and WCI, and they consider both PE questions in conjunction with weights* $\frac{1}{2}$*, then they do not give an interval that is contained in the interior of their other interval, and:* $\max\{u_R, u_L\} + \min\{l_L, l_R\} = 1$, $\min\{u_R, u_L\} \leq \max\{l_L, l_R\}$*, and either* $\min\{u_R, u_L\} = \max\{l_L, l_R\}$ *or* $\min\{u_R, u_L\} + \max\{l_L, l_R\} = 1$.

Proof. If for all $x \in (\frac{1}{2}, 1]$ the DM's preferences are such that $\frac{1}{4}x \succ \frac{1}{4}R$ and $\frac{1}{4}x \succ \frac{1}{4}L$, then COM, CON, and WCI, tell us $\frac{1}{2}x \succ \frac{1}{2}R$ and $\frac{1}{2}x \succ \frac{1}{2}L$ for all $x \in (\frac{1}{2}, 1]$, because if not $\frac{1}{2}R \succeq \frac{1}{2}x \Rightarrow \frac{1}{4}R + \frac{1}{4}x \succeq \frac{1}{2}x \Rightarrow \frac{1}{4}R \succeq \frac{1}{4}x$ and $\frac{1}{2}L \succeq \frac{1}{2}x \Rightarrow \frac{1}{4}L + \frac{1}{4}x \succeq \frac{1}{2}x \Rightarrow \frac{1}{4}L \succeq \frac{1}{4}x$, so we would already have a contradiction, thus, COM and CON tell us $\frac{1}{4}R + \frac{1}{4}x \succeq \frac{1}{2}R$ and $\frac{1}{4}L + \frac{1}{4}x \succeq \frac{1}{2}L$, and thus, since this is true for all such x, STR tells us $\frac{1}{4}R + \frac{1}{4}x \succ \frac{1}{2}R$ and $\frac{1}{4}L + \frac{1}{4}x \succ \frac{1}{2}L$ for all $x \in (\frac{1}{2}, 1]$ since for all $\tilde{x} \in (\frac{1}{2}, x)$ $\frac{1}{4}R + \frac{1}{4}\tilde{x} \succeq \frac{1}{2}R$ and $\frac{1}{4}L + \frac{1}{4}\tilde{x} \succeq \frac{1}{2}L$.

So, if $\frac{1}{4}x \succ \frac{1}{4}R$ and $\frac{1}{4}x \succ \frac{1}{4}L$ for all $x \in (\frac{1}{2}, 1]$ (assumed throughout this paragraph) then $l_L = u_L = l_R = u_R = \frac{1}{2}$. This is true because, to begin with, if $\min(l_L, l_R) < \frac{1}{2}$ there exists small $\epsilon > 0$ such that $\min(l_L, l_R) + \epsilon < \frac{1}{2}$ and for all $y \in [\min(l_L, l_R), \min(l_L, l_R) + \epsilon]$ COM, STR, and WCI, tell us:

$$\frac{1}{2} \succ \frac{1}{4}R + \frac{1}{2}\frac{1}{2} + \frac{1}{4}y \text{ because } \frac{1}{4}(1-y) \succ \frac{1}{4}R,$$

$$\frac{1}{2} \succ \frac{1}{4}L + \frac{1}{2}\frac{1}{2} + \frac{1}{4}y \text{ because } \frac{1}{4}(1-y) \succ \frac{1}{4}L,$$

$$\text{and } \frac{1}{2} \succ \frac{1}{2}\frac{1}{2} + \frac{1}{2}y,$$

so if the minimum lower bound is strictly less than the minimum upper bound the DM could strictly benefit from increasing their minimum lower bound (if $l_R > l_L$ then increasing l_L a small amount changes the chosen act locally from $\frac{1}{4}R + \frac{1}{2}(\frac{1}{2}R + \frac{1}{2}L) + \frac{1}{4}y = \frac{1}{4}R + \frac{1}{2}\frac{1}{2} + \frac{1}{4}y$ to $\frac{1}{2}R + \frac{1}{2}L = \frac{1}{2}$, while if $l_R < l_L$ then increasing l_R a small amount similarly changes the chosen act locally from $\frac{1}{4}L + \frac{1}{2}\frac{1}{2} + \frac{1}{4}y$ to $\frac{1}{2}$), or by increasing both if they are equal (if $l_R = l_L$ then increasing both l_R and l_L the same small amount changes the chosen act locally from $\frac{1}{2}(\frac{1}{2}R + \frac{1}{2}L) + \frac{1}{2}y = \frac{1}{2}\frac{1}{2} + \frac{1}{2}y$ to $\frac{1}{2}R + \frac{1}{2}L = \frac{1}{2}$), and COM, STR, and WCI, tell us (for $y < \frac{1}{2}$):

$$\frac{1}{2}\frac{1}{2} + \frac{1}{2}y \succ y,$$

$$\frac{1}{2}\frac{1}{2} + \frac{1}{2}y \succ \frac{1}{4}R + \frac{3}{4}y \text{ again because } \frac{1}{4}(1-y) \succ \frac{1}{4}R,$$

$$\frac{1}{2}\frac{1}{2} + \frac{1}{2}y \succ \frac{1}{4}L + \frac{3}{4}y \text{ again because } \frac{1}{4}(1-y) \succ \frac{1}{4}L,$$

$$\frac{1}{4}R + \frac{1}{2}\frac{1}{2} + \frac{1}{4}y \succ \frac{1}{2}R + \frac{1}{2}y \text{ because } \frac{1}{4}R + \frac{1}{4}(1-y) \succ \frac{1}{2}R,$$
$$\text{and } \frac{1}{4}L + \frac{1}{2}\frac{1}{2} + \frac{1}{4}y \succ \frac{1}{2}L + \frac{1}{2}y \text{ because } \frac{1}{4}L + \frac{1}{4}(1-y) \succ \frac{1}{2}L,$$

so the DM could strictly benefit from increasing their minimum upper bound(s) if one or more of them is equal to $\min(l_L,\, l_R)$ (if $u_L = u_R = \min(l_L,\, l_R) < \frac{1}{2}$ then increasing u_L and u_R the same small amount changes the chosen act locally from y to $\frac{1}{2}(\frac{1}{2}R + \frac{1}{2}L) + \frac{1}{2}y = \frac{1}{2}\frac{1}{2} + \frac{1}{2}y$, if $u_L = \min(l_L,\, l_R) < u_R$ then increasing u_L a small amount changes the chosen act locally, depending on if $u_L \geq l_R$ or $u_L < l_R$, from $\frac{1}{4}R + \frac{3}{4}y$ to $\frac{1}{2}(\frac{1}{2}R + \frac{1}{2}L) + \frac{1}{2}y = \frac{1}{2}\frac{1}{2} + \frac{1}{2}y$ or from $\frac{1}{2}R + \frac{1}{2}y$ to $\frac{1}{4}R + \frac{1}{2}(\frac{1}{2}R + \frac{1}{2}L) + \frac{1}{4}y = \frac{1}{4}R + \frac{1}{2}\frac{1}{2} + \frac{1}{4}y$, and if $u_R = \min(l_L,\, l_R) < u_L$ then increasing u_R a small amount changes the chosen act locally, depending on if $u_R \geq l_L$ or $u_R < l_L$, from $\frac{1}{4}L + \frac{3}{4}y$ to $\frac{1}{2}(\frac{1}{2}R + \frac{1}{2}L) + \frac{1}{2}y = \frac{1}{2}\frac{1}{2} + \frac{1}{2}y$ or from $\frac{1}{2}L + \frac{1}{2}y$ to $\frac{1}{4}L + \frac{1}{2}(\frac{1}{2}R + \frac{1}{2}L) + \frac{1}{4}y = \frac{1}{4}L + \frac{1}{2}\frac{1}{2} + \frac{1}{4}y$). Further, if $\max(l_L,\, l_R) > \frac{1}{2}$, since there exists small $\epsilon > 0$ such that $\max(l_L,\, l_R) - \epsilon > \frac{1}{2}$ and for all $y \in [\max(l_L,\, l_R) - \epsilon,\, \max(l_L,\, l_R)]$ COM, STR, and WCI, tell us (given what we showed in the previous paragraph):

$$\frac{1}{2}y + \frac{1}{2}\frac{1}{2} \succ \frac{1}{2},$$

$$\frac{1}{2}y + \frac{1}{2}\frac{1}{2} \succ \frac{1}{4}R + \frac{1}{2}\frac{1}{2} + \frac{1}{4}y \text{ because now (for these } y\text{) } \frac{1}{4}y \succ \frac{1}{4}R,$$
$$\frac{1}{2}y + \frac{1}{2}\frac{1}{2} \succ \frac{1}{4}L + \frac{1}{2}\frac{1}{2} + \frac{1}{4}y \text{ because now (for these } y\text{) } \frac{1}{4}y \succ \frac{1}{4}L,$$
$$\frac{1}{4}R + \frac{3}{4}y \succ \frac{1}{2}R + \frac{1}{2}y \text{ because now (for these } y\text{) } \frac{1}{4}R + \frac{1}{4}y \succ \frac{1}{2}R$$
$$\text{and } \frac{1}{4}L + \frac{3}{4}y \succ \frac{1}{2}L + \frac{1}{2}y \text{ because now (for these } y\text{) } \frac{1}{4}L + \frac{1}{4}y \succ \frac{1}{2}L,$$

so the DM could do strictly better by lowering $\max(l_L,\, l_R)$ (if $l_R > l_L$ then decreasing l_R a small amount changes the chosen act locally, depending on if $l_R > u_L$ or $l_R \leq u_L$, from $\frac{1}{2}R + \frac{1}{2}y$ to $\frac{1}{4}R + \frac{3}{4}y$ or from $\frac{1}{4}R + \frac{1}{2}(\frac{1}{2}R + \frac{1}{2}L) + \frac{1}{4}y = \frac{1}{4}R + \frac{1}{2}\frac{1}{2} + \frac{1}{4}y$ to $\frac{1}{2}y + \frac{1}{2}(\frac{1}{2}R + \frac{1}{2}L) = \frac{1}{2}y + \frac{1}{2}\frac{1}{2}$, and if $l_R < l_L$ then decreasing l_L a small amount changes the chosen act locally, depending on if $l_L > u_R$ or $l_L \leq u_R$, from $\frac{1}{2}L + \frac{1}{2}y$ to $\frac{1}{4}L + \frac{3}{4}y$ or from $\frac{1}{4}L + \frac{1}{2}(\frac{1}{2}R + \frac{1}{2}L) + \frac{1}{4}y = \frac{1}{4}L + \frac{1}{2}\frac{1}{2} + \frac{1}{4}y$ to $\frac{1}{2}y + \frac{1}{2}(\frac{1}{2}R + \frac{1}{2}L) = \frac{1}{2}y + \frac{1}{2}\frac{1}{2}$), or both if they are equal (if $l_R = l_L$ then decreasing both l_R and l_L the same small amount changes the chosen act locally from $\frac{1}{2}R + \frac{1}{2}L = \frac{1}{2}$ to $\frac{1}{2}(\frac{1}{2}R + \frac{1}{2}L) + \frac{1}{2}y = \frac{1}{2}\frac{1}{2} + \frac{1}{2}y$), so we can conclude that $\max(l_L,\, l_R) \leq \frac{1}{2}$, and if $\max(u_L,\, u_R) > \frac{1}{2}$ then we know $\max(u_L,\, u_R) > \max(l_L,\, l_R)$ and since there exists small $\epsilon > 0$ such that $\max(u_L,\, u_R) - \epsilon > \frac{1}{2}$ and for all $z \in [\max(u_L,\, u_R) - \epsilon,\, \max(u_L,\, u_R)]$ COM, STR, and WCI, tell us:

$$z \succ \frac{1}{2}\frac{1}{2} + \frac{1}{2}z, \text{ because } z > \frac{1}{2}$$

$$z \succ \frac{1}{4}R + \frac{3}{4}z, \text{ because (for these } z) \ \frac{1}{4}z \succ \frac{1}{4}R$$

$$\text{and } z \succ \frac{1}{4}L + \frac{3}{4}z \text{ because (for these } z) \ \frac{1}{4}z \succ \frac{1}{4}L,$$

so the DM could do strictly better by lowering $\max(u_L, u_R)$ (if $u_R > u_L$ then decreasing u_R a small amount changes the chosen act locally from $\frac{1}{4}R + \frac{3}{4}z$ to z and if $u_R < u_L$ then decreasing u_L a small amount changes the chosen act locally from $\frac{1}{4}L + \frac{3}{4}z$ to z) or both if they are equal (if $u_R = u_L$ then decreasing both u_R and u_L the same small amount changes the chosen act locally from $\frac{1}{2}(\frac{1}{2}R + \frac{1}{2}L) + \frac{1}{2}z = \frac{1}{2}\frac{1}{2} + \frac{1}{2}z$ to z).

Assume for the rest of the proof $\exists x \in (\frac{1}{2}, 1]$ and $f \in \{L, R\}$ such that $\frac{1}{4}f \succeq \frac{1}{4}x$, then for $e \in \{L, R\}\backslash\{f\}$ we know $\frac{1}{4}y \succ \frac{1}{4}e$ for all $y \in (\frac{1}{2}, 1]$ because if not COM, STR, and CON, tell us $\frac{1}{8}f + \frac{1}{8}e = \frac{1}{4}\frac{1}{2} \succeq \frac{1}{4}\min(x, y)$, which violates COM and STR. Assume without loss that $\frac{1}{4}R \succeq \frac{1}{4}x$ for some $x \in (\frac{1}{2}, 1]$, thus STR tells us $\frac{1}{4}R \succ \frac{1}{4}x$ for some $x \in (\frac{1}{2}, 1]$ and $\frac{1}{4}y \succ \frac{1}{4}L$ for all $y \in (\frac{1}{2}, 1]$, and thus COM, CON, and WCI, tell us that for all such y that $\frac{1}{4}L + \frac{1}{4}y \succeq \frac{1}{2}L$ (because we cannot have that $\frac{1}{2}L \succ \frac{1}{4}L + \frac{1}{4}y$ and $\frac{1}{2}y \succ \frac{1}{4}L + \frac{1}{4}y$), and since this is true for all such y STR tells us that for all such y that $\frac{1}{4}L + \frac{1}{4}y \succ \frac{1}{2}L$, and thus COM, CON, and WCI tell us for all such y that $\frac{1}{2}y \succeq \frac{1}{2}L$ (because, otherwise, COM implies $\frac{1}{2}L \succ \frac{1}{2}y$, and then COM and CON imply $\frac{1}{4}L + \frac{1}{4}y \succeq \frac{1}{2}y$, and then WCI implies $\frac{1}{4}L \succeq \frac{1}{4}y$, which contradicts $\frac{1}{4}y \succ \frac{1}{4}L$) and since this is true for all such y STR tells us that for all such y that $\frac{1}{2}y \succ \frac{1}{2}L$.

It must then be that $u_R \geq u_L$ because otherwise (if $u_R < u_L$) COM, STR, and WCI, tell us, since for all $y \in (\frac{1}{2}, 1]$:

$$y \succ \frac{1}{4}L + \frac{3}{4}y \text{ because (for these } y\text{, based on previous paragraph) } \frac{1}{4}y \succ \frac{1}{4}L$$

$$\frac{1}{4}L + \frac{3}{4}y \succ \frac{1}{2}L + \frac{1}{2}y \text{ because (for these } y\text{, based on previous paragraph)}$$
$$\frac{1}{4}L + \frac{1}{4}y \succ \frac{1}{2}L,$$

$$\text{and } y \succ \frac{1}{2}L + \frac{1}{2}y \text{ because (for these } y\text{, based on previous paragraph)}$$
$$\frac{1}{2}y \succ \frac{1}{2}L,$$

if $u_L > \frac{1}{2}$ the DM could do strictly better by decreasing u_L if $u_L > l_L$ (if $u_L > l_L$ then decreasing u_L a small amount changes the chosen act locally from $\frac{1}{4}L + \frac{3}{4}y$ to y) and by decreasing both u_L and l_L if $l_L = u_L$ (if $l_L = u_L$ then decreasing both u_L and l_L the same small amount changes the chosen act locally, depending on if the location is equal to the new u_L or the old u_L or between the two, from $\frac{1}{2}L+\frac{1}{2}y$ to $\frac{1}{4}L+\frac{3}{4}y$ or from $\frac{1}{4}L+\frac{3}{4}y$ to y or from $\frac{1}{2}L+\frac{1}{2}y$ to y) while, for similar reasons, if $u_L \leq \frac{1}{2}$ the DM could do strictly better by increasing u_R (if $u_R < u_L \leq \frac{1}{2}$ then increasing u_R a small amount changes the chosen act locally, depending on if $u_R < l_L$ or $u_R \geq l_L$, from $\frac{1}{2}L + \frac{1}{2}y$ to $\frac{1}{4}L + \frac{1}{2}(\frac{1}{2}R + \frac{1}{2}L) + \frac{1}{4}y = \frac{1}{4}L + \frac{1}{2}\frac{1}{2} + \frac{1}{4}y$ for $y < \frac{1}{2}$ or from $\frac{1}{4}L + \frac{3}{4}y$ to $\frac{1}{2}(\frac{1}{2}R + \frac{1}{2}L) + \frac{1}{2}y = \frac{1}{2}\frac{1}{2} + \frac{1}{2}y$ for $y < \frac{1}{2}$).

It must also then be that $l_R \geq l_L$ because, again, COM, STR, and WCI tell us that, since:

$$\frac{1}{2} \succ \frac{1}{4}L + \frac{1}{2}\frac{1}{2} + \frac{1}{4}y \text{ for } y < \frac{1}{2} \text{ because we know } \frac{1}{4}(1-y) \succ \frac{1}{4}L,$$
$$\text{and } \frac{1}{2}\frac{1}{2} + \frac{1}{2}y \succ \frac{1}{4}L + \frac{1}{2}\frac{1}{2} + \frac{1}{4}y \text{ for } y > \frac{1}{2} \text{ because we know } \frac{1}{4}y \succ \frac{1}{4}L,$$

if $l_R < l_L$ (so $l_R < u_R$ since we showed $u_R \geq u_L$) and $l_L \leq \frac{1}{2}$ then the DM could do strictly better by increasing l_R (locally this changes the chosen act from $\frac{1}{4}L + \frac{1}{2}(\frac{1}{2}R + \frac{1}{2}L) + \frac{1}{4}y = \frac{1}{4}L + \frac{1}{2}\frac{1}{2} + \frac{1}{4}y$ to $\frac{1}{2}R + \frac{1}{2}L = \frac{1}{2}$), while if $l_R < l_L$ (so $l_R < u_R$ since we showed $u_R \geq u_L$) and $l_L > \frac{1}{2}$ then the DM could do strictly better by decreasing l_L (locally this changes the chosen act from $\frac{1}{4}L + \frac{1}{2}(\frac{1}{2}R + \frac{1}{2}L) + \frac{1}{4}y = \frac{1}{4}L + \frac{1}{2}\frac{1}{2} + \frac{1}{4}y$ to $\frac{1}{2}(\frac{1}{2}R + \frac{1}{2}L) + \frac{1}{2}y = \frac{1}{2}\frac{1}{2} + \frac{1}{2}y$). It must also then be that $l_R \geq u_L$, because COM, STR, and WCI, tell us:

$$\frac{1}{2} \succ \frac{1}{4}L + \frac{1}{2}\frac{1}{2} + \frac{1}{4}y \text{ for } y < \frac{1}{2} \text{ because we know } \frac{1}{4}(1-y) \succ \frac{1}{4}L,$$
$$\frac{1}{4}R + \frac{1}{2}\frac{1}{2} + \frac{1}{4}y \succ \frac{1}{2}\frac{1}{2} + \frac{1}{2}y \text{ for } y < \frac{1}{2} \text{ because we know } \frac{1}{4}R \succ \frac{1}{4}y,$$
$$\text{and } \frac{1}{4}R + \frac{3}{4}y \succ \frac{1}{2}\frac{1}{2} + \frac{1}{2}y \text{ for } y > \frac{1}{2} \text{ because we know } \frac{1}{4}R \succ \frac{1}{4}(1-y),$$

so the DM could otherwise (if $l_R < u_L \leq u_R$) do strictly better from at least one of increasing l_R (if $u_L \leq \frac{1}{2}$ then increasing l_R locally changes the chosen act, since $l_R \geq l_L$, from $\frac{1}{2}(\frac{1}{2}R + \frac{1}{2}L)+\frac{1}{2}y = \frac{1}{2}\frac{1}{2}+\frac{1}{2}y$ to $\frac{1}{4}R+\frac{1}{2}(\frac{1}{2}R+\frac{1}{2}L)+\frac{1}{4}y = \frac{1}{4}R+\frac{1}{2}\frac{1}{2}+\frac{1}{4}y$) or decreasing u_L (if $u_L > \frac{1}{2}$ then decreasing u_L locally changes the chosen act from $\frac{1}{2}(\frac{1}{2}R + \frac{1}{2}L) + \frac{1}{2}y = \frac{1}{2}\frac{1}{2} + \frac{1}{2}y$ to $\frac{1}{4}R + \frac{3}{4}y$).

So, $l_R \geq u_L$ (notice that this establishes that we cannot have an interval that is contained in the interior of the other interval). If $l_R > u_L$ then the fact that the DM does not decrease l_R (which would locally change the chosen act from $\frac{1}{2}R+\frac{1}{2}z$ to $\frac{1}{4}R+\frac{3}{4}z$), COM, STR, and WCI, tell us for all $\epsilon > 0$ there is $z \in (l_R - \epsilon,\, l_R)$ such that (using WCI and then COM and CON):

$$\frac{1}{2}R + \frac{1}{2}z \succ \frac{1}{4}R + \frac{3}{4}z$$
$$\Rightarrow \frac{1}{2}R \succ \frac{1}{4}R + \frac{1}{4}z \Rightarrow \frac{1}{2}R \succ \frac{1}{2}z \text{ and } \frac{1}{4}R + \frac{1}{4}z \succeq \frac{1}{2}z,$$

and the fact that the DM does not increase u_L (which would locally change the chosen act from $\frac{1}{2}R+\frac{1}{2}y$ to $\frac{1}{4}R+\frac{1}{2}(\frac{1}{2}R+\frac{1}{2}L)+\frac{1}{4}y=\frac{1}{4}R+\frac{1}{2}\frac{1}{2}+\frac{1}{4}y)$, COM, STR, and WCI, tell us for all $\epsilon>0$ there is $q\in(u_L,\, u_L+\epsilon)$ such that (using WCI then COM and CON):

$$\frac{1}{2}R+\frac{1}{2}q\succ\frac{1}{4}R+\frac{1}{2}\frac{1}{2}+\frac{1}{4}q$$

$$\Rightarrow\frac{1}{2}R\succ\frac{1}{4}R+\frac{1}{4}(1-q)\Rightarrow\frac{1}{2}R\succ\frac{1}{2}(1-q)\text{ and }\frac{1}{4}R+\frac{1}{4}(1-q)\succeq\frac{1}{2}(1-q),$$

so if $l_R>1-u_L$, which thus implies $l_R>\frac{1}{2}$, then STR and WCI tell us the DM could do strictly better by decreasing u_L if $u_L>l_L$ (which locally changes the chosen act from $\frac{1}{4}R+\frac{1}{2}(\frac{1}{2}R+\frac{1}{2}L)+\frac{1}{4}y=\frac{1}{4}R+\frac{1}{2}\frac{1}{2}+\frac{1}{4}y$ to $\frac{1}{2}R+\frac{1}{2}y)$ and by decreasing u_L and l_L if $l_L=u_L$ (which locally changes the chosen act, depending on if the location is equal to the new u_L the old u_L or between the two, from $\frac{1}{2}R+\frac{1}{2}L=\frac{1}{2}$ to $\frac{1}{4}R+\frac{1}{2}(\frac{1}{2}R+\frac{1}{2}L)+\frac{1}{4}y=\frac{1}{4}R+\frac{1}{2}\frac{1}{2}+\frac{1}{4}y$ or from $\frac{1}{4}R+\frac{1}{2}(\frac{1}{2}R+\frac{1}{2}L)+\frac{1}{4}y=\frac{1}{4}R+\frac{1}{2}\frac{1}{2}+\frac{1}{4}y$ to $\frac{1}{2}R+\frac{1}{2}y$ or from $\frac{1}{2}R+\frac{1}{2}L=\frac{1}{2}$ to $\frac{1}{2}R+\frac{1}{2}y)$, while if $l_R<1-u_L$, which thus implies $u_L<\frac{1}{2}$, then STR and WCI tell us the DM could do strictly better by increasing l_R if $l_R<u_R$ (which locally changes the chosen act from $\frac{1}{4}R+\frac{3}{4}y$ to $\frac{1}{2}R+\frac{1}{2}y)$ and by increasing l_R and u_R if $l_R=u_R$ (which locally changes the chosen act, depending on if the location is that of the new u_R or the old u_R or between the two, from y to $\frac{1}{4}R+\frac{3}{4}y$ or from $\frac{1}{4}R+\frac{3}{4}y$ to $\frac{1}{2}R+\frac{1}{2}y$ or from y to $\frac{1}{2}R+\frac{1}{2}y)$. So, if $l_R>u_L$ then $l_R=1-u_L$.

If $u_R=1$ then COM, STR, CON, and WCI, tell us it cannot be that $l_L=l_R=u_R$ as then the DM could do strictly better by reducing l_L and l_R (locally this changes the chosen act from $\frac{1}{2}R+\frac{1}{2}L=\frac{1}{2}$ to $\frac{1}{2}(\frac{1}{2}R+\frac{1}{2}L)+\frac{1}{2}y=\frac{1}{2}\frac{1}{2}+\frac{1}{2}y)$, and thus if $u_R=1$, for all small $\epsilon>0$ there is $x\in(1-\epsilon,\,1)$ such that:

$$\frac{1}{4}R\succ\frac{1}{4}x,$$

since otherwise there is small $\delta>0$ such that for all $x\in(1-\delta,\,1)$:

$$\frac{1}{4}x\succ\frac{1}{4}R,\ \frac{1}{2}x\succ\frac{1}{2}R\text{ and }\frac{1}{4}R+\frac{1}{4}x\succ\frac{1}{2}R,$$

and the DM could do strictly better by decreasing u_R if $u_R>l_R$ (locally this changes the chosen act from $\frac{1}{4}R+\frac{3}{4}y$ to y), by decreasing l_R if $u_R=l_R>u_L$ (locally this changes the chosen act from $\frac{1}{2}R+\frac{1}{2}y$ to $\frac{1}{4}R+\frac{3}{4}y$), and by decreasing l_R if $l_R=u_R=u_L>l_L$ (locally this changes the chosen act from $\frac{1}{4}R+\frac{1}{2}(\frac{1}{2}R+\frac{1}{2}L)+\frac{1}{4}y=\frac{1}{4}R+\frac{1}{2}\frac{1}{2}+\frac{1}{4}y$ to $\frac{1}{2}(\frac{1}{2}R+\frac{1}{2}L)+\frac{1}{2}y=\frac{1}{2}\frac{1}{2}+\frac{1}{2}y)$, and thus $l_L=0$ because otherwise the DM could do strictly better by reducing l_L (locally this changes the chosen act from $\frac{1}{2}R+\frac{1}{2}L=\frac{1}{2}$ to $\frac{1}{4}R+\frac{1}{2}(\frac{1}{2}R+\frac{1}{2}L)+\frac{1}{4}y=\frac{1}{4}R+\frac{1}{2}\frac{1}{2}+\frac{1}{4}y)$. If $l_L=0$ then COM, STR, CON, and WCI, tell us it cannot be that $u_R=u_L=l_L$ since then the DM could do strictly better by increasing u_L and u_R (locally this changes the chosen act from y to $\frac{1}{2}(\frac{1}{2}R+\frac{1}{2}L)+\frac{1}{2}y=\frac{1}{2}\frac{1}{2}+\frac{1}{2}y$), and thus for all small $\epsilon>0$ there is $y\in(0,\,\epsilon)$ such that:

$$\frac{1}{4}R+\frac{1}{4}y\succ\frac{1}{2}\frac{1}{2}\Rightarrow\frac{1}{4}R\succ\frac{1}{4}(1-y),$$

since otherwise there is small $\delta>0$ such that for all $y\in(0,\,\delta)$:

$$\frac{1}{2}\frac{1}{2}\succ\frac{1}{4}R+\frac{1}{4}y,\ \frac{1}{2}\succ\frac{1}{2}R+\frac{1}{2}y\text{ and }\frac{1}{4}R+\frac{1}{2}\frac{1}{2}+\frac{1}{4}y\succ\frac{1}{2}R+\frac{1}{2}y,$$

and the DM could do strictly better by increasing l_L if $l_L<u_L$ (locally this changes the chosen act from $\frac{1}{4}R+\frac{1}{2}(\frac{1}{2}R+\frac{1}{2}L)+\frac{1}{4}y=\frac{1}{4}R+\frac{1}{2}\frac{1}{2}+\frac{1}{4}y$ to $\frac{1}{2}R+\frac{1}{2}L=\frac{1}{2}$), by increasing u_L if $l_L=u_L<l_R$ (locally this changes the chosen act from $\frac{1}{2}R+\frac{1}{2}y$ to $\frac{1}{4}R+\frac{1}{2}(\frac{1}{2}R+\frac{1}{2}L)+\frac{1}{4}y=\frac{1}{4}R+\frac{1}{2}\frac{1}{2}+\frac{1}{4}y$), and by increasing u_L if $u_L=l_R=l_L<u_R$ (locally this changes the chosen act from $\frac{1}{4}R+\frac{3}{4}y$ to $\frac{1}{2}(\frac{1}{2}R+\frac{1}{2}L)+\frac{1}{2}y=\frac{1}{2}\frac{1}{2}+\frac{1}{2}y$), and thus $u_R=1$ because otherwise the DM could do strictly better by increasing u_R (locally this changes the chosen act from y to $\frac{1}{4}R+\frac{3}{4}y$). If $u_R<1$ then, as is implied by our work above, $l_L>0$, and the fact that the DM does not increase u_R (which locally changes the chosen act from y to $\frac{1}{4}R+\frac{3}{4}y$) or decrease l_L (which locally changes the chosen act from $\frac{1}{2}R+\frac{1}{2}L=\frac{1}{2}$ to $\frac{1}{4}R+\frac{1}{2}(\frac{1}{2}R+\frac{1}{2}L)+\frac{1}{4}y=\frac{1}{4}R+\frac{1}{2}\frac{1}{2}+\frac{1}{4}y$), COM, STR, and WCI, tell us for all small $\epsilon>0$ there is $y\in[u_R,\,u_R+\epsilon]$ such that:

$$y\succ\frac{1}{4}R+\frac{3}{4}y,$$

and there is $z\in[l_L-\epsilon,\,l_L]$ such that:

$$\frac{1}{2}\succ\frac{1}{4}R+\frac{1}{2}\frac{1}{2}+\frac{1}{4}z.$$

Thus, COM, STR, CON, and WCI, tell us that if $u_R<1-l_L$ (which, given what we have shown, implies $l_L<\frac{1}{2}$) the DM could do strictly better by increasing l_L if $u_L>l_L$ (which locally changes the chosen act from $\frac{1}{4}R+\frac{1}{2}(\frac{1}{2}R+\frac{1}{2}L)+\frac{1}{4}y=\frac{1}{4}R+\frac{1}{2}\frac{1}{2}+\frac{1}{4}y$ to $\frac{1}{2}R+\frac{1}{2}L=\frac{1}{2}$), or by increasing u_L if $l_L=u_L<u_R$ (which locally changes the chosen act from $\frac{1}{4}R+\frac{3}{4}y$ to $\frac{1}{2}(\frac{1}{2}R+\frac{1}{2}L)+\frac{1}{2}y=\frac{1}{2}\frac{1}{2}+\frac{1}{2}y$ since $u_L=l_L<1-u_R\Rightarrow l_R=u_L$ given that we showed above that if $l_R>u_L$ then $l_R+u_L=1\Rightarrow u_R+u_L\geq 1$), or by increasing u_R and u_L if $l_L=u_L=u_R$ (which locally changes the chosen act from y to $\frac{1}{2}(\frac{1}{2}R+\frac{1}{2}L)+\frac{1}{2}y=\frac{1}{2}\frac{1}{2}+\frac{1}{2}y$). If, instead, $u_R>1-l_L$ (which, given what we have shown, implies $u_R>\frac{1}{2}$) COM, STR, CON, and WCI, tell us that the DM could do strictly better by decreasing u_R if $u_R>l_R$ (which locally changes the chosen act from $\frac{1}{4}R+\frac{3}{4}y$ to y), or by decreasing l_R if $u_R=l_R>l_L$ (which locally changes the chosen act from $\frac{1}{4}R+\frac{1}{2}(\frac{1}{2}R+\frac{1}{2}L)+\frac{1}{4}y=\frac{1}{4}R+\frac{1}{2}\frac{1}{2}+\frac{1}{4}y$ to

$\frac{1}{2}(\frac{1}{2}R + \frac{1}{2}L) + \frac{1}{2}y = \frac{1}{2}\frac{1}{2} + \frac{1}{2}y$ since $u_R = l_R > 1 - l_L \Rightarrow l_R = u_L$ given what we showed above that if $l_R > u_L$ then $l_R + u_L = 1 \Rightarrow l_R + l_L \leq 1$), or by decreasing l_R and l_L if $u_R = l_R = l_L$ (which locally changes the chosen lottery from $\frac{1}{2}R + \frac{1}{2}L = \frac{1}{2}$ to $\frac{1}{2}(\frac{1}{2}R + \frac{1}{2}L) + \frac{1}{2}y = \frac{1}{2}\frac{1}{2} + \frac{1}{2}y$). So, $u_R = 1 - l_L$. □

It follows that a DM who integrates the two PE problems (with equal weights) and whose preferences are consistent with MPP must choose intervals that satisfy the constraints in Theorem 7.

Theorem 8. *If the DM's preferences satisfy COM, STR, CON, TR, and CI, and they consider both PE questions in conjunction with weights* $\frac{1}{2}$*, then* $l_L = u_L$*,* $l_R = u_R$*, and* $u_R + u_L = 1$*.*

Proof. **Proof of Theorem 8.** Notice that the implications of Theorem 7 hold in this setting since CI implies WCI when COM holds. Suppose the DM reports a non-degenerate interval and thus, given the result in Theorem 7, we must either have a unique maximum bound (and) or a unique minimum bound. Further, notice that the DM would never report one interval that is contained in the interior of their other interval, as is shown in the poof of Theorem 7. We can thus assume without loss of generality that $u_R > l_L$. If u_R is the unique maximal bound, by which we mean $u_R > \max(u_L, l_R)$, then the fact that the DM does not decrease u_R, COM, STR, and WCI, tell us that for all small $\epsilon > 0$ there is $y \in [u_R - \epsilon, u_R]$ such that:

$$\frac{1}{4}R + \frac{3}{4}y \succ y \Rightarrow \frac{1}{4}R \succ \frac{1}{4}y,$$

and then STR tells us there is a $z \in [u_R - \epsilon, 1]$ such that if $x < z$ then $\frac{1}{4}R \succ \frac{1}{4}x$ and if $x > z$ then $\frac{1}{4}x \succ \frac{1}{4}R$ (making sure in both cases that $\frac{1}{4}x \in X$), and the fact that the DM does not increase l_R (which, since Theorem 7 implies it must be that $u_L \leq l_R$, locally changes the chosen act strictly above u_L and weakly above l_R from $\frac{1}{4}R + \frac{3}{4}y$ to $\frac{1}{2}R + \frac{1}{2}y$ and potentially changes the chosen act at u_L, if $l_R = u_L$, from $\frac{1}{2}(\frac{1}{2}R + \frac{1}{2}L) + \frac{1}{2}l_R = \frac{1}{2}\frac{1}{2} + \frac{1}{2}l_R$ to $\frac{1}{4}R + \frac{1}{2}(\frac{1}{2}R + \frac{1}{2}L) + \frac{1}{4}l_R = \frac{1}{4}R + \frac{1}{2}\frac{1}{2} + \frac{1}{4}l_R$), COM, STR, and WCI, tell us that for all $\epsilon > 0$ there is $w \in [l_R, l_R + \epsilon]$ such that:

$$\frac{1}{4}R + \frac{3}{4}w \succ \frac{1}{2}R + \frac{1}{2}w,$$

because if for all $w \in [l_R, l_R + \epsilon]$:

$$\frac{1}{2}R + \frac{1}{2}w \succeq \frac{1}{4}R + \frac{3}{4}w$$

then for all $w \in [l_R,\, l_R + \epsilon)$:

$$\frac{1}{2}R + \frac{1}{2}w \succ \frac{1}{4}R + \frac{3}{4}w \Rightarrow \frac{1}{4}R + \frac{1}{4}w \succeq \frac{1}{2}w,$$

and then the DM could strictly benefit from increasing l_R, so thus TR and CI tell us that for all small $\epsilon > 0$ that:

$$\frac{1}{4}(z+\epsilon) + \frac{3}{4}w \succ \frac{1}{2}(z-\epsilon) + \frac{1}{2}w,$$

and taking ϵ to zero we get a contradiction with COM and STR. So, we must have $l_L < u_L \leq l_R = u_R$ (because we assumed there was a non-degenerate interval) and further Theorem 7 then requires $u_L = l_R$, but then l_L is the unique minimum bound and the fact that the DM does not increase l_L, COM, STR, and WCI, tell us that for all small $\epsilon > 0$ there is $q \in [l_L,\, l_L + \epsilon]$ such that:

$$\frac{1}{4}R + \frac{1}{2}\frac{1}{2} + \frac{1}{4}q \succ \frac{1}{2} \Rightarrow \frac{1}{4}R \succ \frac{1}{4}(1-q),$$

and then STR tell us there is a $k \in [0,\, l_L + \epsilon]$ such that if $x > k$ then $\frac{1}{4}R \succ \frac{1}{4}(1-x)$ and if $x < k$ then $\frac{1}{4}(1-x) \succ \frac{1}{4}R$ (making sure in both cases that $\frac{1}{4}(1-x) \in X$), and the fact that the DM does not decrease u_L (which, since $u_L = l_R$, locally changes the chosen act below u_L from $\frac{1}{4}R + \frac{1}{2}(\frac{1}{2}R + \frac{1}{2}L) + \frac{1}{4}y = \frac{1}{4}R + \frac{1}{2}\frac{1}{2} + \frac{1}{4}y$ to $\frac{1}{2}R + \frac{1}{2}y$ and changes the chosen act at u_L from $\frac{1}{2}(\frac{1}{2}R + \frac{1}{2}L) + \frac{1}{2}u_L = \frac{1}{2}\frac{1}{2} + \frac{1}{2}u_L$ to $\frac{1}{4}R + \frac{3}{4}u_L$), COM, STR, and WCI, tell us that for all $\epsilon > 0$ there is $n \in [u_L - \epsilon,\, u_L]$ such that:

$$\frac{1}{4}R + \frac{1}{2}\frac{1}{2} + \frac{1}{4}n \succ \frac{1}{2}R + \frac{1}{2}n,$$

because if for all $n \in [u_L - \epsilon,\, u_L]$:

$$\frac{1}{2}R + \frac{1}{2}n \succeq \frac{1}{4}R + \frac{1}{2}\frac{1}{2} + \frac{1}{4}n,$$

then for all $n \in (u_L - \epsilon,\, u_L]$:

$$\frac{1}{2}R + \frac{1}{2}n \succ \frac{1}{4}R + \frac{1}{2}\frac{1}{2} + \frac{1}{4}n \Rightarrow \frac{1}{4}R + \frac{1}{4}(1-n) \succeq \frac{1}{2}(1-n) \Rightarrow \frac{1}{4}R + \frac{3}{4}n \succeq \frac{1}{2}\frac{1}{2} + \frac{1}{2}n,$$

and then the DM could strictly benefit from decreasing u_L, so thus TR and CI tell us that for all small $\epsilon > 0$ that:

$$\frac{1}{4}(1-k+\epsilon) + \frac{1}{2}\frac{1}{2} + \frac{1}{4}n \succ \frac{1}{2}(1-k-\epsilon) + \frac{1}{2}n \Rightarrow \frac{1}{4}(1-k+\epsilon) + \frac{1}{4}(1-n) \succ \frac{1}{2}(1-k-\epsilon),$$

and taking ϵ to zero we get a contradiction with COM and STR. Thus, $u_R = l_R$ and $u_L = l_L$, and Theorem 7 tells us $\max(u_R, u_L) + \min(l_R, l_L) = 1$, so $u_R + u_L = 1$. □

It follows that a DM who integrates the two PE problems (with equal weights) and has preferences represented by the Maxmin Expected Utility Model (Gilboa & Schmeidler, 1989) makes choices as if they are probabilistically sophisticated (see also Baillon et al. (2022a)).

B.2 Discrete Interval and Isolation Results

This subsection and the next present amended results that accommodate the discrete nature of the double-sliders in the experiment. If the continuous version of a result is called Theorem Y, then the discrete version found here is called Theorem $Y.1$. Similarly, the discrete version of Corollary 1 is presented here as Corollary 1.1.

To match the set of possible outcomes for the sliders in the experiment, we thus, as a minor abuse of notation, change the set of consequences X to be $X = [0, 100]$, but use the same incentivization of questions as is described in Section 2.1, with $x \in X$ now representing the constant act that produces a $x\%$ chance of winning the monetary prize m in each of the two states. Correspondingly, let $l_L, u_L \in \{0, 1, \ldots, 100\}$ with $l_L \leq u_L$ denote the lower and upper bound reported by the DM for the left shape, and let $l_R, u_R \in \{0, 1, \ldots, 100\}$ with $l_R \leq u_R$ denote the lower and upper bound reported by the DM for the right shape. Further, the random lottery r is now distributed over $[0, 100]$, and we relax the assumption that the DM assigns a positive probability to each open interval of potential r, and instead only assume that the DM assigns a positive probability to each interval of potential r that contains an integer. This means that our results are robust to subjects in the experiment having strange beliefs about how the random lottery is drawn.

Our results in this section only produce a rejection of the axioms if the DM can change their response to one of the questions and do weakly better for all r and strictly better for at least one integer $r \in \{0, 1, \ldots, 100\}$. To define this notion formally, for each $r \in [0, 100]$ let $Q(r) \in \mathcal{F}$ denote the act assigned to the DM when the question they are answering is used for payment and the random lottery is r. We then impose, given some sub-set of axioms on their preferences, that the DM's answer is not such that there is an alternative way of answering the question that would result in alternate acts $\tilde{Q}(r) \in \mathcal{F}$ for each $r \in [0, 100]$ if the question is used for payment such that $\exists\, r \in \{0, 1, \ldots 100\}$ with: $\tilde{Q}(r) \succ Q(r)$, and $\forall\, r \in [0, 100]$: $\tilde{Q}(r) \succeq Q(r)$.

Theorem 1.1. If the DM's preferences satisfy WP, COM, CON, and WCI, and the DM isolates the three choice problems, then $u_L + u_R \leq 101$ unless: $u_L + u_R = 102$, $u_L = l_L$, and $u_R = l_R$.

Proof. Assume $u_L + u_R \geq 102$. The upper bounds, COM, and CON, imply (since the DM would not strictly benefit from lowering either upper bound and $u_L + u_R \geq 102 \Rightarrow \min(u_L, u_R) \geq 2$) that there is $z_R \in [u_R - 1, u_R]$ and $z_L \in [u_L - 1, u_L]$ such that:

$$\frac{1}{2}z_R + \frac{1}{2}R \succeq z_R \text{ and } \frac{1}{2}z_L + \frac{1}{2}L \succeq z_L.$$

This is evident if neither interval is degenerate, but it is also true if there is a degenerate interval since COM and CON imply:

$$z_R \succ \frac{1}{2}z_R + \frac{1}{2}R \Rightarrow z_R \succ R,\ \frac{1}{2}z_R + \frac{1}{2}R \succeq R,$$

$$\text{and } z_L \succ \frac{1}{2}z_L + \frac{1}{2}L \Rightarrow z_L \succ L,\ \frac{1}{2}z_L + \frac{1}{2}L \succeq L,$$

and otherwise the DM could do strictly better by lowering both bounds of the degenerate interval by one. Thus, WCI tells us for all such z_R and z_L:

$$\frac{1}{2}z_L + \frac{1}{2}R \succeq \frac{1}{2}z_R + \frac{1}{2}z_L \text{ and } \frac{1}{2}z_R + \frac{1}{2}L \succeq \frac{1}{2}z_L + \frac{1}{2}z_R.$$

But, then CON tells us for all such z_R and z_L:

$$\frac{1}{4}L + \frac{1}{4}R + \frac{1}{4}z_R + \frac{1}{4}z_L \succeq \frac{1}{2}z_R + \frac{1}{2}z_L.$$

Notice that $\frac{1}{4}L(s) + \frac{1}{4}R(s) = 25$ for all $s \in S$, so $\frac{1}{4}L + \frac{1}{4}R = 25 \in X$. Thus, WP tells us:

$$\frac{1}{4}L + \frac{1}{4}R + \frac{1}{4}z_R + \frac{1}{4}z_L = \frac{1}{2}\left(\frac{100}{2}\right) + \frac{1}{2}\left(\frac{1}{2}z_R + \frac{1}{2}z_L\right) \geq \frac{1}{2}z_R + \frac{1}{2}z_L$$

$$\Rightarrow z_R + z_L \leq 100 \Rightarrow u_R + u_L = 102 \text{ (since } u_R + u_L \geq 102) \Rightarrow z_R + z_L = 100.$$

Thus, since for all z_R and z_L such that:

$$\frac{1}{2}z_R + \frac{1}{2}R \succeq z_R \text{ and } \frac{1}{2}z_L + \frac{1}{2}L \succeq z_L,$$

we know $z_R + z_L = 100$, COM tells us it must be that for all $z_R \in (u_R - 1, u_R]$ and $z_L \in (u_L - 1, u_L]$ that:

$$z_R \succ \frac{1}{2}z_R + \frac{1}{2}R \text{ and } z_L \succ \frac{1}{2}z_L + \frac{1}{2}L,$$

and therefore $l_L = u_L$ and $l_R = u_R$, because otherwise the DM could do strictly better by lowering an upper bound. □

Corollary 1.1. If the DM's preferences satisfy WP, COM, CON, and WCI, $l_L < u_L$ or $l_R < u_R$, and the DM isolates the three choice problems, then:

$$b_{av} \leq 50.5 - \frac{1}{2}s_{av}.$$

Proof. If the DM's preferences satisfy WP, COM, CON, and WCI, and $l_L < u_L$ or $l_R < u_R$, then Theorem 2.1 tells us:

$$b_{av} = \frac{1}{2}(u_L + u_R) - \frac{1}{2}s_{av} \leq 50.5 - \frac{1}{2}s_{av}.$$

□

Theorem 2.1. If the DM assigns weight $\alpha \geq \frac{3}{4}$ to $f \in \{L, R\}$ when asked how they would like to randomize over L and R, their preferences satisfy COM, STR, CON, and WCI, and the DM isolates the three choice problems, then $u_f \geq 49$.

If the DM assigns weight $\alpha \leq \frac{1}{2}$ to $f \in \{L, R\}$ when asked how they would like to randomize over L and R, their preferences satisfy COM, STR, CON, and WCI, and the DM isolates the three choice problems, then $u_f \leq 50$.

Proof. Without loss of generality assume $f = R$. COM, CON, and the DM's choice of how to randomize over L and R tells us $\frac{3}{4}R+\frac{1}{4}L = \frac{1}{2}R+\frac{1}{2}\frac{100}{2} \succeq \frac{1}{2}R+\frac{1}{2}L = \frac{100}{2}$. Then, STR and WCI tell us $\frac{1}{2}R + \frac{1}{2}z \succ z$ for $z \in [0, 50)$, and thus the DM can strictly benefit by increasing u_R to 49 if $u_R < 49$. COM, CON, and the DM's choice of how to randomize over L and R tells us $\frac{1}{2}R+\frac{1}{2}L = \frac{100}{2} \succeq \frac{3}{4}R+\frac{1}{4}L = \frac{1}{2}R+\frac{1}{2}\frac{100}{2}$, $\frac{1}{2}R+\frac{1}{2}L = \frac{100}{2} \succeq R$, and $\frac{3}{4}R+\frac{1}{4}L = \frac{1}{2}R+\frac{1}{2}\frac{100}{2} \succeq R$. Then, STR and WCI tell us $z \succ \frac{1}{2}R + \frac{1}{2}z$, $z \succ R$, and $\frac{1}{2}R + \frac{1}{2}z \succ R$ for $z \in (50, 100]$, and thus the DM can strictly benefit by decreasing u_R to 50 (and l_R if $l_R > 50$) if $u_R > 50$. □

Theorem 3.1. If the DM selects $f \in \{L, R\}$ when asked how they would like to randomize over L and R, their preferences satisfy COM, STR, and CON, and the DM isolates the three choice problems, then the relevant lower bound l_f is such that $l_f \geq 50$. If the DM assigns weight $\alpha \leq \frac{3}{4}$ to $f \in \{L, R\}$ when asked how they would like to randomize over L and R, their preferences satisfy COM, STR, and CON, and the DM isolates the three choice problems, then the relevant lower bounds l_f is such that $l_f \leq 51$.

Proof. Assume not, and $l_f < 50$. Without loss of generality assume $f = R$. The DM's selection of R tells us $R \succeq \frac{1}{2}L + \frac{1}{2}R = \frac{100}{2}$ by COM, thus $\frac{1}{2}R + \frac{1}{2}\frac{100}{2} \succeq \frac{100}{2}$ by COM and CON, $R \succ x$ for $x < 50$ by STR, and the DM's selection of R also tells us $R \succeq \frac{1}{4}L + \frac{3}{4}R = \frac{1}{2}\frac{100}{2} + \frac{1}{2}R$ by COM, so, $R \succ \frac{1}{2}x + \frac{1}{2}R$, for $x < 50$ by STR. But, then the DM can do strictly better by increasing l_R (and perhaps u_R) to 50. Assume not, and $l_f > 51$ for $f \in \{L, R\}$. Without loss of generality assume $f = R$. The DM's selection of $\alpha R + (1-\alpha)L$ tells us $\alpha R + (1-\alpha)L \succeq R$ by COM, which means $\frac{3}{4}R + \frac{1}{4}L = \frac{1}{2}R + \frac{1}{2}\frac{100}{2} \succeq R$ by COM and CON. So, $\frac{1}{2}R + \frac{1}{2}x \succ R$ for $x > 50$ by STR, and the DM could strictly benefit from reducing l_R to 51. □

Theorem 4.1. If the preferences of the DM satisfy WP, COM, WSTR, and CON, and the DM isolates the three choice problems, then their upper bounds u_L and u_R are such that:

$$\frac{u_L + u_R - 1}{4} + 25 \geq \min\{u_L,\, u_R\} - 1.$$

Proof. First of all, if the inequality does not hold then $u_L,\, u_R > 0$, and for $f \in \{L, R\}$ there is $z \in [u_f - 1, u_f]$ such that $\frac{1}{2}f + \frac{1}{2}z \succeq z$. To see why, we will break things down into cases. If the associated lower bound l_f is such that $l_f < u_f$, then, given COM, the DM could do strictly better by reducing their upper bound by one unless there is $z \in (u_f - 1, u_f]$ such that $\frac{1}{2}f + \frac{1}{2}z \succeq z$. If $l_f = u_f$, then, given COM, there is $z \in [u_f - 1, u_f]$ such that $f \succeq \frac{1}{2}f + \frac{1}{2}z$, otherwise the DM could do strictly better by reducing their lower bound, and by WSTR we can then infer $f \succeq \frac{1}{2}f + \frac{1}{2}(u_f - 1)$. If $f \succ \frac{1}{2}f + \frac{1}{2}(u_f - 1)$, then COM and CON tell us $\frac{1}{2}f + \frac{1}{2}(u_f - 1) \succeq u_f - 1$. If instead $f \sim \frac{1}{2}f + \frac{1}{2}(u_f - 1)$ and further $u_f - 1 \succ f$, then it must be $\frac{1}{2}f + \frac{1}{2}u_f \succeq u_f$ otherwise the DM could do strictly better by lowering both bounds by one by COM and WSTR. So, if $f \sim \frac{1}{2}f + \frac{1}{2}(u_f - 1)$ then either $\frac{1}{2}f + \frac{1}{2}u_f \succeq u_f$ or $f \succeq u_f - 1$, and in the latter case COM and CON then tell us $\frac{1}{2}f + \frac{1}{2}(u_f - 1) \succeq u_f - 1$. Thus, there is $z \in [u_f - 1, u_f]$ such that $\frac{1}{2}f + \frac{1}{2}z \succeq z$.

Now, for $f,\, g \in \{L, R\}$ with $f \neq g$, let z_f denote a constant act in $[u_f - 1, u_f]$ such that $\frac{1}{2}f + \frac{1}{2}z_f \succeq z_f$ and let z_g denote a constant act in $[u_g - 1, u_g]$ such that $\frac{1}{2}g + \frac{1}{2}z_g \succeq z_g$. It is without loss to assume $z_f \geq z_g$ and $u_f \geq u_g$. It is then the case that $\frac{1}{2}f + \frac{1}{2}z_f \succeq z_g$ by WSTR. CON then tells us $\frac{1}{4}L + \frac{1}{4}R + \frac{1}{4}z_f + \frac{1}{4}z_g \succeq z_g$, but $\frac{1}{4}L + \frac{1}{4}R + \frac{1}{4}z_f + \frac{1}{4}z_g = \frac{1}{2}50 + \frac{z_f + z_g}{4} = 25 + \frac{z_f + z_g}{4}$, so WP tells us $25 + \frac{z_f + z_g}{4} \geq z_g \Rightarrow \frac{u_f + z_g}{4} + 25 \geq z_g \Rightarrow \frac{u_f + u_g - 1}{4} + 25 \geq u_g - 1$. □

Theorem 5.1. If the DM selects $f \in \{L, R\}$ when asked how they would like to randomize over L and R, their preferences satisfy WP, COM, WSTR, CON, and TR, and the DM isolates the three choice problems, then $u_f \geq 49$.

Proof. Assume not, and $u_f < 49$. Without loss of generality assume $f = R$. The DM's selection of R tells us $R \succeq \frac{1}{4}L + \frac{3}{4}R = \frac{1}{2}\frac{100}{2} + \frac{1}{2}R$ by COM. So, $R \succeq \frac{1}{2}x + \frac{1}{2}R$, for $x \leq 50$ by WSTR. The DM's selection of R also tells us $R \succeq \frac{1}{2}L + \frac{1}{2}R = \frac{100}{2}$ by COM, thus $\frac{1}{2}\frac{100}{2} + \frac{1}{2}R \succeq \frac{100}{2}$ by COM and CON. Further, $R \succ x$, for $x < 50$ by WP and TR. But, then we have the desired contradiction since the DM can do strictly better by increasing both u_R and l_R to 50. □

Theorem 6.1. If the preferences of the DM satisfy COM, STR, TR, and CI, and the DM isolates the three choice problems, then $l_L \geq u_L - 1$ and $l_R \geq u_R - 1$.

Proof. Suppose the DM responds to the PE question about $f \in \{L, R\}$ with $u_f > l_f + 1$ (notice that this implies $u_f \geq 2$ and $l_f \leq 98$). COM, STR, CI, and the fact the the DM does not wish to decrease u_f tell us that $\frac{1}{2}f + \frac{1}{2}(u_f - 1) \succ u_f - 1 = \frac{1}{2}(u_f - 1) + \frac{1}{2}(u_f - 1) \Rightarrow f \succ u_f - 1$ because if not $u_f - 1 \succeq \frac{1}{2}f + \frac{1}{2}(u_f - 1) \Rightarrow \frac{1}{2}(u_f - 1) \succeq \frac{1}{2}f$ and then for all $y \in (u_f - 1, u_f]$ we have $\frac{1}{2}y \succ \frac{1}{2}f \Rightarrow y \succ \frac{1}{2}f + \frac{1}{2}y$ and the DM could do strictly better by lowering u_f. Similarly, COM, STR, and the fact the the DM does not wish to increase l_f tell us that $\frac{1}{2}(l_f + 1) + \frac{1}{2}f \succ f$. It cannot be that $f \succ 99.9$ because then we have a contradiction with COM, STR, TR, and $\frac{1}{2}(l_f + 1) + \frac{1}{2}f \succ f \succ 99.9$ because 99.9 statewise dominates $\frac{1}{2}(l_f + 1) + \frac{1}{2}f$ since $l_f + 1 \leq 99$. It cannot be that $0.1 \succeq f$ because then we have a contradiction with COM, STR, TR, and $0.1 \succeq f \succ u_f - 1$ because $u_f - 1$ statewise dominates 0.1 since $u_f - 1 \geq 1$. Thus, WP, COM, and TR, tell us there is $x \in (0, 100)$ such that if $y > x$ then $y \succ f$ and if $y < x$ then $f \succ y$, and thus using CI if $y > x$ then $\frac{1}{2}y \succ \frac{1}{2}f$ and $y \succ \frac{1}{2}f + \frac{1}{2}y$, and if $y < x$ then $\frac{1}{2}f \succ \frac{1}{2}y$ and $\frac{1}{2}f + \frac{1}{2}y \succ y$. Picking arbitrarily small $\epsilon > 0$, and using WP, TR, and CI, we thus have $\frac{1}{2}(x + \epsilon) \succ \frac{1}{2}(u_f - 1) \Rightarrow x \geq u_f - 1$ and $\frac{1}{2}(l_f + 1) + \frac{1}{2}(x + \epsilon) \succ x - \epsilon = \frac{1}{2}(x - 3\epsilon) + \frac{1}{2}(x + \epsilon) \Rightarrow \frac{1}{2}(l_f + 1) \succ \frac{1}{2}(x - 3\epsilon) \Rightarrow l_f + 1 \geq x$, so it must be that $x = u_f - 1 = l_f + 1$, but then the DM could do strictly better by lowering u_f to $u_f - 1$. □

B.3 Discrete Interval and Integration Results

We now turn to our results that address the potential for the DM to be "integrating" (Baillon, Halevy, & Li, 2022b) and answering multiple questions in conjunction instead of "isolating" and answering each question as if it is the only question they face. When we say the DM is answering a set of questions in conjunction we mean that for each random lottery they are trying to answer each of said questions in a way that maximizes their expected payoff given the weights that they think are the probabilities of each of said question being used for payment if one of the questions from the set is being used for payment. To define integration formally we need to introduce some new notation. For each random lottery $r \in [0, 100]$, let $Q_i(r) \in \mathcal{F}$ for $i \in \{1, 2, 3\}$ denote the act assigned to the DM when question i is used for payment and the random lottery is r. We then say the DM answers a subset of questions $\mathcal{Q} \subseteq \{1, 2, 3\}$ in **conjunction** with **weights** $\beta_j \geq 0$ for each $j \in \mathcal{Q}$ such that $\sum_{j \in \mathcal{Q}} \beta_j = 1$ if there is not an alternative way of answering the questions that would result in alternate acts $\tilde{Q}_j(r) \in \mathcal{F}$ for each $r \in [0, 100]$ and $j \in \mathcal{Q}$ such that $\exists\, r \in \{0, 1, \ldots, 100\}$ such that:

$$\sum_{j \in \mathcal{Q}} \beta_j \tilde{Q}_j(r) \succ \sum_{j \in \mathcal{Q}} \beta_j Q_j(r),$$

and $\forall\, r \in [0, 100]$:

$$\sum_{j \in \mathcal{Q}} \beta_j \tilde{Q}_j(r) \succeq \sum_{j \in \mathcal{Q}} \beta_j Q_j(r).$$

If we were to instead assume a particular distribution of r, for instance the uniform distribution over $\{0, 1, \ldots, 100\}$ that is actually used in the experiment, then rejection would be easier as we could reject the behavior of the DM if the they did not maximize their preference relation across the potential realizations of the random lottery and the question that is used for payment, and we are thus setting a relatively high bar for rejection of DM behavior. We also do not assume that $\beta_i = \frac{1}{3}\, \forall i \in \{1, 2, 3\}$ as is the reality in the experiment, but because of their symmetry we do assume in our results that the weights on the two PE (double-slider) questions are the same and strictly positive when the DM is answering them.

To ease exposition, we introduce continuity below, which is a feature of the models of Variational Preferences (Maccheroni et al., 2006) (axiom A.3 in their paper), Uncertainty Averse Preferences (Cerreia-Vioglio et al., 2011) (axiom A.5 in their paper), and Smooth Ambiguity Preferences (Klibanoff et al., 2005; Denti & Pomatto, 2022) (by Lemma 6 in the work of Denti and Pomatto (2022)). We then introduce three more simple lemmas, Lemma 4, Lemma 5, and Lemma 6, that build upon continuity.

Continuity. If $f, g, h \in \mathcal{F}$, the sets $\{\alpha \in [0, 1] : \alpha f + (1-\alpha) g \succeq h\}$ and $\{\alpha \in [0,1] : h \succeq \alpha f + (1-\alpha) g\}$ are closed.

Lemma 4. If the preferences of the DM satisfy WP, COM, WSTR, CON, TR, WCI, and continuity, then they satisfy STR, and for all $\beta \in [0, 1]$, and $e, j \in \mathcal{F}$ there is a $w \in [0, 100]$ such that $\beta e + (1-\beta) j \sim \beta w + (1-\beta) j$, and if $\beta > 0$ then such w is unique.

Proof. First, notice that Proposition 1 tells us STR holds since, given WP, it is trivial to show that Axiom 1 (COM) through continuity imply axioms A.1 through A.6 from the model of Maccheroni et al. (2006), the only one that is perhaps not trivial is axiom A.4, but it is not hard to show it is implied by WSTR and continuity.

Let $x = 100 \in X$ and $y = 0 \in X$. If $\beta = 0$ then COM tells us we are done. Next, COM, STR, and continuity tell us for $\beta \in (0, 1]$ and $e, j \in \mathcal{F}$, that $\beta x + (1-\beta)j \succeq \beta e + (1-\beta)j$ because for all $\gamma \in (0, \beta)$ we know $\beta x + (1-\beta)j \succ \gamma e + (\beta-\gamma)y + (1-\beta)j$ and the set $\{\alpha \in [0, 1] : \beta x + (1-\beta)j \succeq \alpha(\beta e + (1-\beta)j) + (1-\alpha)(\beta y + (1-\beta)j)\}$ is closed, and that $\beta e + (1-\beta)j \succeq \beta y + (1-\beta)j$, because for all $\gamma \in (0, \beta)$ we know $\gamma e + (\beta-\gamma)x + (1-\beta)j \succ \beta y + (1-\beta)j$ and the set $\{\alpha \in [0, 1] : \alpha(\beta e + (1-\beta)j) + (1-\alpha)(\beta x + (1-\beta)j) \succeq \beta y + (1-\beta)j\}$ is closed. Thus, letting f, g, and h, from the statement of continuity be defined $f = \beta x + (1-\beta)j$, $g = \beta y + (1-\beta)j$, and $h = \beta e + (1-\beta)j$, it must be that there is a $w \in [0, 100]$ such that $\beta e + (1-\beta)j \sim \beta w + (1-\beta)j$, and COM and STR tell us such w must be unique if $\beta > 0$. $\square$

Lemma 5. If the preferences of the DM satisfy WP, COM, WSTR, CON, TR, WCI, and continuity, then if $\beta f + (1-\beta)g \succeq \beta w + (1-\beta)g$ for $\beta \in [0, 1]$, $w \in X$, and $f, g \in \mathcal{F}$, then for all $\alpha \in [0, \beta) : \alpha f + (1-\beta)g \succeq \alpha w + (1-\beta)g$, and if $\beta f + (1-\beta)g \succ \beta w + (1-\beta)g$ for $\beta \in (0, 1]$, $w \in X$, and $f, g \in \mathcal{F}$, then for all $\alpha \in (0, \beta) : \alpha f + (1-\beta)g \succ \alpha w + (1-\beta)g$.

Proof. Suppose $\beta f + (1-\beta)g \succeq \beta w + (1-\beta)g$ for $\beta \in [0, 1]$, $w \in X$, and $f, g \in \mathcal{F}$, then COM, CON, and WCI, tell us for all $\alpha \in [0, \beta)$:

$$\frac{\alpha}{\beta}(\beta f + (1-\beta)g) + \frac{\beta-\alpha}{\beta}(\beta w + (1-\beta)g) = \alpha f + (\beta-\alpha)w + (1-\beta)g$$

$$\succeq \beta w + (1-\beta)g = \alpha w + (\beta-\alpha)w + (1-\beta)g$$

$$\Rightarrow \alpha f + (1-\beta)g \succeq \alpha w + (1-\beta)g.$$

If, instead, $\beta f + (1-\beta)g \succ \beta w + (1-\beta)g$ for $\beta \in (0, 1]$, $w \in X$, and $f, g \in \mathcal{F}$, then Lemma 4 tells us $\exists \tilde{w} \in X$ such that $\beta f + (1-\beta)g \sim \beta \tilde{w} + (1-\beta)g$, and COM and STR (which is satisfied by Lemma 4) tell us $\tilde{w} > w$, then as we have just shown for all $\alpha \in (0, \beta)$: $\alpha f + (1-\beta)g \succeq \alpha \tilde{w} + (1-\beta)g$, and thus STR tells us $\alpha f + (1-\beta)g \succ \alpha w + (1-\beta)g$. $\square$

Lemma 6. If the preferences of the DM satisfy WP, COM, WSTR, CON, TR, WCI, and continuity, then if $\beta w + (1-\beta-\gamma)g \succeq \beta f + (1-\beta-\gamma)g$ for $\beta \in (0, 1)$, $\gamma \in [0, 1-\beta]$, $w \in X$, and $f, g \in \mathcal{F}$, then for all $\alpha \in [0, 1] : \alpha(\beta+\gamma)f + (1-\alpha)(\beta+\gamma)w + (1-\beta-\gamma)g \succeq (\beta+\gamma)f + (1-\beta-\gamma)g$, and if $\beta w + (1-\beta-\gamma)g \succ \beta f + (1-\beta-\gamma)g$ for $\beta \in (0, 1)$, $\gamma \in [0, 1-\beta]$, $w \in X$, and $f, g \in \mathcal{F}$, then for all $\alpha \in [0, 1) : \alpha(\beta+\gamma)f + (1-\alpha)(\beta+\gamma)w + (1-\beta-\gamma)g \succ (\beta+\gamma)f + (1-\beta-\gamma)g$.

Proof. If $\beta w + (1-\beta-\gamma)g \succeq \beta f + (1-\beta-\gamma)g$ for $\beta \in (0, 1)$, $\gamma \in [0, 1-\beta]$, $w \in X$, and $f, g \in \mathcal{F}$, then COM and Lemma 5 tell us $(\beta+\gamma)w+(1-\beta-\gamma)g \succeq (\beta+\gamma)f+(1-\beta-\gamma)g$, and COM and CON thus tell us for all $\alpha \in [0, 1] : \alpha(\beta+\gamma)f+(1-\alpha)(\beta+\gamma)w+(1-\beta-\gamma)g \succeq (\beta+\gamma)f + (1-\beta-\gamma)g$.

If $\beta w+(1-\beta-\gamma)g \succ \beta f+(1-\beta-\gamma)g$ for $\beta \in (0, 1)$, $\gamma \in [0, 1-\beta]$, $w \in X$, and $f, g \in \mathcal{F}$, then Lemma 4 tells us there is a $\tilde{w} \in X$ such that $\beta\tilde{w} + (1-\beta-\gamma)g \sim \beta f + (1-\beta-\gamma)g$, COM and STR (which is satisfied by Lemma 4) tell us $\tilde{w} < w$, and Lemma 5 tells us $(\beta+\gamma)\tilde{w} + (1-\beta-\gamma)g \succeq (\beta+\gamma)f + (1-\beta-\gamma)g$. Then COM and CON tell us that for all $\alpha \in [0, 1] : \alpha(\beta+\gamma)f+(1-\alpha)(\beta+\gamma)\tilde{w}+(1-\beta-\gamma)g \succeq (\beta+\gamma)f+(1-\beta-\gamma)g$, and thus STR tells us for $\alpha \in [0, 1) : \alpha(\beta+\gamma)f+(1-\alpha)(\beta+\gamma)w+(1-\beta-\gamma)g \succ (\beta+\gamma)f+(1-\beta-\gamma)g$. □

Lemma 7. If the preferences of the DM satisfy WP, COM, WSTR, CON, TR, WCI, and continuity, and they consider all three question in conjunction and assign equal and strictly positive weights to the PE questions, then they do not give an interval that is contained in the interior of their other interval.

Proof. Let $\alpha \in \{0, 0.01, 0.02, \ldots, 1\}$ be the chance that the DM selects to assign to betting on R as opposed to L in the binary choice question. Let $g = \alpha R + (1-\alpha)L$. The specific composition of the act g does not impact our argument because all we need to do to show our desired result is focus on the answers to the PE questions. Assume that the DM gave an interval that is contained in the interior of their other interval and we shall reach a contradiction. Assume without loss of generality that $u_R > u_L$ and $l_R < l_L$ (this is without loss as the nature of g is irrelevant to the argument). Let $\beta \in [0, 1)$ denote the DM's weight on the binary choice question when they answer the PE questions.

Since the DM does not want to decrease u_R COM, STR (which is satisfied by Lemma 4), TR, and WCI tell us:

$$(1-\beta)\Big(\frac{1}{4}R + \frac{3}{4}(u_R - 1)\Big) + \beta g \succ (1-\beta)(u_R - 1) + \beta g,$$

because otherwise:

$$(1-\beta)(u_R - 1) + \beta g \succeq (1-\beta)\Big(\frac{1}{4}R + \frac{3}{4}(u_R - 1)\Big) + \beta g \Rightarrow \forall x \in (u_R - 1, u_R] :$$

$$(1-\beta)x + \beta g \succ (1-\beta)\Big(\frac{1}{4}R + \frac{3}{4}(x)\Big) + \beta g,$$

and the DM could do strictly better by reducing u_R by 1 (doing so does not change the chosen act at $u_R - 1$),

$$\text{so (using WCI)} \;\Rightarrow \frac{1-\beta}{4}R + \beta g \succ \frac{1-\beta}{4}(u_R - 1) + \beta g.$$

Since the DM does not want to increase l_R COM, STR, TR, and WCI tell us:

$$(1-\beta)\Big(\frac{1}{4}L + \frac{1}{2}\frac{100}{2} + \frac{1}{4}(l_R+1)\Big) + \beta g \succ (1-\beta)\frac{100}{2} + \beta g,$$

because otherwise:

$$(1-\beta)\frac{100}{2} + \beta g \succeq (1-\beta)\Big(\frac{1}{4}L + \frac{1}{2}\frac{100}{2} + \frac{1}{4}(l_R+1)\Big) + \beta g \Rightarrow \forall x \in [l_R,\, l_R+1):$$

$$(1-\beta)\frac{100}{2} + \beta g \succ (1-\beta)\Big(\frac{1}{4}L + \frac{1}{2}\frac{100}{2} + \frac{1}{4}(x)\Big) + \beta g,$$

and the DM could do strictly better by increasing l_R by 1 (doing so does not change the chosen act at $l_R + 1$),

$$\text{so (using WCI)} \;\Rightarrow \frac{1-\beta}{4}L + \beta g \succ \frac{1-\beta}{4}(100 - l_R - 1) + \beta g.$$

Since the DM neither benefits from switching their upper bounds with each other or switching their lower bounds with each other, which would replace a $\frac{1-\beta}{4}R$ with a $\frac{1-\beta}{4}L$ or replace a $\frac{1-\beta}{4}L$ with a $\frac{1-\beta}{4}R$ respectively, COM tells us (using CON and TR):

$$\frac{1-\beta}{4}R + \beta g \sim \frac{1-\beta}{4}L + \beta g,$$

$$\Rightarrow \frac{1-\beta}{4}\Big(\frac{1}{2}R + \frac{1}{2}L\Big) + \beta g = \frac{1-\beta}{4}\frac{100}{2} + \beta g \succ \frac{1-\beta}{4}(u_R - 1) + \beta g$$
$$\text{and } \frac{1-\beta}{4}\Big(\frac{1}{2}R + \frac{1}{2}L\Big) + \beta g = \frac{1-\beta}{4}\frac{100}{2} + \beta g \succ \frac{1-\beta}{4}(100 - l_R - 1) + \beta g.$$

So, if $u_R \geq 51$ we have a contradiction with COM and STR, and if $u_R \leq 50$ then $l_R < 50$ and we again have a contradiction with COM and STR. □

Lemma 7 is interesting because it means that there are certain responses to the PE questions that are admissible if the DM is not believed to be integrating, but can be rejected if they are integrating. The same is true for Lemma 8.

Lemma 8. If the preferences of the DM satisfy WP, COM, WSTR, CON, TR, WCI, and continuity, and they answer all three questions in conjunction and assign equal and strictly positive weights to the PE questions, then they would never give upper bounds with $\max(u_R, u_L) < 50$, or lower bounds $l_R = l_L < 50$ unless $l_R = l_L = \min(u_L, u_R)$, and if $\max(u_L, u_R) = 50$ then $\min(l_R, l_L) \geq 49$.

Proof. Let $\alpha \in \{0, 0.01, 0.02, \ldots, 1\}$ be the chance that the DM selects to assign to betting on R as opposed to L in the binary choice question. Let $g = \alpha R + (1-\alpha)L$. The specific composition of the act g does not impact our argument because all we need to do to show our desired result is focus on the answers to the PE questions. Let $\beta \in [0, 1)$ denote the DM's weight on the binary choice question when they answer the PE questions. Assume without loss of generality that $u_R \geq u_L$ (this is without loss as the nature of g is irrelevant to the argument).

If $u_R = u_L < 49$, COM and STR (which is satisfied by Lemma 4) tell us the DM could strictly benefit from increasing their upper bounds. If $l_R = l_L < \min(u_L, 50)$ COM and STR tell us the DM could strictly benefit from increasing their lower bounds. If $u_R = u_L = 49$ then if $l_R = l_L = 49$ then COM and STR tell us the DM could strictly benefit from increasing their lower and upper bounds to 50. If $u_R = u_L = 49$ and $\min(l_L, l_R) < \max(l_L, l_R)$, in which case we can assume without loss of generality that $\min(l_L, l_R) = l_R$, and thus COM, STR, and WCI tell us that since the DM does not increase l_R:

$$(1-\beta)\Big(\frac{1}{4}L + \frac{1}{2}\frac{100}{2} + \frac{1}{4}(l_R + 1)\Big) + \beta g \succ (1-\beta)\frac{100}{2} + \beta g,$$

but then the DM could do strictly better by increasing u_L by one since $l_R + 1 \leq 49$. If $50 > u_R > u_L$ then Lemma 7 tells us $l_R \geq l_L$ and COM, STR, and WCI tell us that since the DM does not increase u_R to 50 (using WCI and Lemma 6):

$$(1-\beta)\frac{100}{2} + \beta g \succeq (1-\beta)\Big(\frac{1}{4}R + \frac{1}{2}\frac{100}{2} + \frac{1}{4}\frac{100}{2}\Big) + \beta g$$

$$\Rightarrow (1-\beta)\frac{1}{4}\frac{100}{2} + \beta g \succeq (1-\beta)\frac{1}{4}R + \beta g$$
$$\Rightarrow (1-\beta)\frac{1}{2}\frac{100}{2} + \beta g \succeq (1-\beta)\frac{1}{2}R + \beta g,$$

and COM and STR tell us the DM could do strictly better by increasing all bounds to 50.

We have thus shown $\max(u_L, u_R) = u_R \geq 50$. For the rest of the proof assume $u_R = 50$. We know from COM, STR, and WCI, that since the DM does not increase u_R:

$$(1-\beta)51 + \beta g \succeq (1-\beta)\Big(\frac{1}{4}R + \frac{3}{4}51\Big) + \beta g$$

$$\Rightarrow (1-\beta)\frac{1}{4}51 + \beta g \succeq (1-\beta)\frac{1}{4}R + \beta g.$$

COM, STR, and Lemma 4 thus tell us there is $w \leq 51$ such that $(1-\beta)\frac{1}{4}R + \beta g \sim (1-\beta)\frac{1}{4}w + \beta g$. We next show $\min(l_L, l_R) \geq 49$. If $\min(l_L, l_R) < 49$ it must be $l_L < 49$, because if not then $l_R < \min(l_L, 49)$, and Lemma 7 tells us $u_L = 50$, and since the DM does not increase l_R COM, STR, and WCI, tell us:

$$(1-\beta)\Big(\frac{1}{4}L + \frac{1}{2}50 + \frac{1}{4}(l_R+1)\Big) + \beta g \succ (1-\beta)50 + \beta g,$$

and thus the DM would strictly benefit from increasing u_L to 51, so if $\min(l_L, l_R) < 49$ it must be $l_L < 49$. If $l_L < 49$, $u_L = l_L$, and $u_R > l_R$, then if $l_R = u_L$ the DM would strictly benefit from increasing l_R, u_L, and l_L, to 49, so if $l_L < 49$, $u_L = l_L$, and $u_R > l_R$, then $l_R > u_L$ (using Lemma 7) and since the DM does not increase l_R COM, STR, and WCI, tell us:

$$(1-\beta)\Big(\frac{1}{4}R + \frac{3}{4}\frac{100}{2}\Big) + \beta g \succ (1-\beta)\Big(\frac{1}{2}R + \frac{1}{2}\frac{100}{2}\Big) + \beta g,$$

and the DM could thus do strictly better by increasing u_L. If $l_L < 49$ and $l_L < u_L$, then since the DM does not benefit from increasing l_L, COM, STR, and WCI, tell us (using TR):

$$(1-\beta)\Big(\frac{1}{4}R + \frac{1}{2}\frac{100}{2} + \frac{1}{4}(l_L+1)\Big) + \beta g \succ (1-\beta)\frac{100}{2} + \beta g$$

$$\Rightarrow (1-\beta)\Big(\frac{1}{4}51 + \frac{1}{2}\frac{100}{2} + \frac{1}{4}(l_L+1)\Big) + \beta g \succ (1-\beta)\frac{100}{2} + \beta g$$

$$\Rightarrow l_L \geq 49,$$

and we have a contradiction. If $l_L < 49$, $l_R = u_R$, and $u_L = l_L$, then since the DM does not benefit from increasing u_L COM, STR, and WCI, tell us (using TR and WCI):

$$(1-\beta)\Big(\frac{1}{2}R + \frac{1}{2}(u_L+1)\Big) + \beta g \succeq (1-\beta)\Big(\frac{1}{4}R + \frac{1}{2}\frac{100}{2} + \frac{1}{4}(u_L+1)\Big) + \beta g.$$

$$\Rightarrow (1-\beta)\frac{1}{2}R + \beta g \succeq (1-\beta)\Big(\frac{1}{4}w + \frac{100-(u_L+1)}{4}\Big) + \beta g,$$

but then Lemma 5, COM, and STR, tell us $w = 51$ and the DM could do strictly better by increasing u_R and l_R to 51. □

Lemma 9. If the preferences of the DM satisfy WP, COM, WSTR, CON, TR, WCI, and continuity, and they answer all three questions in conjunction and assign equal and strictly positive weights to the PE questions, then if $\max(u_L,\, u_R) > 50$ and $\min(u_L,\, u_R) \leq 50$, then $\max(l_L,\, l_R) \geq \min(u_L,\, u_R)$, $\min(l_L,\, l_R) + \max(u_L,\, u_R) \in [99,\, 101]$, and either $\min(u_L,\, u_R) \in [\max(l_L,\, l_R) - 1,\, \max(l_L,\, l_R)]$ or $\max(l_L,\, l_R) + \min(u_L,\, u_R) \in [99,\, 101]$.

Proof. Let $\alpha \in \{0,\, 0.01,\, 0.02,\, \ldots,\, 1\}$ be the chance that the DM selects to assign to betting on R as opposed to L in the binary choice question. Let $g = \alpha R + (1-\alpha)L$. The specific composition of the act g does not impact our argument because all we need to do to show our desired result is focus on the answers to the PE questions. Let $\beta \in [0,\, 1)$ denote the DM's weight on the binary choice question when they answer the PE questions. Assume without loss of generality that $u_R \geq u_L$ (this is without loss as the nature of g is irrelevant to the argument), and then assume $u_R > 50$ and $u_L \leq 50$. Lemma 7 tells us $l_R \geq l_L$.

First, $\max(l_L,\, l_R) = l_R \geq \min(u_L,\, u_R) = u_L$, because otherwise COM, STR (which is satisfied by Lemma 4), WCI, and the fact that the DM does not lower u_R tells us:

$$(1-\beta)\Big(\frac{1}{4}R + \frac{3}{4}\frac{100}{2}\Big) + \beta g \succ (1-\beta)\frac{100}{2} + \beta g$$

$$\Rightarrow (1-\beta)\frac{1}{4}R + \beta g \succ (1-\beta)\frac{1}{4}\frac{100}{2} + \beta g,$$

and the DM could do strictly better by increasing l_R to u_L (which locally changes the chosen act from $(1-\beta)(\frac{1}{2}(\frac{1}{2}R + \frac{1}{2}L) + \frac{1}{2}y) + \beta g = (1-\beta)(\frac{1}{2}\frac{100}{2} + \frac{1}{2}y) + \beta g$ to $(1-\beta)(\frac{1}{4}R + \frac{1}{2}(\frac{1}{2}R + \frac{1}{2}L) + \frac{1}{4}y) + \beta g = (1-\beta)(\frac{1}{4}R + \frac{1}{2}\frac{100}{2} + \frac{1}{4}y) + \beta g)$.

If $u_R > l_R$ then COM, STR, WCI, and the fact that the DM does not lower u_R tells us:

$$(1-\beta)\Big(\frac{1}{4}R + \frac{3}{4}(u_R - 1)\Big) + \beta g \succ (1-\beta)(u_R - 1) + \beta g$$

$$\Rightarrow (1-\beta)\frac{1}{4}R + \beta g \succ (1-\beta)\frac{1}{4}(u_R - 1) + \beta g,$$

but if $u_R = l_R$ then COM, STR, TR, WCI, Lemma 6, and the fact that the DM does not lower u_R and l_R (which locally changes the chosen act, depending on if the act is equal to the original u_R or the new u_R which is more than u_L or the new u_R which is equal to $u_L = 50$ or between the original and new u_R, from $(1-\beta)(\frac{1}{4}R + \frac{3}{4}u_R) + \beta g$ to $(1-\beta)u_R + \beta g$ or from $(1-\beta)(\frac{1}{2}R + \frac{1}{2}(u_R - 1)) + \beta g$ to $(1-\beta)(\frac{1}{4}R + \frac{3}{4}(u_R - 1)) + \beta g$ or from $(1-\beta)(\frac{1}{4}R + \frac{3}{4}(50)) + \beta g$ to $(1-\beta)50 + \beta g$ or from $(1-\beta)(\frac{1}{2}R + \frac{1}{2}y) + \beta g$ to $(1-\beta)y + \beta g)$ tells us the same thing:

$$(1-\beta)\Big(\frac{1}{4}R + \frac{3}{4}(u_R - 1)\Big) + \beta g \succ (1-\beta)(u_R - 1) + \beta g$$

$$\Rightarrow (1-\beta)\frac{1}{4}R + \beta g \succ (1-\beta)\frac{1}{4}(u_R - 1) + \beta g,$$

because if:

$$(1-\beta)(u_R-1)+\beta g \succeq (1-\beta)\Big(\frac{1}{4}R+\frac{3}{4}(u_R-1)\Big)+\beta g$$

then WCI and Lemma 6 tell us:

$$(1-\beta)\frac{1}{2}(u_R-1)+\beta g \succeq (1-\beta)\frac{1}{2}R+\beta g \text{ and}$$
$$(1-\beta)\Big(\frac{1}{4}R+\frac{1}{4}(u_R-1)\Big)+\beta g \succeq (1-\beta)\frac{1}{2}R+\beta g.$$

If $u_L > l_L$ then COM, STR, WCI, and the fact that the DM does not increase l_L tells us:

$$(1-\beta)\Big(\frac{1}{4}R+\frac{1}{2}\frac{100}{2}+\frac{1}{4}(l_L+1)\Big)+\beta g \succ (1-\beta)\frac{100}{2}+\beta g$$
$$\Rightarrow (1-\beta)\Big(\frac{1}{4}R+\frac{1}{4}(l_L+1)\Big)+\beta g \succ (1-\beta)\Big(\frac{100}{4}\Big)+\beta g,$$

but if $u_L = l_L < l_R$ then the fact that the DM does not increase u_L and l_L (which locally changes the chosen act, depending on if the act is equal to the original u_L or the new u_L which is less than l_R or the new u_L which is equal to l_R or between the original and new u_L, from $(1-\beta)(\frac{1}{4}R+\frac{1}{2}\frac{100}{2}+\frac{1}{4}l_L)+\beta g$ to $(1-\beta)\frac{100}{2}+\beta g$ or from $(1-\beta)(\frac{1}{2}R+\frac{1}{2}(l_L+1))+\beta g$ to $(1-\beta)(\frac{1}{4}R+\frac{1}{2}\frac{100}{2}+\frac{1}{4}(l_L+1))+\beta g$ or from $(1-\beta)(\frac{1}{4}R+\frac{3}{4}(l_L+1))+\beta g$ to $(1-\beta)\Big(\frac{1}{2}\frac{100}{2}+\frac{1}{2}(l_L+1)\Big)+\beta g$ or from $(1-\beta)(\frac{1}{2}R+\frac{1}{2}y)+\beta g$ to $(1-\beta)\frac{100}{2}+\beta g$), COM, STR, TR, WCI, and Lemma 6, tell us the same thing:

$$(1-\beta)\Big(\frac{1}{4}R+\frac{1}{2}\frac{100}{2}+\frac{1}{4}(l_L+1)\Big)+\beta g \succ (1-\beta)\frac{100}{2}+\beta g$$

$$\Rightarrow (1-\beta)\Big(\frac{1}{4}R+\frac{1}{4}(l_L+1)\Big)+\beta g \succ (1-\beta)\Big(\frac{100}{4}\Big)+\beta g,$$

because if (using WCI):

$$(1-\beta)\Big(\frac{100}{4}\Big)+\beta g \succeq (1-\beta)\Big(\frac{1}{4}R+\frac{1}{4}(l_L+1)\Big)+\beta g$$

$$\Rightarrow (1-\beta)\Big(\frac{100-(l_L+1)}{4}\Big)+\beta g \succeq (1-\beta)\frac{1}{4}R+\beta g$$

then WCI and Lemma 6 tell us:

$$(1-\beta)\frac{100-(l_L+1)}{2}+\beta g \succeq (1-\beta)\frac{1}{2}R+\beta g$$

$$\text{and } (1-\beta)\Big(\frac{1}{4}R+\frac{100-(l_L+1)}{4}\Big)+\beta g \succeq (1-\beta)\frac{1}{2}R+\beta g,$$

while if $u_L = l_L = l_R$, then either $u_L = l_L = l_R \geq 49$ and $u_R = 51$, in which case we are done, or, COM, STR, and WCI, tell us we can again reach the same conclusion because otherwise, if:

$$(1-\beta)\Big(\frac{100}{4}\Big) + \beta g \succeq (1-\beta)\Big(\frac{1}{4}R + \frac{1}{4}(l_L+1)\Big) + \beta g$$

$$\Rightarrow (1-\beta)\Big(\frac{100-(l_L+1)}{4}\Big) + \beta g \succeq (1-\beta)\frac{1}{4}R + \beta g$$

$$\Rightarrow (1-\beta)\Big(\frac{100-(l_L+1)}{2}\Big) + \beta g \succeq (1-\beta)\Big(\frac{1}{4}R + \frac{100-(l_L+1)}{4}\Big) + \beta g,$$

then either $u_L = l_L = l_R < 49$ and thus for $y < l_L + 1 \leq 49$ (using STR and WCI):

$$(1-\beta)\Big(\frac{100-y}{2}\Big) + \beta g \succ (1-\beta)\Big(\frac{1}{4}R + \frac{y}{4}\Big) + \beta g$$

$$\Rightarrow (1-\beta)\frac{100}{2} + \beta g \succ (1-\beta)\Big(\frac{1}{4}R + \frac{3}{4}y\Big) + \beta g$$

and so the DM can strictly benefit from increasing u_L, l_L, and l_R, by one (which locally changes the chosen act, depending on if the act is equal to the original l_L or the new l_L or between them, from $(1-\beta)(\frac{1}{2}\frac{100}{2} + \frac{1}{2}l_L) + \beta g$ to $(1-\beta)\frac{100}{2} + \beta g$ or from $(1-\beta)\Big(\frac{1}{4}R + \frac{3}{4}(l_L + 1)\Big) + \beta g$ to $(1-\beta)\Big(\frac{1}{2}\frac{100}{2} + \frac{1}{2}(l_L+1)\Big) + \beta g$ or from $(1-\beta)\Big(\frac{1}{4}R + \frac{3}{4}y\Big) + \beta g$ to $(1-\beta)\frac{100}{2} + \beta g$), or $u_L = l_L = l_R \geq 49$ and $u_R > 51$ in which case for $y > u_R - 1 \geq 51$ (using STR and WCI):

$$(1-\beta)\Big(\frac{100-(l_L+1)}{4}\Big) + \beta g \succeq (1-\beta)\frac{1}{4}R + \beta g$$

$$\Rightarrow (1-\beta)y + \beta g \succ (1-\beta)\Big(\frac{1}{4}R + \frac{3}{4}y\Big) + \beta g,$$

and so the DM can strictly benefit from decreasing u_R by one (which locally changes the chosen act from $(1-\beta)(\frac{1}{4}R + \frac{3}{4}y) + \beta g$ to $(1-\beta)y + \beta g$). So, we know:

$$(1-\beta)\frac{1}{4}R + \beta g \succ (1-\beta)\frac{1}{4}(u_R - 1) + \beta g \text{ (from the previous paragraph)}$$

$$\text{and } (1-\beta)\Big(\frac{1}{4}R + \frac{1}{4}(l_L+1)\Big) + \beta g \succ (1-\beta)\Big(\frac{100}{4}\Big) + \beta g.$$

If $u_R = 100$ then either $l_L = 0$ (so $l_L + u_R = 100$), or $l_L > 0$ and COM, STR, WCI, and the fact that the DM does not lower l_L tells us (using COM, STR, TR, and WCI):

$$(1-\beta)\frac{100}{2} + \beta g \succeq (1-\beta)\Big(\frac{1}{4}R + \frac{1}{2}\frac{100}{2} + \frac{1}{4}(l_L - 1)\Big) + \beta g,$$

and because we showed:

$$(1-\beta)\frac{1}{4}R+\beta g \succ (1-\beta)\frac{1}{4}(u_R-1)+\beta g,$$

we then know:

$$(1-\beta)\frac{100}{4}+\beta g \succ (1-\beta)\Big(\frac{1}{4}(u_R-1)+\frac{1}{4}(l_L-1)\Big)+\beta g$$

$$\Rightarrow l_L+u_R=101,$$

and if $u_R<100$ then COM, STR, WCI, and the fact that the DM does not increase u_R tells us (using COM, STR, TR, and WCI):

$$(1-\beta)(u_R+1)+\beta g \succeq (1-\beta)\Big(\frac{1}{4}R+\frac{3}{4}(u_R+1)\Big)+\beta g$$

$$\Rightarrow (1-\beta)\frac{1}{4}(u_R+1)+\beta g \succeq (1-\beta)\frac{1}{4}R+\beta g$$

and because we showed:

$$(1-\beta)\Big(\frac{1}{4}R+\frac{1}{4}(l_L+1)\Big)+\beta g \succ (1-\beta)\Big(\frac{100}{4}\Big)+\beta g,$$

we then know:

$$(1-\beta)\frac{1}{4}(u_R+1)+\beta g \succ (1-\beta)\frac{1}{4}(100-(l_L+1))+\beta g$$

$$\Rightarrow u_R+l_L>98,$$

and either $l_L=0$ (so $l_L+u_R=99$), or $l_L>0$ and COM, STR, WCI, and the fact that the DM does not lower l_L tells us (using COM, STR, TR, and WCI):

$$(1-\beta)\frac{100}{2}+\beta g \succeq (1-\beta)\Big(\frac{1}{4}R+\frac{1}{2}\frac{100}{2}+\frac{1}{4}(l_L-1)\Big)+\beta g$$

and because we showed:

$$(1-\beta)\frac{1}{4}R+\beta g \succ (1-\beta)\frac{1}{4}(u_R-1)+\beta g,$$

we then know:

$$(1-\beta)\frac{100}{4}+\beta g \succ (1-\beta)\Big(\frac{1}{4}(u_R-1)+\frac{1}{4}(l_L-1)\Big)+\beta g$$

$$\Rightarrow l_L + u_R < 102,$$

so $l_L + u_R \in [99, 101]$.

For the rest of the proof assume $l_R > u_L + 1$. If $l_R < u_R$ then the fact that the DM does not increase l_R, COM, STR, and WCI, tell us:

$$(1-\beta)\Big(\frac{1}{4}R + \frac{3}{4}(l_R+1)\Big) + \beta g \succ (1-\beta)\Big(\frac{1}{2}R + \frac{1}{2}(l_R+1)\Big) + \beta g$$

$$\Rightarrow (1-\beta)\Big(\frac{1}{4}R + \frac{1}{4}(l_R+1)\Big) + \beta g \succ (1-\beta)\frac{1}{2}R + \beta g,$$

but if $l_R = u_R$ then either $u_R = 100$ (in which case the same conclusion can be reached with COM and STR) or COM, STR, TR, WCI, and Lemma 6 tell us, since:

$$(1-\beta)\frac{1}{2}R + \beta g \succeq (1-\beta)\Big(\frac{1}{4}R + \frac{1}{4}(l_R+1)\Big) + \beta g,$$

implies that (for $y < l_R + 1$):

$$(1-\beta)\frac{1}{4}R + \beta g \succeq (1-\beta)\Big(\frac{1}{4}(l_R+1)\Big) + \beta g \text{ and } (1-\beta)\frac{1}{2}R + \beta g \succ (1-\beta)\frac{1}{2}y + \beta g,$$

and the fact that the DM does not increase l_R and u_R (which locally changes the chosen act, depending on if the act is equal to the original l_R or the new l_R or between them, from $(1-\beta)\Big(\frac{1}{4}R + \frac{3}{4}(l_R)\Big) + \beta g$ to $(1-\beta)\Big(\frac{1}{2}R + \frac{1}{2}(l_R)\Big) + \beta g$ or from $(1-\beta)(l_R+1) + \beta g$ to $(1-\beta)\Big(\frac{1}{4}R + \frac{3}{4}(l_R+1)\Big) + \beta g$ or from $(1-\beta)y + \beta g$ to $(1-\beta)\Big(\frac{1}{2}R + \frac{1}{2}y\Big) + \beta g$) tells us the same thing:

$$(1-\beta)\Big(\frac{1}{4}R + \frac{3}{4}(l_R+1)\Big) + \beta g \succ (1-\beta)\Big(\frac{1}{2}R + \frac{1}{2}(l_R+1)\Big) + \beta g$$

$$\Rightarrow (1-\beta)\Big(\frac{1}{4}R + \frac{1}{4}(l_R+1)\Big) + \beta g \succ (1-\beta)\frac{1}{2}R + \beta g,$$

and either way the fact that the DM does not decrease l_R, COM, STR, and WCI, tell us

$$(1-\beta)\Big(\frac{1}{2}R + \frac{1}{2}(l_R-1)\Big) + \beta g \succeq (1-\beta)\Big(\frac{1}{4}R + \frac{3}{4}(l_R-1)\Big) + \beta g$$

$$\Rightarrow (1-\beta)\frac{1}{2}R + \beta g \succeq (1-\beta)\Big(\frac{1}{4}R + \frac{1}{4}(l_R-1)\Big) + \beta g.$$

If $u_L > l_L$ then the fact that the DM does not decrease u_L, COM, STR, and WCI, tell us:

$$(1-\beta)\Big(\frac{1}{4}R + \frac{1}{2}\frac{100}{2} + \frac{1}{4}(u_L-1)\Big) + \beta g \succ (1-\beta)\Big(\frac{1}{2}R + \frac{1}{2}(u_L-1)\Big) + \beta g$$

$$\Rightarrow (1-\beta)\Big(\frac{1}{4}R + \frac{1}{4}(100-(u_L-1))\Big) + \beta g \succ (1-\beta)\frac{1}{2}R + \beta g,$$

but if $u_L = l_L$ then either $u_L = 0$ (in which case the same conclusion can be reached with COM and STR) or COM, STR, TR, WCI, and Lemma 6 tell us, since:

$$(1-\beta)\frac{1}{2}R + \beta g \succeq (1-\beta)\Big(\frac{1}{4}R + \frac{1}{4}(100-(u_L-1))\Big) + \beta g$$

implies that (for $y > u_L - 1$):

$$(1-\beta)\Big(\frac{1}{4}R + \frac{1}{2}\frac{100}{2} + \frac{1}{4}(u_L-1)\Big) + \beta g \succeq (1-\beta)\frac{100}{2} + \beta g,$$

$$\text{and } (1-\beta)\Big(\frac{1}{2}R + \frac{1}{2}y\Big) + \beta g \succ (1-\beta)\frac{100}{2} + \beta g$$

and the fact that the DM does not decrease u_L and l_L (which locally changes the chosen act, depending on if the act is equal to the original u_L or the new u_L or between them, from $(1-\beta)\Big(\frac{1}{4}R + \frac{1}{2}\frac{100}{2} + \frac{1}{4}(u_L)\Big) + \beta g$ to $(1-\beta)\Big(\frac{1}{2}R + \frac{1}{2}(u_L)\Big) + \beta g$ or from $(1-\beta)\frac{100}{2} + \beta g$ to $(1-\beta)\Big(\frac{1}{4}R + \frac{1}{2}\frac{100}{2} + \frac{1}{4}(u_L-1)\Big) + \beta g$ or from $(1-\beta)\frac{100}{2} + \beta g$ to $(1-\beta)\Big(\frac{1}{2}R + \frac{1}{2}y\Big) + \beta g)$ tells us the same thing:

$$(1-\beta)\Big(\frac{1}{4}R + \frac{1}{2}\frac{100}{2} + \frac{1}{4}(u_L-1)\Big) + \beta g \succ (1-\beta)\Big(\frac{1}{2}R + \frac{1}{2}(u_L-1)\Big) + \beta g$$

$$\Rightarrow (1-\beta)\Big(\frac{1}{4}R + \frac{1}{4}(100-(u_L-1))\Big) + \beta g \succ (1-\beta)\frac{1}{2}R + \beta g,$$

and either way the fact that the DM does not increase u_L, COM, STR, and WCI, tell us:

$$(1-\beta)\Big(\frac{1}{2}R + \frac{1}{2}(u_L+1)\Big) + \beta g \succeq (1-\beta)\Big(\frac{1}{4}R + \frac{1}{2}\frac{100}{2} + \frac{1}{4}(u_L+1)\Big) + \beta g$$

$$\Rightarrow (1-\beta)\frac{1}{2}R + \beta g \succeq (1-\beta)\Big(\frac{1}{4}R + \frac{1}{4}(100-(u_L+1))\Big) + \beta g.$$

Thus, because we showed:

$$(1-\beta)\Big(\frac{1}{4}R + \frac{1}{4}(l_R+1)\Big) + \beta g \succ (1-\beta)\frac{1}{2}R + \beta g,$$

$$(1-\beta)\frac{1}{2}R + \beta g \succeq (1-\beta)\Big(\frac{1}{4}R + \frac{1}{4}(l_R-1)\Big) + \beta g,$$

$$(1-\beta)\Big(\frac{1}{4}R + \frac{1}{4}(100-(u_L-1))\Big) + \beta g \succ (1-\beta)\frac{1}{2}R + \beta g,$$

$$\text{and } (1-\beta)\frac{1}{2}R + \beta g \succeq (1-\beta)\Big(\frac{1}{4}R + \frac{1}{4}(100-(u_L+1))\Big) + \beta g,$$

COM, STR, and TR, tell us $l_R + u_L \in [99, 101]$. $\square$

Theorem 8.1. If the DM answers all three questions in conjunction and assigns equal and strictly positive weights to the PE questions, and their preferences satisfy WP, COM, WSTR, CON, TR, WCI, and continuity, then they do not give an interval that is contained in the interior of their other interval, and: $\max(u_R, u_L) + \min(l_L, l_R) \in [99, 101]$, $\min(u_R, u_L) \leq \max(l_R, l_L)$, and either $\min(u_R, u_L) \in [\max(l_L, l_R) - 1, \max(l_L, l_R)]$ or $\min(u_R, u_L) + \max(l_R, l_L) \in [99, 101]$.

Proof. Let $\alpha \in \{0, 0.01, 0.02, \ldots, 1\}$ be the chance that the DM selects to assign to betting on R as opposed to L in the binary choice question. Let $g = \alpha R + (1 - \alpha)L$. The specific composition of the act g does not impact our argument because all we need to do to show our desired result is focus on the answers to the PE questions. Let $\beta \in [0, 1)$ denote the DM's weight on the binary choice question when they answer the PE questions. If $u_L + u_R \leq 101$ then we are done by Lemma 7, Lemma 8, and Lemma 9. Assume for the rest of the proof that $u_L + u_R \geq 102$, and assume without loss of generality that $u_R \geq u_L$ (this is without loss as the nature of g is irrelevant to the argument, and thus $u_R \geq 51$ and $u_L \geq 2$). Next we will address all of the potential cases for our bounds.

Case 1: If, $u_R > u_L > l_R$ then, the L interval is non-degenerate (in addition to the R interval being non-degenerate given the conditions of this case) because Lemma 7 tells us $l_R \geq l_L$. Given COM, STR (which is satisfied by Lemma 4), and WCI, the upper bounds thus imply (since the DM would not strictly benefit from lowering an upper bound):

$$(1 - \beta)\Big(\frac{1}{4}R + \frac{3}{4}(u_R - 1)\Big) + \beta g \succ (1 - \beta)(u_R - 1) + \beta g,$$

$$\text{and } (1 - \beta)\Big(\frac{1}{2}\frac{100}{2} + \frac{1}{2}(u_L - 1)\Big) + \beta g \succ (1 - \beta)\Big(\frac{1}{4}R + \frac{3}{4}(u_L - 1)\Big) + \beta g.$$

Thus, WCI tells us:

$$(1 - \beta)\Big(\frac{1}{4}R + \frac{3}{8}(u_R - 1) + \frac{3}{8}(u_L - 1)\Big) + \beta g$$

$$\succ (1 - \beta)\Big(\frac{3}{8}(u_R - 1) + \frac{1}{8}(u_L - 1) + \frac{4}{8}\frac{(u_R - 1) + (u_L - 1)}{2}\Big) + \beta g$$

$$\text{and } (1 - \beta)\Big(\frac{100}{4} + \frac{3}{8}(u_R - 1) + \frac{1}{8}(u_L - 1)\Big) + \beta g$$

$$\succ (1 - \beta)\Big(\frac{1}{4}R + \frac{3}{8}(u_R - 1) + \frac{3}{8}(u_L - 1)\Big) + \beta g.$$

Putting these together using TR:

$$(1-\beta)\Big(\frac{100}{4}+\frac{3}{8}(u_R-1)+\frac{1}{8}(u_L-1)\Big)+\beta g$$

$$\succ (1-\beta)\Big(\frac{3}{8}(u_R-1)+\frac{1}{8}(u_L-1)+\frac{4}{8}\frac{(u_R-1)+(u_L-1)}{2}\Big)+\beta g,$$

which contradicts COM, STR, and TR. This all means that Case 1 is not possible.

Case 2: If, instead, $u_R > u_L = l_R$ then COM, STR, WCI, and the fact that the DM does not change u_R tells us (using WCI):

$$(1-\beta)(u_R+1)+\beta g \succeq (1-\beta)\Big(\frac{1}{4}R+\frac{3}{4}(u_R+1)\Big)+\beta g,$$

and

$$(1-\beta)\Big(\frac{1}{4}R+\frac{3}{4}(u_R-1)\Big)+\beta g \succ (1-\beta)(u_R-1)+\beta g$$

$$\Rightarrow (1-\beta)\frac{1}{4}(u_R+1)+\beta g \succeq (1-\beta)\frac{1}{4}R+\beta g \succ (1-\beta)\frac{1}{4}(u_R-1)+\beta g.$$

(This argument works unless $u_R = 100$, in which case COM, STR, and continuity, tell us $(1-\beta)\frac{1}{4}u_R+\beta g \succeq (1-\beta)\frac{1}{4}R+\beta g \succ (1-\beta)\frac{1}{4}(u_R-1)+\beta g$, and the rest of the argument becomes easier). COM, STR, WCI, and the fact that the DM does not decrease l_L tells us (using TR and WCI):

$$(1-\beta)\frac{100}{2}+\beta g \succeq (1-\beta)\Big(\frac{1}{4}R+\frac{1}{2}\frac{100}{2}+\frac{1}{4}(l_L-1)\Big)+\beta g$$

$$\Rightarrow (1-\beta)\frac{1}{2}\frac{100}{2}+\beta g \succeq (1-\beta)\Big(\frac{1}{4}R+\frac{1}{4}(l_L-1)\Big)+\beta g$$

$$\Rightarrow (1-\beta)\frac{100}{4}+\beta g \succ (1-\beta)\frac{1}{4}(u_R-1+l_L-1)+\beta g.$$

(This argument works unless $l_L = 0$, in which case it must be that $l_R = u_L > l_L$ because $u_L + u_R \geq 102$, and then COM, STR, WCI, and the fact that the DM does not increase l_L tells us $(1-\beta)(\frac{1}{4}R+\frac{1}{2}\frac{100}{2}+\frac{1}{4})+\beta g \succ (1-\beta)50+\beta g$, and then COM, STR, TR, and WCI, tell us $u_R \geq 99$ and we are done.) Thus COM and STR tell us $l_L + u_R < 102$, and since $u_R + u_L \geq 102$, we know $l_L < u_L$, and we can conclude from COM, STR, WCI, and the DM's choice of l_L that (using COM, STR, TR, and WCI):

$$(1-\beta)\Big(\frac{1}{4}R+\frac{1}{2}\frac{100}{2}+\frac{1}{4}(l_L+1)\Big)+\beta g \succ (1-\beta)\frac{100}{2}+\beta g$$

$$\Rightarrow (1-\beta)\Big(\frac{1}{4}(u_R+1)+\frac{1}{4}(l_L+1)\Big)+\beta g \succ (1-\beta)\frac{50}{2}+\beta g$$
$$\Rightarrow u_R + l_L > 98.$$

Together, this all means that if we are in Case 2 then $u_R + l_L \in [99, 101]$.

Case 3: Suppose, instead, $u_R \geq l_R = u_L + 1$. Since the DM does not want to increase u_R COM, STR, and WCI, tell us (using WCI):

$$(1-\beta)(u_R+1)+\beta g \succeq (1-\beta)\Big(\frac{1}{4}R+\frac{3}{4}(u_R+1)\Big)+\beta g$$

$$\Rightarrow (1-\beta)\frac{1}{4}(u_R+1)+\beta g \succeq (1-\beta)\frac{1}{4}R+\beta g. \tag{1}$$

(This argument works unless $u_R = 100$, in which case COM, STR, and continuity, tell us $(1-\beta)\frac{1}{4}u_R+\beta g \succeq (1-\beta)\frac{1}{4}R+\beta g$ and the rest of the argument is easier.) Since the DM does not lower l_R, COM, STR, and WCI, tell us (using COM, CON, and WCI) either the DM prefers the act they chose at u_L to the one they would get instead if they lowered l_R by one:

$$(1-\beta)\Big(\frac{1}{4}R+\frac{1}{2}\frac{100}{2}+\frac{1}{4}u_L\Big)+\beta g \succeq (1-\beta)\Big(\frac{1}{2}\frac{100}{2}+\frac{1}{2}u_L\Big)+\beta g$$
$$\Rightarrow (1-\beta)\frac{1}{4}R+\beta g \succeq (1-\beta)\frac{1}{4}u_L+\beta g,$$

or the DM strictly prefers one of the acts between u_L and l_R to the one they would get instead if they lowered l_R by one:

$$(1-\beta)\Big(\frac{1}{2}R+\frac{1}{2}u_L\Big)+\beta g \succ (1-\beta)\Big(\frac{1}{4}R+\frac{3}{4}u_L\Big)+\beta g$$

$$\Rightarrow (1-\beta)\Big(\frac{1}{4}R+\frac{3}{4}u_L\Big)+\beta g \succeq (1-\beta)u_L+\beta g$$

$$\Rightarrow (1-\beta)\frac{1}{4}R+\beta g \succeq (1-\beta)\frac{1}{4}u_L+\beta g.$$

Further, COM and Lemma 5 tell us:

$$\text{if } (1-\beta)\frac{1}{4}R+\beta g \sim (1-\beta)\frac{1}{4}u_L+\beta g \text{ then } (1-\beta)\frac{1}{2}u_L+\beta g \succeq (1-\beta)\frac{1}{2}R+\beta g,$$

and the DM could do strictly better by lowering both u_R and l_R to u_L by COM, STR, and WCI, so:

$$(1-\beta)\frac{1}{4}R+\beta g \succ (1-\beta)\frac{1}{4}u_L+\beta g. \tag{2}$$

If $u_R = l_R = u_L + 1$ it must then be that $l_L < u_L$ since $u_L > 50$ and so otherwise the DM could do strictly better by lowering l_L to 50, and since the DM does not want to increase l_L COM tells us (using COM, STR, TR, and WCI, and equation (1) above):

$$(1-\beta)\Big(\frac{1}{4}R+\frac{1}{2}\frac{100}{2}+\frac{1}{4}(l_L+1)\Big)+\beta g \succ (1-\beta)\frac{100}{2}+\beta g$$

$$\Rightarrow u_R+l_L \geq 99.$$

If $u_R = l_R$, then if $l_L = 0$ we have $u_R + l_L \leq 101$, while if $l_L > 0$, since the DM does not want to decrease l_L, COM, STR, and WCI, tell us (using equation (2) above, COM, STR, TR, and WCI):

$$(1-\beta)\frac{100}{2}+\beta g \succeq (1-\beta)\Big(\frac{1}{4}R+\frac{1}{2}\frac{100}{2}+\frac{1}{4}(l_L-1)\Big)+\beta g$$

$$\Rightarrow l_L+u_R \leq 101.$$

If, instead $u_R > l_R = u_L + 1$, then since the DM does not want to decrease u_R COM, STR, and WCI tell us:

$$(1-\beta)\Big(\frac{1}{4}R+\frac{3}{4}(u_R-1)\Big)+\beta g \succ (1-\beta)(u_R-1)+\beta g$$

$$\Rightarrow (1-\beta)\frac{1}{4}R+\beta g \succ (1-\beta)\frac{1}{4}(u_R-1)+\beta g,$$

and either $l_L = 0$ so $u_R + l_L \leq 100$, or $l_L > 0$ and then since the DM does not want to decrease l_L COM, STR, and WCI, tell us (using COM, STR, TR, and WCI):

$$(1-\beta)\frac{100}{2}+\beta g \succeq (1-\beta)\Big(\frac{1}{4}R+\frac{1}{2}\frac{100}{2}+\frac{1}{4}(l_L-1)\Big)+\beta g$$

$$\Rightarrow l_L+u_R \leq 101.$$

Thus, if $u_R > l_R = u_L + 1$, $l_L < u_L$ since $u_R + u_L \geq 102$ and $u_R + l_L \leq 101$, and since the DM does not benefit from increasing l_L, COM, STR, and WCI, tell us (using COM, STR, TR, WCI, and equation (1) above):

$$(1-\beta)\Big(\frac{1}{4}R+\frac{1}{2}\frac{100}{2}+\frac{1}{4}(l_L+1)\Big)+\beta g \succ (1-\beta)\frac{100}{2}+\beta g$$

$$\Rightarrow u_R+l_L \geq 99.$$

Case 4: Suppose, instead, $u_R \geq l_R > u_L + 1$. If $u_R = l_R$ then, since the DM does not want to increase u_R or decrease l_R, COM, STR, and WCI tell us:

$$(1-\beta)(u_R+1)+\beta g \succeq (1-\beta)\Big(\frac{1}{4}R+\frac{3}{4}(u_R+1)\Big)+\beta g,$$

$$\text{and } (1-\beta)\Big(\frac{1}{2}R+\frac{1}{2}(u_R-1)\Big)+\beta g \succeq (1-\beta)\Big(\frac{1}{4}R+\frac{3}{4}(u_R-1)\Big)+\beta g,$$

further, COM, WCI, and Lemma 6 tell us:

$$(1-\beta)(u_R+1)+\beta g \succ (1-\beta)\Big(\frac{1}{4}R+\frac{3}{4}(u_R+1)\Big)+\beta g$$

$$\Rightarrow (1-\beta)\Big(\frac{1}{4}R+\frac{3}{4}(u_R+1)\Big)+\beta g \succ (1-\beta)\Big(\frac{1}{2}R+\frac{1}{2}(u_R+1)\Big)+\beta g,$$

$$\text{and } (1-\beta)\Big(\frac{1}{2}R+\frac{1}{2}(u_R-1)\Big)+\beta g \succ (1-\beta)\Big(\frac{1}{4}R+\frac{3}{4}(u_R-1)\Big)+\beta g$$

$$\Rightarrow (1-\beta)\Big(\frac{1}{4}R+\frac{3}{4}(u_R-1)\Big)+\beta g \succ (1-\beta)(u_R-1)+\beta g,$$

while COM, STR, TR, WCI, and the fact that the DM does not benefit from increasing or decreasing both u_R and l_R tells us:

$$(1-\beta)(u_R+1)+\beta g \sim (1-\beta)\Big(\frac{1}{4}R+\frac{3}{4}(u_R+1)\Big)+\beta g$$

$$\Rightarrow (1-\beta)\Big(\frac{1}{4}R+\frac{3}{4}(u_R+1)\Big)+\beta g \succ (1-\beta)\Big(\frac{1}{2}R+\frac{1}{2}(u_R+1)\Big)+\beta g,$$

because if:

$$(1-\beta)(u_R+1)+\beta g \sim (1-\beta)\Big(\frac{1}{4}R+\frac{3}{4}(u_R+1)\Big)+\beta g$$

$$\text{and } (1-\beta)\Big(\frac{1}{2}R+\frac{1}{2}(u_R+1)\Big)+\beta g \succeq (1-\beta)\Big(\frac{1}{4}R+\frac{3}{4}(u_R+1)\Big)+\beta g,$$

then:

$$(1-\beta)\Big(\frac{1}{2}R+\frac{1}{2}u_R\Big)+\beta g \succ (1-\beta)\Big(\frac{1}{4}R+\frac{3}{4}u_R\Big)+\beta g$$

$$\text{and for } x \in (u_R,\, u_R+1) : (1-\beta)\Big(\frac{1}{2}R+\frac{1}{2}x\Big)+\beta g \succ (1-\beta)x+\beta g$$

so the DM could strictly benefit from increasing both u_R and l_R by 1, and further:

$$(1-\beta)\Big(\frac{1}{2}R+\frac{1}{2}(u_R-1)\Big)+\beta g \sim (1-\beta)\Big(\frac{1}{4}R+\frac{3}{4}(u_R-1)\Big)+\beta g$$

$$\Rightarrow (1-\beta)\Big(\frac{1}{4}R+\frac{3}{4}(u_R-1)\Big)+\beta g \succ (1-\beta)(u_R-1)+\beta g,$$

because if:

$$(1-\beta)\Big(\frac{1}{2}R+\frac{1}{2}(u_R-1)\Big)+\beta g\sim(1-\beta)\Big(\frac{1}{4}R+\frac{3}{4}(u_R-1)\Big)+\beta g$$

$$\text{and } (1-\beta)(u_R-1)+\beta g\succeq(1-\beta)\Big(\frac{1}{4}R+\frac{3}{4}(u_R-1)\Big)+\beta g,$$

then:

$$(1-\beta)u_R+\beta g\succ(1-\beta)\Big(\frac{1}{4}R+\frac{3}{4}u_R\Big)+\beta g$$

$$\text{and for } x\in(u_R-1,\,u_R):(1-\beta)x+\beta g\succ(1-\beta)\Big(\frac{1}{2}R+\frac{1}{2}x\Big)+\beta g$$

so the DM could strictly benefit from decreasing both u_R and l_R by one, so, either way, using TR and WCI:

$$(1-\beta)\frac{1}{4}(u_R+1)+\beta g\succeq(1-\beta)\frac{1}{4}R+\beta g\succ(1-\beta)\frac{1}{4}(u_R-1)+\beta g,$$

$$\begin{aligned}\text{and } (1-\beta)\Big(\frac{1}{4}R+\frac{1}{4}(l_R+1)\Big)+\beta g&\succ(1-\beta)\frac{1}{2}R+\beta g\\&\succeq(1-\beta)\Big(\frac{1}{4}R+\frac{1}{4}(l_R-1)\Big)+\beta g.\end{aligned}$$

(This argument works unless $u_R=100$, in which case we can argue in a similar fashion that $(1-\beta)\frac{1}{4}u_R+\beta g\succeq(1-\beta)\frac{1}{4}R+\beta g\succ(1-\beta)\frac{1}{4}(u_R-1)+\beta g$, $(1-\beta)(\frac{1}{4}R+\frac{1}{4}l_R)+\beta g\succ(1-\beta)\frac{1}{2}R+\beta g\succeq(1-\beta)(\frac{1}{4}R+\frac{1}{4}(l_R-1))+\beta g$, and the rest of the argument becomes easier, or $(1-\beta)(\frac{1}{4}R+\frac{1}{4}l_R)+\beta g\sim(1-\beta)\frac{1}{2}R+\beta g$, in which case COM, STR, TR, Lemma 4, and Lemma 5, tell us $(1-\beta)\frac{1}{2}R+\beta g\sim(1-\beta)\frac{1}{2}100+\beta g$ because if $(1-\beta)\frac{1}{2}100+\beta g\succ(1-\beta)\frac{1}{2}R+\beta g$ then $(1-\beta)\frac{1}{2}R+\beta g\sim(1-\beta)\frac{1}{2}x+\beta g$ with $x<100$ and we have $(1-\beta)\frac{1}{2}x+\beta g\sim(1-\beta)\frac{1}{4}R+\frac{1}{4}100+\beta g\succeq(1-\beta)\frac{1}{4}x+\frac{1}{4}100+\beta g$ which violates COM and STR while if $(1-\beta)\frac{1}{2}R+\beta g\succ(1-\beta)\frac{1}{2}100+\beta g$ then $(1-\beta)\frac{1}{2}R+\beta g\sim(1-\beta)\frac{1}{2}x+\beta g$ with $x>100$ which violates COM and STR, and thus WCI and TR tell us $(1-\beta)\frac{1}{4}R+\beta g\sim(1-\beta)\frac{1}{4}100+\beta g$, and so $u_L\leq 1$ because otherwise the DM could strictly benefit from lowering u_L and or l_L, and we are done.)

If, instead, $u_R>l_R$ then we can reach the same conclusions (summarized in equations (3) and (4) below) because the upper and lower bounds for R, COM, STR, TR, and WCI, tell us:

$$(1-\beta)(u_R+1)+\beta g\succeq(1-\beta)\Big(\frac{1}{4}R+\frac{3}{4}(u_R+1)\Big)+\beta g$$

and

$$(1-\beta)\Big(\frac{1}{4}R+\frac{3}{4}(u_R-1)\Big)+\beta g\succ(1-\beta)(u_R-1)+\beta g$$

$$\Rightarrow (1-\beta)\frac{1}{4}(u_R+1)+\beta g \succeq (1-\beta)\frac{1}{4}R+\beta g \succ (1-\beta)\frac{1}{4}(u_R-1)+\beta g, \tag{3}$$

and

$$(1-\beta)\Big(\frac{1}{4}R+\frac{3}{4}(l_R+1)\Big)+\beta g \succ \Big(\frac{1}{2}R+\frac{1}{2}(l_R+1)\Big)+\beta g$$

and

$$(1-\beta)\Big(\frac{1}{2}R+\frac{1}{2}(l_R-1)\Big)+\beta g \succeq (1-\beta)\Big(\frac{1}{4}R+\frac{3}{4}(l_R-1)\Big)+\beta g$$

$$\begin{aligned}\Rightarrow (1-\beta)\Big(\frac{1}{4}R+\frac{1}{4}(l_R+1)\Big)+\beta g &\succ (1-\beta)\frac{1}{2}R+\beta g\\ &\succeq (1-\beta)\Big(\frac{1}{4}R+\frac{1}{4}(l_R-1)\Big)+\beta g.\end{aligned} \tag{4}$$

(Again, this argument works unless $u_R = 100$, in which case COM, STR, and continuity, tell us $(1-\beta)\frac{1}{4}u_R+\beta g \succeq (1-\beta)\frac{1}{4}R+\beta g \succ (1-\beta)\frac{1}{4}(u_R-1)+\beta g$, in addition to $(1-\beta)(\frac{1}{4}R+\frac{1}{4}(l_R+1))+\beta g \succ (1-\beta)\frac{1}{2}R+\beta g \succeq (1-\beta)(\frac{1}{4}R+\frac{1}{4}(l_R-1))+\beta g$, and the rest of the argument becomes easier.)

Similarly, since the DM does not want to increase u_L or decrease l_L, COM, STR, CON, TR, and WCI, tell us:

$$(1-\beta)\Big(\frac{1}{2}R+\frac{1}{2}(u_L+1)\Big)+\beta g \succeq (1-\beta)\Big(\frac{1}{4}R+\frac{1}{2}\frac{100}{2}+\frac{1}{4}(u_L+1)\Big)+\beta g,$$

$$\text{and } (1-\beta)\frac{100}{2}+\beta g \succeq (1-\beta)\Big(\frac{1}{4}R+\frac{1}{2}\frac{100}{2}+\frac{1}{4}(l_L-1)\Big)+\beta g, \tag{5}$$

$$\Rightarrow (1-\beta)\frac{1}{2}R+\beta g \succeq (1-\beta)\Big(\frac{1}{4}R+\frac{1}{4}(100-u_L-1)\Big)+\beta g, \tag{6}$$

$$\text{and } (1-\beta)\frac{1}{4}(100-l_L+1)+\beta g \succeq (1-\beta)\frac{1}{4}R+\beta g. \tag{7}$$

(This argument works unless $l_L = 0$, in which case COM, STR, and continuity, tell us $(1-\beta)\frac{1}{2}R+\beta g \succeq (1-\beta)(\frac{1}{4}R+\frac{1}{4}(100-u_L-1))+\beta g$, and $(1-\beta)\frac{1}{4}(100)+\beta g \succeq (1-\beta)\frac{1}{4}R+\beta g$, and the rest of the argument is easier.) Thus, equations (3) and (7), COM, STR, and TR, tell us:

$$(1-\beta)\frac{1}{4}(100-l_L+1)+\beta g \succ (1-\beta)\frac{1}{4}(u_R-1)+\beta g \Rightarrow 101 \geq u_R+l_L.$$

Thus $l_L < u_L$ since $u_R+u_L \geq 102$. Next, notice that COM, STR, WCI, and the fact that the DM does not want to decrease u_L or increase l_L, tell us:

$$(1-\beta)\Big(\frac{1}{4}R+\frac{1}{2}\frac{100}{2}+\frac{1}{4}(u_L-1)\Big)+\beta g \succ (1-\beta)\Big(\frac{1}{2}R+\frac{1}{2}(u_L-1)\Big)+\beta g, \tag{8}$$

$$\text{and } (1-\beta)\Big(\frac{1}{4}R + \frac{1}{2}\frac{100}{2} + \frac{1}{4}(l_L+1)\Big) + \beta g \succ (1-\beta)\frac{100}{2} + \beta g. \tag{9}$$

Thus, TR and WCI and equations (8), (6), (7), and (9), tell us:

$$\begin{aligned}(1-\beta)\Big(\frac{1}{4}R + \frac{1}{4}(100-u_L+1)\Big) + \beta g &\succ (1-\beta)\frac{1}{2}R + \beta g \\ &\succeq (1-\beta)\Big(\frac{1}{4}R + \frac{1}{4}(100-u_L-1)\Big) + \beta g\end{aligned} \tag{10}$$

$$\text{and } (1-\beta)\frac{1}{4}(100-l_L+1) + \beta g \succeq (1-\beta)\frac{1}{4}R + \beta g \succ (1-\beta)\frac{1}{4}(100-l_L-1) + \beta g. \tag{11}$$

So, using both equation (3) and equation (11) we get, using COM, STR, and TR, $u_R + l_L \in [99, 101]$. Next, using both equation (4) and equation (10) we get, using COM, STR, and TR:

$$(1-\beta)\frac{1}{4}(R + l_R + 1) + \beta g \succ (1-\beta)\frac{1}{4}(R + 100 - u_L - 1) + \beta g,$$

and

$$(1-\beta)\frac{1}{4}(R + 100 - u_L + 1) + \beta g \succ (1-\beta)\frac{1}{4}(R + l_R - 1) + \beta g,$$

$$\Rightarrow u_L + l_R \in [99, 101].$$

Case 5: Finally, if $u_R = u_L = u$ (which implies $u \geq 51$), then if both intervals are non-degenerate COM, STR, WCI, and the upper bounds, imply (since the DM does not lower both upper bounds):

$$(1-\beta)\Big(\frac{1}{2}\frac{100}{2} + \frac{1}{2}(u-1)\Big) + \beta g \succ (1-\beta)(u-1) + \beta g.$$

This results in a contradiction with COM and STR immediately. If, instead, both intervals are degenerate then similarly, COM,STR, WCI, tells us the DM could do strictly better by decreasing all four bounds by one since $u \geq 51$.

Further, if one interval is degenerate and one is non-degenerate, then assume without loss of generality (given $u_R = u_L = u$) that $l_L < u$. Then, COM, STR, WCI, and the fact that the DM does not benefit from increasing u_R tells us (using WCI):

$$(1-\beta)(u+1) + \beta g \succeq (1-\beta)\Big(\frac{1}{4}R + \frac{3}{4}(u+1)\Big) + \beta g$$

$$\Rightarrow (1-\beta)\frac{1}{4}(u+1) + \beta g \succeq (1-\beta)\frac{1}{4}R + \beta g,$$

COM, STR, WCI, and the fact that the DM does not change l_L, thus tell us (using COM, STR, TR, and WCI):

$$(1-\beta)\Big(\frac{1}{4}(l_L+1)+\frac{1}{2}\frac{100}{2}+\frac{1}{4}R\Big)+\beta g \succ (1-\beta)\frac{100}{2}+\beta g$$

$$\Rightarrow u_R + l_L \geq 99,$$

$$\text{and } (1-\beta)\frac{100}{2}+\beta g \succeq (1-\beta)\Big(\frac{1}{4}(l_L-1)+\frac{1}{2}\frac{100}{2}+\frac{1}{4}R\Big)+\beta g.$$

(This argument works unless $l_L = 0$, in which case $u_R + l_L \leq 101$ in addition to $u_R + l_L \geq 99$ and we are done.) COM, STR, WCI, and the fact that the DM does not benefit from lowering l_R, tell us (using COM, STR, TR, and WCI):

$$(1-\beta)\Big(\frac{1}{4}R+\frac{1}{4}(u-1)+\frac{1}{2}\frac{100}{2}\Big)+\beta g \succ (1-\beta)\Big(\frac{1}{2}(u-1)+\frac{1}{2}\frac{100}{2}\Big)+\beta g$$

$$\Rightarrow (1-\beta)\frac{1}{4}R+\beta g \succ (1-\beta)\frac{1}{4}(u-1)+\beta g \Rightarrow u_R + l_L \leq 101.$$

(It cannot be that $(1-\beta)(\frac{1}{4}R+\frac{1}{4}(u-1)+\frac{1}{2}\frac{100}{2})+\beta g \sim (1-\beta)(\frac{1}{2}(u-1)+\frac{1}{2}\frac{100}{2})+\beta g$ because then the DM could strictly benefit from either lowering l_R or from lowering all three of u_R, u_L, and l_R by one by COM, STR, TR, and WCI.) □

Theorem 9.1 If the preferences of the DM satisfy WP, COM, WSTR, CON, TR, CI, and continuity, and they answer all three questions in conjunction and assign equal and strictly positive weights to the PE questions, then $l_L \geq u_L - 1$, $l_R \geq u_R - 1$, $u_L + u_R \leq 101$, and $l_L + l_R \geq 99$.

Proof. Let $\alpha \in \{0, 0.01, 0.02, \ldots, 1\}$ be the chance that the DM selects to assign to betting on R as opposed to L in the binary choice question. Let $\beta \in [0, 1)$ denote the DM's weight on the binary choice question when they answer the PE questions. Lemma 4 tells us there is $x_L \in X$ and $x_R \in X$ such that $L \sim x_L$ and $R \sim x_R$. COM, STR (which is satisfied by Lemma 4), and CI, tell us to consider $\max(x_L, x_R) > 50$, because otherwise $l_L = u_L = l_R = u_R = 50$ and we are done. It is thus without loss to assume $\max(x_L, x_R) = x_R > 50$, and thus WP, COM, CON, and TR tell us $x_L \leq 50$, so $\alpha = 1$ if $\beta > 0$, $u_R \geq u_L$, and $l_R \geq l_L$.

Assume that the DM did report an interval of size two or more and we will reach a contradiction. Lemma 8 thus tells us $u_R \geq 51$. If $u_R = u_L$ then $l_R = u_R$ otherwise COM and STR tell us the DM could strictly benefit from lowering u_R and u_L by one, but then COM, STR, TR, and CI tell us the DM should lower u_L by one. So, $u_R > u_L$. If $l_R < u_R$ then the fact that the DM does not lower u_R, COM, STR, and WCI, tell us:

$$(1-\beta)\Big(\frac{1}{4}R+\frac{3}{4}(u_R-1)\Big)+\beta R \succ (1-\beta)(u_R-1)+\beta R$$

$$\Rightarrow (1-\beta)\frac{1}{4}R+\beta R \succ (1-\beta)\frac{1}{4}(u_R-1)+\beta R \Rightarrow x_R > u_R-1,$$

but then Lemma 7, COM, STR, TR, and CI, tell us $l_R \geq u_L$ because if $l_R < u_L$ then $l_R+1 \leq u_R-1 < x_R$ and increasing l_R (since Lemma 7 tells us $l_L \leq l_R$) changes the chosen act for $y \in [l_R,\, l_R+1)$ from:

$$(1-\beta)\Big(\frac{1}{2}\frac{100}{2}+\frac{1}{2}y\Big)+\beta R$$

to:

$$(1-\beta)\Big(\frac{1}{4}R+\frac{1}{2}\frac{100}{2}+\frac{1}{4}y\Big)+\beta R,$$

which then strictly benefits them and thus, also, $l_R \geq u_R-1$ because if $l_R < u_R-1$ then $l_R+1 \leq u_R-1 < x_R$ and increasing l_R changes the chosen act for $y \in (l_R,\, l_R+1)$ from:

$$(1-\beta)\Big(\frac{1}{4}R+\frac{3}{4}y\Big)+\beta R$$

to:

$$(1-\beta)\Big(\frac{1}{2}R+\frac{1}{2}y\Big)+\beta R,$$

and changes the chosen act for $y = l_R$, depending on if $l_R > u_L$ or $l_R = u_L$, from:

$$(1-\beta)\Big(\frac{1}{4}R+\frac{3}{4}l_R\Big)+\beta R$$

to:

$$(1-\beta)\Big(\frac{1}{2}R+\frac{1}{2}l_R\Big)+\beta R,$$

or from:

$$(1-\beta)\Big(\frac{1}{2}\frac{100}{2}+\frac{1}{2}l_R\Big)+\beta R$$

to:

$$(1-\beta)\Big(\frac{1}{4}R+\frac{1}{2}\frac{100}{2}+\frac{1}{4}l_R\Big)+\beta R,$$

and thus since one interval was assumed to be of size two or more we require $l_L \leq u_L-2$, and thus to prevent the DM from strictly benefiting from either lowering u_L or increasing l_L

COM, STR, and CI, tell us we require:

$$(1-\beta)\Big(\frac{1}{4}x_R+\frac{1}{2}\frac{100}{2}+\frac{1}{4}(u_L-1)\Big)+\beta x_R\sim(1-\beta)\Big(\frac{1}{4}R+\frac{1}{2}\frac{100}{2}+\frac{1}{4}(u_L-1)\Big)+\beta R$$

$$\succ(1-\beta)\Big(\frac{1}{2}R+\frac{1}{2}(u_L-1)\Big)+\beta R\sim(1-\beta)\Big(\frac{1}{2}x_R+\frac{1}{2}(u_L-1)\Big)+\beta x_R,$$

and

$$(1-\beta)\Big(\frac{1}{4}x_R+\frac{1}{2}\frac{100}{2}+\frac{1}{4}(l_L+1)\Big)+\beta x_R\sim(1-\beta)\Big(\frac{1}{4}R+\frac{1}{2}\frac{100}{2}+\frac{1}{4}(l_L+1)\Big)+\beta R$$

$$\succ(1-\beta)\frac{100}{2}+\beta R\sim(1-\beta)\frac{100}{2}+\beta x_R,$$

which contradicts COM, STR, TR, and CI since then:

$$(1-\beta)\Big(\frac{100}{4}\Big)>(1-\beta)\Big(\frac{1}{4}x_R+\frac{1}{4}(u_L-1)\Big)\text{ and}$$
$$(1-\beta)\Big(\frac{1}{4}x_R+\frac{1}{4}(l_L+1)\Big)>(1-\beta)\frac{100}{4}.$$

If, instead, $l_R=u_R$, then once again $l_L\leq u_L-2$, and considering the conditions required for the DM to not strictly benefit from lowering u_L or increasing l_L above, COM, STR, TR, and CI, create the same contradiction.

Further, Theorem 3.1 then tells us that $u_R+u_L\leq 102$, and if $u_R+u_L=102$ it cannot be that $u_R=u_L=51$ as then $\min(l_L,\,l_R)\leq 50$ and COM, STR, TR, and CI, tell us the DM can do strictly better by either lowering u_L by one if $\max(l_L,\,l_R)=51$ (remember $l_R\geq l_L$) or by decreasing both u_R and u_L by one if $\max(l_L,\,l_R)\leq 50$, so if $u_R+u_L=102$ it must then be that $u_L\leq u_R-2$, $u_L=l_L+1$, $u_R=l_R+1$, and since we thus have $u_L\leq l_R$, then $u_L<l_R$, $u_L\leq 50$, $u_R\geq 52$, and the fact that the DM does not lower u_R, COM, STR, TR, and WCI, tell us:

$$(1-\beta)\Big(\frac{1}{4}R+\frac{3}{4}(l_R)\Big)+\beta R\succ(1-\beta)(l_R)+\beta R\Rightarrow x_R>l_R=100-l_L,$$

and COM, STR, TR, and CI, tell us the DM should lower u_L, so $u_L+u_R\leq 101$.

Theorem 3.1 also then tells us that $l_R+l_L\geq 98$, and if $l_R+l_L=98$ it cannot be that $l_R=l_L=49$ as then $\max(u_L,\,u_R)\geq 50$ and COM, STR, TR, and CI, tell us the DM can do strictly better by either increasing l_R by one if $\min(u_L,\,u_R)=49$ (remember $u_R\geq u_L$) or by increasing both l_R and l_L by one if $\min(u_L,\,u_R)\geq 50$, so if $l_R+l_L=98$ it must then be that $l_L\leq l_R-2$, $u_L=l_L+1$, $u_R=l_R+1$, and since $u_L\leq l_R$, then $u_L<l_R$, $l_L\leq 48$,

$l_R \geq 50$, and the fact that the DM does not increase l_L, COM, STR, TR, and WCI, tell us:

$$(1-\beta)\Big(\frac{1}{4}R + \frac{1}{4}\frac{100}{2} + \frac{1}{4}(u_L)\Big) + \beta R \succ (1-\beta)(\frac{100}{2}) + \beta R \Rightarrow x_R > 100 - u_L = u_R,$$

and COM, STR, TR, and CI, tell us the DM should increase l_R, so $l_L + l_R \geq 99$. □

C Additional Experimental Details

This subsection provides more details on the experiment. Subjects were invited to participate and register for the experiment using ORSEE (Greiner, 2015), and all sessions were done remotely between November 21st and December 2nd 2021. Subjects registered for a particular 10 hour time slot and had to start the experiment before the deadline. Before starting the experiment, subjects were required to read and sign a consent waiver.

For each type of round, Big-Shape and Lottery, subjects are first trained on the interface before completing a quiz with 5 questions: some are multiple choice questions and others involve subjects correctly using the interface sliders, in order to verify their understanding of the interface. If a subject finishes a quiz they receive \$2.50 (these are Canadian dollars, as with all other dollar amounts referred to in this paper) and, further, for each quiz question they answer correctly on the first attempt they are rewarded with an additional \$0.50. There was thus a maximum bonus payment of \$2.50+\$2.50 per quiz, or \$10 in total across quizzes. Subjects who do not answer a quiz question correctly are not able to advance to the next question without first providing a correct response. Further, in the multiple choice questions the answers are randomly permuted each time the question is answered incorrectly, so the subject cannot simply guess the possible answers in order. We observe how many attempts it takes each subject to answer each quiz question correctly. In general, subjects invested a great deal of time into the training with a median time spent on the two trainings and quizzes of 37.5 minutes. The training and quiz questions can be found in Section E.

In total, 250 subjects completed at least one quiz, and out of these subjects 218 ended up completing the entire experiment (both quizzes and all 24 rounds of decision problems). The 218 subjects that completed the entire experiment constitute the dataset we use for analysis.

The attrition rate does not seem concerning given that the experiment was conducted on-line during a busy part of the semester and subjects had the option to exit the experiment at the bottom of the screen in every round. During the training subjects are made aware of this feature of the experiment, and are informed that if they exit before the round with the question that was randomly selected to determine if they won the prize then they will not

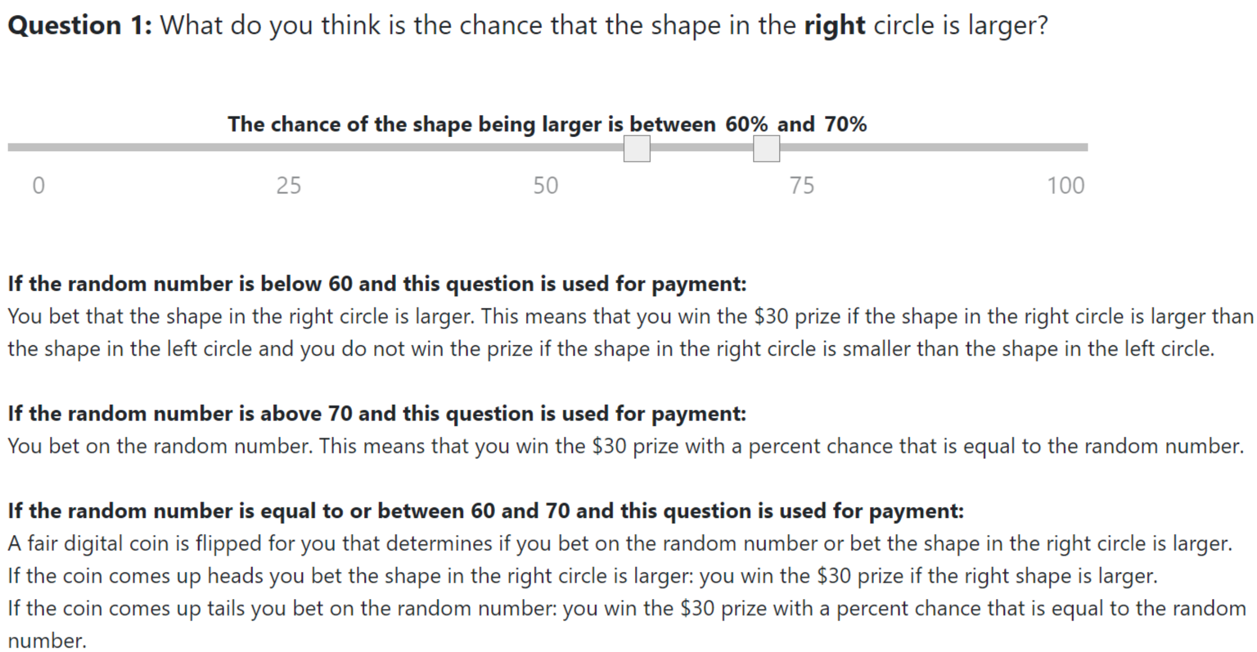


Figure 4: Graphical Interface for "PE" Problems

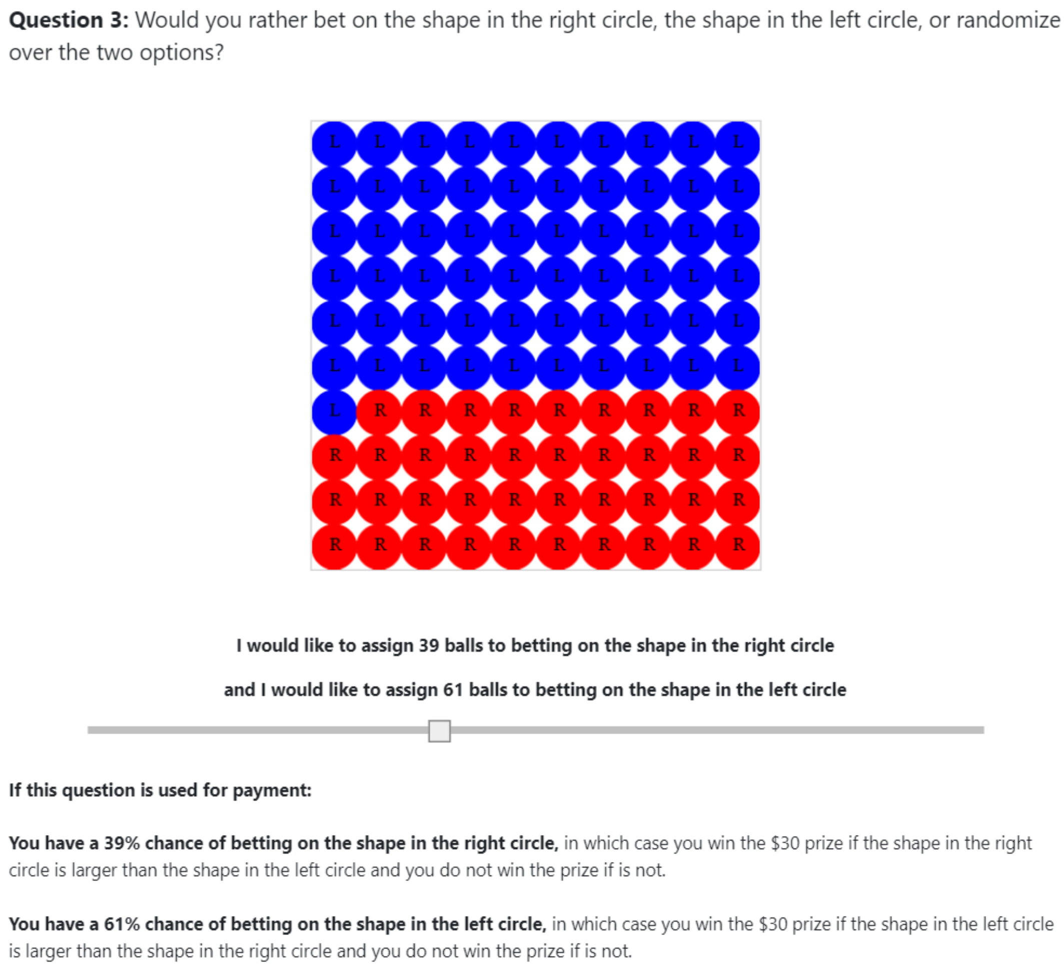
An example of a "PE" problem. Note that the text below the double-sliders updates automatically when the sliders are moved, and explains to the participant the payoff consequences of their choices.

win the prize, but would still receive any payments they earned from their performance on a quiz, and if they answered the question that was randomly selected to determine if they won the prize before they exited then choosing to exit would not impact whether or not they won the prize. We gave the subjects this forgiving exit option because we want their choices to be indicative of their preferences, not simply a bi-product of them trying to finish the experiment so that whatever benefit they have already earned would not be lost from exiting.

Subjects that did not finish the first quiz are also not very concerning. Subjects signed up for sessions several days in advance and knew that if they did not attempt the experiment then it could negatively impact their ability to participate in future experiments, so they may have started the experiment just to avoid being excluded from future experiments, or realized that the experiment was a larger time commitment than they had anticipated and stopped before the investment of time required to get through the initial training. Also, remember that subjects could not progress past a quiz question without first answering it correctly, which may have encouraged some of the subjects that did not understand the experimental design as well to quit.

Screenshots of the interface used in the Big-Shape rounds are provided in Figures 4 and 5. The initial position of the double-sliders for the PE questions are 0 and 100 for the

Figure 5: Graphical Interface for Randomization Problem



In order to visualize the randomization between betting on the left shape or the right shape, the participant chooses a composition of an urn that contains 100 balls. A randomly chosen ball determines the bet. Note that the participant can choose to bet on the right or the left shape (for sure) by choosing all red or all blue balls. The dynamically changing text below the urn explains the implications of choice to the participant.

lower and upper bound respectively. The initial position of the slider for the binary choice question is uniformly distributed over $\{0, 1, \dots, 100\}$ and is recorded in our data.

We collected complete process data for all 5 sliders in each round. We see when each change of each slider is made, and from what position to what position it is moved.

C.1 Lottery Rounds

In the section of the experiment that includes the "Lottery" rounds, subjects compare the same shapes as in the Big-Shape rounds, but with different questions. In these rounds, subjects respond to the following three questions:

1. What percent of the right circle do you think is covered by the shape in the right circle?
2. What percent of the left circle do you think is covered by the shape in the left circle?
3. Would you rather bet on the shape in the right circle, the shape in the left circle, or randomize over the two options?

The elicitation mechanism in the Lottery rounds is almost identical to the mechanism in the Big-Shape rounds. The difference in the incentive scheme of the questions is that when a Lottery round question is used for payment a bet on the left or right shape results in the subject winning with a chance that is equal to the proportion of the circle covered by the shape (a proportion that is not known to the participant) instead of winning if the shape is larger (as would be the case if a Big-Shape round question is used for payment instead). The interface in an example Lottery round can be found by clicking here.

To answer questions 1 and 2 the subjects use double-sliders similar to the ones in the Big-Shape rounds. The initial position of the double-sliders for both shapes are 0 and 100 for the lower and upper bound respectively. If question 1 or 2 above is used for payment then the double-slider response of the subject is compared to a random lottery r between 0 and 100. If r is below both sliders then the subject bets on the shape that is having its relative size elicited, which means they win the \$30 prize with a chance that is equal to the percent of the circle that is covered by the shape, if r is above both sliders then they bet on r, which means they win the prize with an $r\%$ chance, and if r is equal to or between the sliders then they bet on the shape or r with equal chances.

To answer question 3, subjects use a single slider to set the color composition of an urn which contains 100 balls, just like in the Big-Shape rounds, but with different consequences. The initial position of the slider for the urn is uniformly distributed over $\{0, 1, \ldots, 100\}$ and is recorded in our data. Suppose they set the slider so that the urn contains x blue balls, and consequently $100 - x$ red balls. If the question is used for payment there is a $x\%$ chance they bet on the left shape and win the prize with a chance that is equal to the percent of the left circle that is covered by the left shape, and there is a $(100 - x)\%$ chance they bet on the right shape and and win the prize with a chance that is equal to the percent of the circle that is covered by the right shape.

In all 12 of the Lottery rounds the subject gets to view the image of the shapes for the round, and the images that they see in the Lottery rounds are the images that they see in the Big-Shape rounds with the exception of the image that is suppressed for the subject in the Big-Shape round where they do not see an image.

We have complete process data for all 5 sliders in each round. We see when each change of each slider is made, and from what position to what position it is moved.

Table 6: Behavior Conditional On Treatment

	Treatment 1	Treatment 2	Treatment 3	Treatment 4
Subjects that violate Theorem 1.1	78% (51/65)	84% (48/57)	69% (34/49)	74% (35/47)
Rounds that violate Theorem 1.1	42%	36%	31%	33%
Rounds with randomization over L and R	64%	61%	64%	62%
Rounds with a non-degenerate PE interval	44%	42%	34%	41%

Treatment 1: Big-shape rounds first, binary choice question at the bottom; Treatment 2: Big-shape rounds first, binary choice question the top. Treatment 3: Lottery rounds first, binary choice question at the bottom. Treatment 4: Lottery rounds first, binary choice question at the top. The percent of subjects and rounds that are inconsistent with MPP (violate Theorem 1.1), the percent of rounds with randomization over betting on the left and right shape, and the percent of rounds with a non-degenerate interval, are roughly the same in all the 4 main order treatments.

C.2 Main Order Treatments

As is mentioned in Section 4, we have four main order treatments, with subjects randomly distributed among them in a two-by-two factorial design. The two dimensions we vary are whether the subjects complete the Big-Shape or Lottery rounds first, and whether the three questions within each type of round are ordered such that the binary choice question appears at the top or at the bottom of the screen. These treatments are meant to control for order effects among the different questions and among the different tasks, but any order effects are inconsequential to our main findings as is evident from Table 6, which describes behavior in the Big-Shape rounds. The training and quiz questions for each of these four main treatments are tailored to the specific treatment, i.e. the number and order of the three questions in the training and the order of the trainings and quizzes differ.

C.3 Images and Order

As mentioned above, for each subject we use the same set of 12 comparisons between shapes for both the Big-Shape and Lottery rounds. Each comparison has a proper name which uniquely identifies it regardless of the order in which it is seen by the subjects: Image 1, Image 2, Image 2B, Image 3, Image 4, Image 4B, Image 5, Image 6, Image 7, Image 8, Image 9, Image 10, Image 10B, Image 11, and Image 12, most of which can be found in Figure 6. Comparisons differ from each other in several ways. Some include comparisons between squares, while others include comparisons between rectangles, while Image 7, for example, includes a comparison between a cross and a square. Moreover, for some of comparisons, we have removed a small slice from both shapes in order to make it more challenging for subjects to measure the objects on their computer using a ruler.[41]

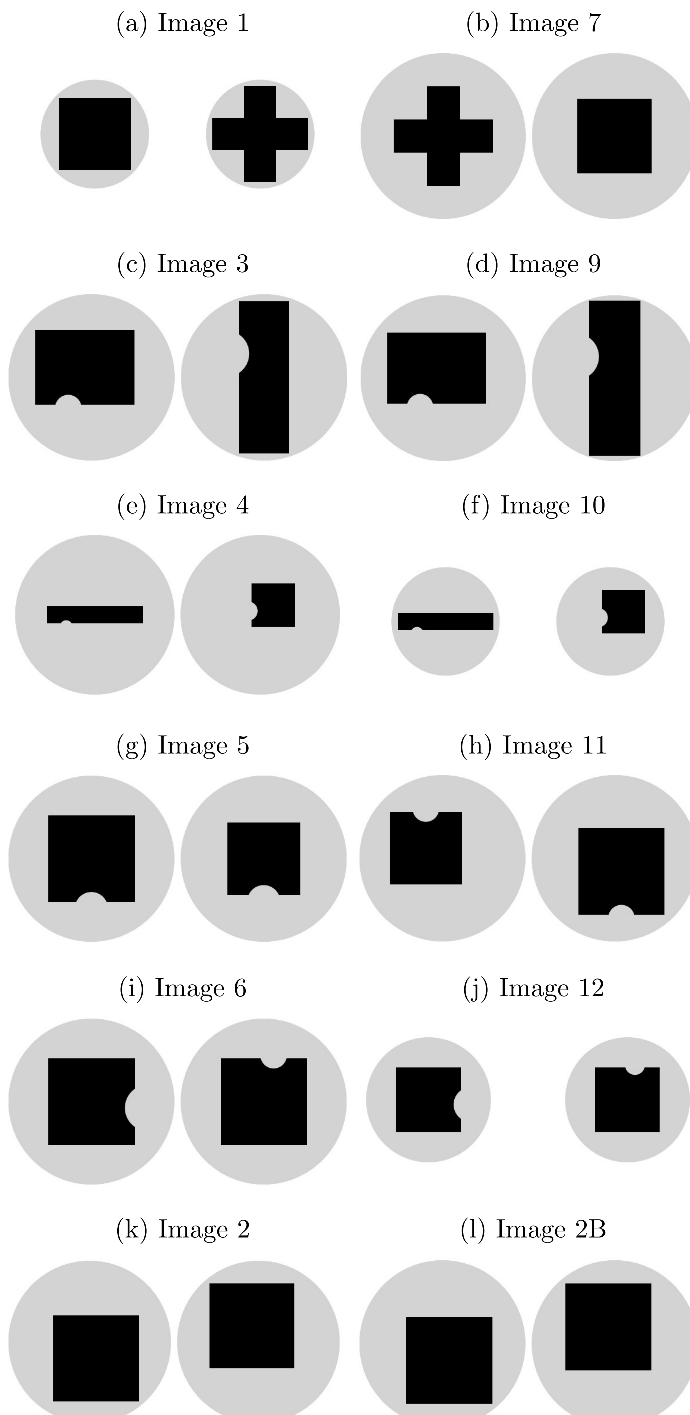
Exceptionally, the first comparison faced by subjects in the Big-Shape rounds does not include any images, yet subjects are still asked to respond to the three questions. This provides a sanity check, to evaluate if the subject reduces compound lotteries and also allows us to detect an inherent desire to randomize which is independent of the characteristics of the choice objects, i.e. subjects may just like to randomize for the sake of it as implied by the model of Fudenberg et al. (2015). In that model a DM can strictly benefit from randomizing over options even if they are identical, could randomize over options that do not provide the same value to the DM as randomization for randomization's sake is valuable, and could thus choose a non-degenerate belief interval even if they posses a degenerate 'point' belief about the state of the world. In our experiment, however, 70% of subjects listed degenerate intervals of 50 as their PEs for both the left and right shape when faced with the Big-Shape round with no image, and thus they did not demonstrate a desire to randomize for the sake of it in general.

The twelve types of images are grouped in ordered blocks of three, called A (Image 1, Image 2 or Image 8, and Image 3), B (Image 4 or Image 4B, Image 5, and Image 6), C (Image 7, Image 8 or Image 2B, and Image 9), and D (Image 10 or Image 10B, Image 11, and Image 12) respectively. Subjects see one of four possible orderings of the blocks: ABCD, BCDA, CDAB, or DABC. These different block orderings allow us to partially control for order effects among the comparisons, but we do not see any substantial evidence of order effects. We chose to change the order in blocks so as to avoid two adjacent comparisons that may appear too similar, leaving the subject with the impression that they are repeating the same comparisons over and over. While it is true that some of the comparisons are, in fact, very similar, there are no two that are identical in either type of round, even if they appear so.

In order to detect any potential bias towards the image on the right or left side, we also exchange both Image 4 and Image 10 for both Image 4B and Image 10B for about half of the subjects, which are 180 degree rotations of Image 4 and Image 10. We do not see evidence of a substantial bias towards one side. Finally, some subjects see Image 2 that has the same shape areas as Image 8 (but which side has the larger shape is switched) while others see Image 2B that is slightly different than Image 2. Subjects that see Image 2 have it in block A and Image 8 in block C, whereas subjects that see Image 2B have it in block C and Image 8 in block A. This allows us to attempt to hold difficulty fixed while varying the

[41]There exists software that allows for users to measure the areas fairly precisely. Even so, any subject who uses this software would be straightforward to identify, hence we do not consider this to be a significant concern.

Figure 6: Images Used in the Experiment



dimensionality of the comparison, and test for substantial order effects (which we do not see evidence of).

There are thus four different image set compositions subjects see, and each composition has four potential block orderings (ABCD, BCDA, CDAB, or DABC), for a total of 16 different sequences of images. Along with the four main treatments, this means there are 64 possible subject experiences.

D Robustness Checks and Additional Results

D.1 More on Squares vs Rectangles

As discussed in Section 5.3, there are two sets of choice problems that are particularly helpful for understanding the circumstances in which one should expect behavior that is inconsistent with MPP and CP, and hence suggests incomplete preferences. Each subject faces a round with two rectangles (Image 8 in Figure 2), which we call the **rectangles of interest**, and a round with two squares (either Image 2 in Figure 2 or Image 2B in Figure 6 in the supplementary materials), which we call the **squares of interest**. Importantly, the areas of the two squares in Image 2 are identical to the areas of the rectangles of interest (but the side with the bigger shape is switched) and the areas of the two squares in Image 2B are essentially identical to the areas of the squares in Image 2 as can be observed in Figure 6.[42]

The goal of these comparisons is to study if and how subjects' choices vary when they confront choice problems that are similar in terms of how challenging it is to identify the larger shape, but differ in terms of "dimensionality" – when sides' length are the relevant attributes of the choice objects. According to this interpretation, it follows that squares have a single attribute, while rectangles have two attributes. We take the chance that a subject bets on the larger shape in the binary choice question as a suitable proxy for how challenging it is to identify the larger shape. The 105 subjects (call them Group 1) that see Image 2B and Image 8 have an average chance of betting correctly of 54% when faced with both the squares and rectangles of interest, and the 113 subjects (call them Group 2) that see Image 2 and Image 8 have average chances of betting correctly of 62% when faced with the squares of interest and 56% when faced with the rectangles of interest, which is not a statistically significant difference.[43] We conclude that identifying the larger shape when faced with the squares and rectangles of interest is similarly challenging.

[42]The squares in Image 2 take up 35.45% and 34.63% of their circles respectively while the squares in Image 2B take up 34.95% and 34.38% of their circles respectively.

[43]The differences in the average chances of betting correctly for the squares and rectangles of interest are not statistically significantly different at the 10% level for either group according to two paired t-tests.

Both groups, however, are much more likely to make choices that are inconsistent with MPP when it is assumed the DM isolates the three choice problems (Theorem 1 – that focuses only on the PE questions, Theorems 2 and 3 – that incorporate the binary choice question) when faced with the rectangles of interest than when faced with the squares of interest. While the proportion of subjects that is not consistent with MPP (assuming the DM isolates the three choice problems) when faced with squares of interest is about 30% for both groups, it increases to about 50% when each group is faced with the rectangles of interest, and for both groups this difference is statistically significant at the 1% level according to two-sided Fisher's exact test. Ignoring the rounds with the 2 easy images (Image 5 and Image 11 in Figure 6), the lowest chance of violating MPP (assuming the DM isolates the three choice problems) is when faced with the squares of interest, at least 6 percentage points lower than any other image they faced for both groups. The probability of the same kind of violations when faced with the rectangles of interest is close to the highest chance (at most 7 percentage point less than the maximal image for both groups).[44] Unsurprisingly, the PE intervals chosen for Image 3 in Figure 6 have the highest chance of being inconsistent with the MPP.[45]

If we instead look for violations of Convex Preferences (CP) when it is assumed the DM isolates the three choice problems (Theorem 3 or Theorem 4 for the squares and rectangles of interest then for both Group 1 and Group 2 it is more likely that CP are rejected when facing the rectangles of interest as opposed to the squares of interest,[46] but the difference is not statistically significant unless we aggregate across the two groups, at which point the chance of violations for the squares versus rectangles of interest is statistically significantly different at the 5% level according to two-sided Fisher's exact test.

Figure 7 demonstrates the markedly different aggregate behavior for the upper and lower bounds for bets on the right shape in rounds in which the subjects choose between bets on the squares of interest versus rounds when the choice is between bets on the rectangles of interest. When faced with the squares of interest subjects frequently put all 5 sliders to 0.5 to indicate their indifference between the squares, whereas when faced with the rectangles of interest they are more likely to report a non-degenerate interval. When faced with choice

[44]The two groups have the images in the rounds that immediately proceed their respective rounds with squares and rectangles of interest swapped, so it is not differences in the image(s) seen before these rounds that is driving differences in behavior. Further, which of the squares and rectangles of interest is seen first is randomly determined for each subject in each group, so it is not order effects that are driving the differences (see Section C.3 for more details).

[45]The chance of betting on the larger shape was only 44%, and 57% of subjects made choices inconsistent with MPP.

[46]Group 1 violates CP 18% of the time when faced with squares of interest and 27% of the time when faced with the rectangles of interest, while Group 2 violates the CP 19% of the time when faced with squares of interest and 29% of the time when faced with the rectangles of interest.

between bets on the squares of interest subjects report a non-degenerate interval 32% of the time, while reporting a non-degenerate interval 54%[47] of the time when faced with the rectangles of interest, even though, as we argued above, how challenging it is to identify the larger shape is essentially the same for the squares and rectangles of interest.

As discussed in Section 5.3, there are two sets of choice problems that are particularly helpful for understanding when one should expect behavior that is inconsistent with MPP and CP models, and hence suggests incompleteness of preferences: the **rectangles of interest** and the **squares of interest**. As is argued in Section 5.3, identifying which shape is larger is equally challenging, but the rectangles of interest simulate a two-dimensional decision problem whereas the squares of interest represent a one-dimensional decision problem.

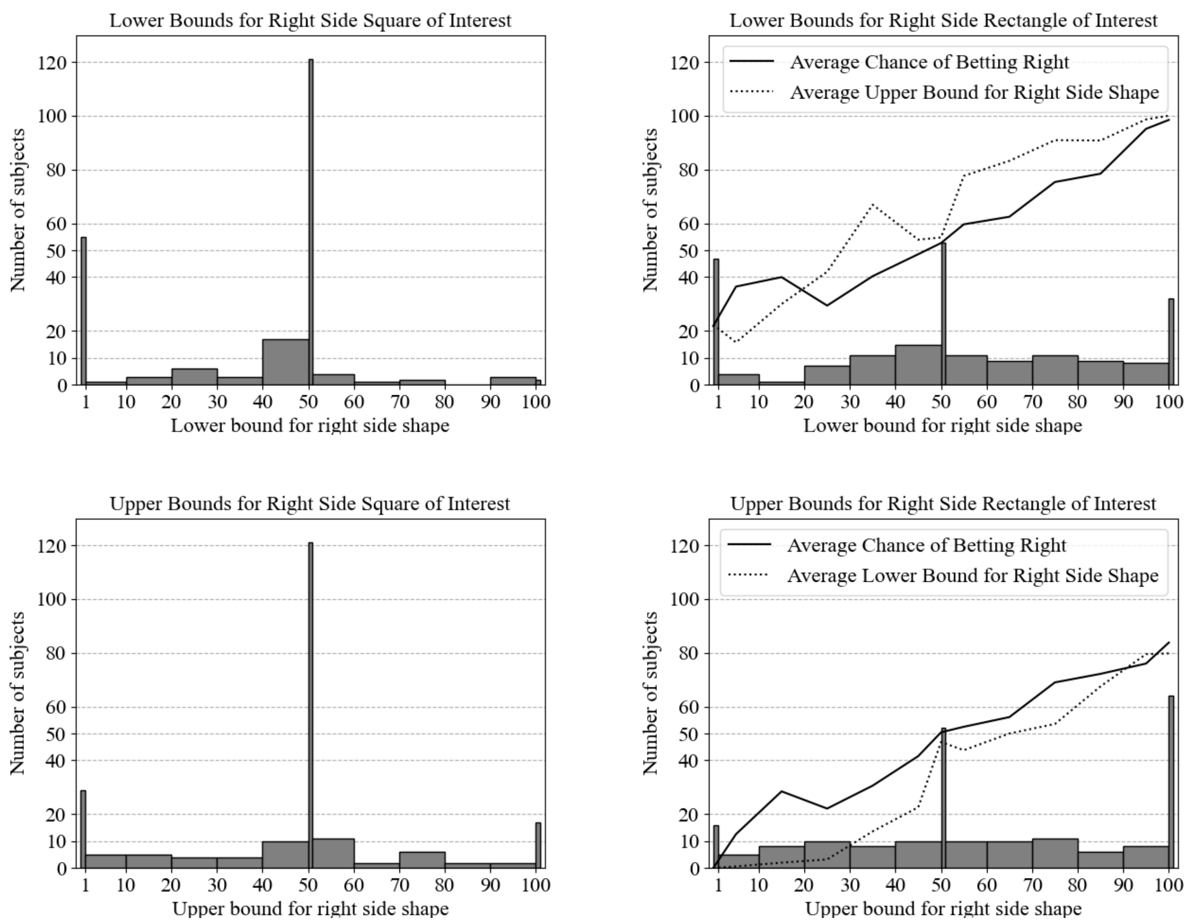
Subjects are much more likely to mix with exactly 50% chance over betting on the left and right-hand side shape when faced with the squares of interest compared to the rectangles of interest: 137 subjects do so when faced with the squares of interest and only 52 do so when faced with the rectangles of interest.

Figure 7 demonstrates subjects' choices of upper and lower bounds for the right-hand side shape when faced with the squares (the two plots on the left-hand side of Figure 7) and rectangles of interest (the two plots on the right-hand side of Figure 7). In Figure 7, the horizontal axes reflect the choices of bounds of subjects and has bins for subjects that chose exactly, 0, 50, or 100, and bins for subjects that chose in each group of 10 (i.e 10 to 19) barring choices of 0, 50, or 100. The vertical axes in Figure 7 indicate the number of subjects that chose bounds in each bin. Further, the dotted line in the plots on the right-hand side of Figure 7 indicate the average choices of the other bound for the interval in the round where subjects faced the rectangles of interest and demonstrate that the upper and lower bounds for the right-hand side rectangle of interest tend to move together, i.e. a low lower bound tends to correspond to a low upper bound as well. The solid line in the plots on the right-hand side of Figure 7 indicate the average choices of the chance of betting on the right-hand side shape in the round with the rectangles of interest and demonstrate that subjects' bets on the shapes in the binary choice question tend to correspond to their "beliefs" about which shape is bigger, i.e. when the upper and lower bounds on the chance of the right-hand side shape being larger are high subjects tend to bet more on the right-hand side shape.[48]

[47] This is a statistically significantly higher chance according to a two-sided Fisher's exact test at the 1% level.

[48] The solid black and dotted black curves that are featured on the right-hand side of Figure 7 are withheld on the left-hand side of Figure 7 because the fewer observations at locations other than 50 cause the curves to look even noisier and, as such, are less interesting.

Figure 7



Subjects are much more likely to pick upper and lower bounds of 50 for the right-hand side shape when faced with the squares of interest compared to the rectangles of interest.

D.2 Integration Data

In an attempt to get the subjects to respond to each of the three questions in each Big-Shape round (the question that asks how they would like to randomize over betting on the left and right-hand shape and the two PE questions that ask the chance of each shape being larger) as if it was the only question they were facing subjects are told that one question has been randomly selected before they start answering questions, and it is only the randomly selected question that determines their chance of winning the prize. Even though we do this, it is possible that subjects "integrate" across the questions, and for a particular random lottery r they behave as if they consider the different questions simultaneously and "hedge" their bets across the questions (Baillon et al., 2022b). In Section B.1 we explore the theoretical implications for such subjects when the Variational Preferences (VP) model (Maccheroni et al., 2006) is imposed, and find necessary conditions that can be rejected by our observable behavior.

Can a combination of integration and MPP explain our data? In short, no, not well at least. If we look for subjects that violate Theorem 7 in at least one of the Big-Shape rounds we find that 65% (142/218) of subjects do so, if we look for subjects that violate Theorem 7 in at least three of the Big-Shape rounds we find that 46% (100/218) of subjects do so, and if we look for subjects that violate Theorem 7 in at least five of the Big-Shape rounds we find that 32% (69/218) of subjects do so.

If we worry some subjects are integrating and others are isolating, we can look for subjects that violate the MPP model under isolation (Theorem 1, Theorem 2, Theorem 3) in at least one round, and also violate Theorem 7 in at least one round, and we find 62% (136/218) of subjects do so. If we instead look for subjects that violate the MPP model under isolation in at least three rounds, and also violate Theorem 7 in at least three rounds, we find 43% (94/218) of subjects do so.

If we try to separate the double-slider behavior of the different subjects into "types" then the double-slider behavior of 18% of subjects (40/218) is consistent with our results on both isolation and integration (do not violate Theorem 1 or Theorem 7 in any round) and would include anyone that behaves in line with standard expected utility, the double slider behavior of 17% (36/218) of subjects is consistent with our results on integration but not isolation (violate Theorem 1 in a round, but do not violate Theorem 7 in any round), the double-slider behavior of 5% (10/218) of subjects is consistent with our results on isolation but not integration (do not violate Theorem 1 in any round, but do violate Theorem 7 in a round), the remaining 61% (132/218) of subjects violate our results on both integration and isolation with their double-sliders (violate Theorem 1 in a round and Theorem 7 in a round).

Further evidence that seems to support that "integrating" is relatively rare can be found in Section D.9.

D.3 Reduction of Compound Lottery Data

The definition we use of a mixture of two acts $\alpha f + (1-\alpha)g$ in the third paragraph of Section 2 is of statewise-mixture. Given the experimental evidence that suggests an empirical association between ambiguity sensitivity and failure to reduce compound lotteries, it is important to pin down how sensitive our results are to the latter. One of the advantages of the axiomatic approach employed in the proofs contained in the current study is that it is evident where reduction of compound lotteries (ROCL) may play a role in them, and the potentially most problematic way it may affect our identification results. In particular, we assume that equal weights assigned to betting on the left and right shape are equivalent to these weights being assigned to a 50% chance of winning. Thus, randomizing equally over betting on the left and right shape is equivalent to a 50% chance of winning, and a three quarter chance of betting on the right shape and a one quarter chance of betting on left shape is equivalent to a one half chance of betting on the right shape and a one half chance of winning with a 50% chance (because half the time the DM is betting on the right shape and the other half of the time they are randomizing equally over the left and right shape).

This is a very specific form of ROCL that is relatively simple, and though ROCL might be a problematic assumption in some instances, our data indicates that this specific form is not the main source of the apparent incompleteness that we observe. First, several of the quiz questions feature compound lotteries and thus it would seem that subjects that perform better on the quiz should be more likely to be reducing compound lotteries, but as is evident from Table 9, it is not subjects with low quiz scores that are driving apparent incompleteness. Second, one of our Big-Shape rounds does not have an image in it and this creates a compound lottery that is particularly relevant to the compound lottery where the agent randomizes with equal chances over betting on the left and right shape. This follows since when one randomizes over betting on the left and right shape with equal chances you have a 50% chance of winning in every state of the world, and in the round with no image there is a 50% chance of winning no matter the shape you bet on. Thus, if a subject reduces compound lotteries a standard model would predict that in the round with no image they should choose a degenerate interval of 50 in both PE questions. 70% of our subjects do exactly this, which indicates that most subjects do seem to be reducing simple compound lotteries. Further, if we restrict the analysis to those subjects that choose a degenerate interval in both PE questions in the Big-Shape round with no image – then 80% of such

Table 7: Number of Subjects that Violate Models Conditional on Order

	All	BSL
Number of subjects (NOS)	218	96
NOS that violate MPP	183 (84%)	79 (82%)
NOS that violate MPP 3 or more times	144 (66%)	61 (64%)
NOS that violate MPP 5 or more times	119 (55%)	49 (51%)
NOS that violate CP	159 (73%)	71 (74%)
NOS that violate CP 3 or more times	92 (42%)	38 (40%)

The last column contains analysis on the subset of subjects that faced the Big-Shape rounds after the Lottery rounds.

subjects have at least one round in which they violate the MPP model, 59% of such subjects have at least three rounds in which they violate the MPP model, and 45% of such subjects have at least five rounds in which they violate the MPP model. Thus, we still observe substantial behavior that rejects quite flexible models of complete preferences even when we restrict analysis to subjects that appear to reduce the compound lottery in the round with no image.

D.4 Fatigue and Mistakes Data

It is possible that as subjects fatigue they begin to make mistakes, and these mistakes are causing the complete models of preferences we consider to be rejected, but our data does not support this hypothesis. It seems that subjects are really careful when using the sliders. It is clear from the left-hand side of Figure 7 that subjects are making very few mistakes when trying to put their slider on 50. Further, some subjects were randomly assigned to face the Big-Shape rounds first and then face the 12 Lottery rounds after (see Section 4), and the rest of subjects face the 12 Lottery rounds first and then face the 12 Big-Shape rounds after (denote these subjects by BSL). If fatigue is the cause of complete models of preference being rejected then we should expect that the subjects that face the 12 Big-Shape rounds at the end of their session should be more likely to reject the complete models of preferences we study, and yet as Table 7, the order in which they face the different rounds seems irrelevant to our main conclusions. In Table 7 the last column features the subjects that had the Big-Shape rounds at the end of their session (BSL), and the percent of subjects that cause rejections do not seem to be systematically impacted.

We can also raise the threshold for rejecting behavior and still get substantial rejections.

Table 8: Subjects that Violate Models Conditional on Passing Sanity Checks

	All	Passed three sanity checks
Number of subjects (NOS)	218	114
NOS that violate MPP	183 (84%)	87 (76%)
NOS that violate MPP 3 or more times	144 (66%)	62 (54%)
NOS that violate MPP 5 or more times	119 (55%)	45 (39%)
NOS that violate CP	159 (73%)	71 (62%)
NOS that violate CP 3 or more times	92 (42%)	34 (30%)

The last column contains analysis on the subset of subjects that passed all three sanity checks.

For instance, if we look for upper bounds that sum to a threshold of 105 or more (instead of the threshold for rejection of 102 in rounds with non-degenerate intervals and 103 in rounds with only degenerate intervals imposed by Theorem 1, see the discrete version of the result in the supplementary materials) we still get that 77% of subjects have at least one of such rounds, 57% of subjects have at least 3 of such rounds, and 46% of subjects have at least 5 of such rounds. If we look for upper bounds that sum to a threshold of 110 or more we still get that 74% of subjects have at least one of such rounds, 51% of subjects have at least 3 of such rounds, and 37% of subjects have at least 5 of such rounds.

D.5 Subjects That Passed All Three Sanity Checks

We can also restrict the analysis to subjects that performed "perfectly" on all three sanity checks. A subject is said to **perform perfectly on all three sanity checks** if when they face the Big-Shape round with no image they submit $l_L = u_L = l_R = u_R = 50$, when they face the Big-Shape round with Image 5 (in Figure 9 in the supplementary materials) they submit $l_L = u_L = 100$, $l_R = u_R = 0$, and bet on L with a 100% chance, and when they face the Big-Shape round with Image 11 (also in Figure 9 in the supplementary materials) they submit $l_L = u_L = 0$, $l_R = u_R = 100$, and bet on R with a 100% chance. The analogue of Table 4 when analysis is instead restricted to such subjects can be found in the last column of Table 8.

Overall, 52% (114/218) of our subjects performed perfectly on all three sanity checks and, surprisingly, this subset of subjects still produce pervasive violations of MPP. If we look for the proportion of subjects that passed all three sanity checks but also violate MPP under isolation (Theorem 1, Theorem 2, Theorem 3), in the Big-Shape rounds we find that 76% (87/114) of them do so in at least one round, 54% (62/114) of them do so in at least three rounds, and 39% (45/114) of them do so in at least five rounds.

Table 9: Number of Subjects That Violate Models Conditional on Quiz Score

	All	Quiz ≥ 4	Quiz ≥ 7
Number of subjects (NOS)	218	191	99
NOS that violate MPP	183 (84%)	159 (83%)	81 (82%)
NOS that violate MPP 3 or more times	144 (66%)	125 (65%)	63 (64%)
NOS that violate MPP 5 or more times	119 (55%)	102 (53%)	52 (53%)
NOS that violate CP	159 (73%)	138 (72%)	66 (67%)
NOS that violate CP 3 or more times	92 (42%)	76 (40%)	35 (35%)

The last and second last column contain analysis on the subset of subjects that got a combined score of 7 or more out of 10 on the two quizzes and 4 or more out of 10 on the two quizzes, respectively.

Subjects that passed the three sanity checks are a bit less likely to report an interval in a round, reporting a non-degenerate interval in 29% of their rounds, and report slightly smaller but still large intervals, with an average length of 19 when an interval is reported. These subjects randomize over L and R in 58% of their rounds and randomize over L and R 94% of the time if their response in the PE questions reject Theorem 1 in a round.

D.6 Controlling For Quiz Performance

As is mentioned in the body of the paper, it is not lack of understanding on the part of subjects that is driving our estimates of incomplete preferences. In Table 9 the second column has all subjects, the third column has all subjects that got 4 or more on the quiz (the passing grade for undergraduate students at the University of Surrey), and the fourth column has subjects that got 7 or more on the quiz (the passing grade for graduate students at the University of Toronto). Unlike, our analysis in the body, here we report the performance on the quiz for the Lottery rounds as well as a measure of overall engagement. Table 9 indicates that the proportion of subjects that contradict our different necessary conditions is essentially unchanged when we remove subjects that did not perform as well on the quiz.

D.7 Breakdown by Image

In this section, we present the results broken down by image. The images themselves can be found in Section C.3 of the Supplemental Appendix. Table 10 presents the results for each image separately. The most notable findings are presented in Section 5.3 of the main body.

Table 10: Breakdown by Question

	1	2	3	4	5	6
Ratio of Areas	1.022	1.024*	1.007	1.104	1.434	1.007
% Rounds Violating MPP	48.4 (42.7)	30.3 (23.7)	56.9 (49.1)	53.0 (44.8)	14.2 (0)	40.4 (31.6)
% Rounds Violating CP	23.2 (20.7)	18.8 (14.0)	32.6 (25.4)	26.8 (24.1)	7.3 (0)	12.8 (7.9)
Median Response Time (secs)	43.6 (51.3)	38.7 (42.9)	42.6 (48.1)	40.3 (48.1)	19.8 (19.9)	31.2 (32.3)
% Rounds w/ Randomization	70.3 (70.7)	85.3 (86.0)	77.1 (73.7)	72.0 (70.1)	8.3 (0)	50.0 (45.6)
% Rounds w/ 50-50	18.1 (24.4)	49.5 (57.9)	16.1 (21.1)	17.9 (20.7)	1.4 (0)	12.4 (13.2)
Chance of Betting on Larger Shape	54.7 (53.4)	58.1 (58.4)	43.7 (43.1)	54.9 (57.6)	90.2 (100)	17.8 (14.4)

	7	8	9	10	11	12
Ratio of Areas	1.022	1.024	1.122	1.104	1.432	1.007
% Rounds Violating MPP	45.9 (35.6)	51.4 (40.4)	55.0 (44.7)	55.3 (43.4)	18.8 (0)	45.0 (41.2)
% Rounds Violating CP	25.0 (21.1)	28.0 (18.4)	30.3 (20.2)	30.2 (20.5)	5.5 (0)	18.3 (14.9)
Median Response Time (secs)	48.3 (48.7)	48.4 (50.0)	39.1 (42.5)	38.9 (47.2)	22.3 (20.1)	34.5 (35.9)
% Rounds w/ Randomization	69.2 (64.4)	71.6 (67.5)	71.6 (65.8)	81.1 (77.1)	15.1 (0)	58.7 (51.8)
% Rounds w/ 50-50	19.8 (23.3)	18.3 (24.6)	14.2 (21.1)	17.6 (22.9)	0.5 (0)	11.5 (11.4)
Chance of Betting on Larger Shape	47.3 (45.6)	55.3 (57.7)	60.3 (63.9)	55.1 (56.1)	94.4 (100)	19.8 (16.9)

Subjects who passed all three sanity checks, excluding the round with no image, in parentheses.

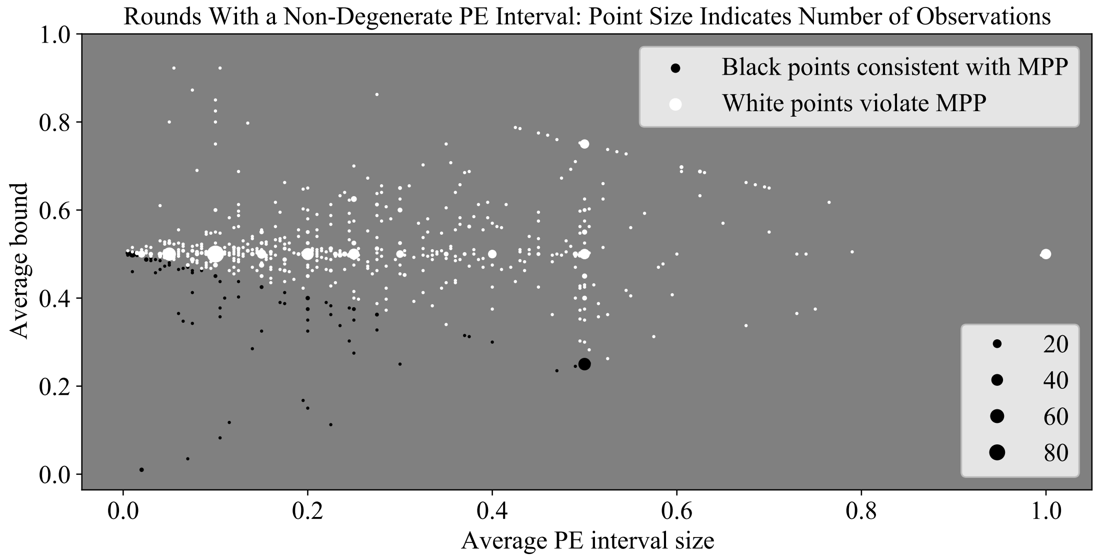
*The results are pooled for images 2 and 2B, however this ratio is for image 2 only. The ratio for 2B is 1.017.

D.8 Systematic Errors in the Interval Predictions of Multiple Prior Preferences

Not only do Multiple Prior Preferences (MPP) fail to account for a substantial proportion of our data, but they are actually systematically incorrect when predicting the relationship between the two intervals in a round, as is demonstrated in this subsection. We believe this is one of the most striking features of our data.

Recall, from Section B, that the average bound, b_{av}, is the average of the four bounds on the PEs for both the left shape and the right shape. As is shown by Corollary 1, MPP impose structure onto the average bound. However, in 87% of rounds with non-degenerate intervals the average bound is inconsistent with the predictions of Corollary 1, as is depicted in Figure 8. The black dots in Figure 8 illustrate the number of rounds in which subjects have a non-degenerate interval and a combination of average bound and average interval size (average of the difference between l_L and u_L and the difference between l_R, u_R) that do not violate Corollary 1. The white dots in Figure 8 illustrate the number of rounds in which subjects have a non-degenerate interval and a combination of average bound and average interval size that violate Corollary 1. As the average interval size increases, it is clear from the black dots that the maximal average bound that is acceptable according to Corollary 1 decreases, but, as is evident from Figure 8, increases in the average interval size are not associated with a reduction in the average bound. On the other hand, as discussed in Section 5.4, the Bewley model of incomplete preferences fits our data quite well.

Figure 8: Empirical distribution of consistency with MPP as a function of average bound and average interval size



PE intervals that are inconsistent with MPP are depicted in white, while those that are consistent with MPP are depicted in black.

D.9 Predicting Intervals with Bewley

Bewley (2002) proposed one of the seminal theories of incomplete preferences, using a model of Knightian uncertainty in which the DM's preference incompleteness stems from their uncertainty about their beliefs concerning the likelihood of states of the world. Application of Bewley's model in our context seems natural as it would be reasonable to assume that subjects may lack confidence in their beliefs about which shape is larger. More recent models of incomplete preferences allow for a "dual" incompleteness due to "tastes" (as opposed to incompleteness due to beliefs as in Knightian uncertainty). For instance, Ok et al. (2012) propose a model in which a DM is either unsure of their beliefs or their tastes, but not both, while Galaabaatar and Karni (2013) allow for a DM to be unsure of both their beliefs and their tastes. In our context, where only two possible prizes are possible, it seems that uncertainty about "tastes" should be irrelevant as long as the agent knows they strictly prefer winning the prize to not winning it.

In the work of Bewley (2002) the DM has a convex set of probabilities over states of the world, and one act is preferred to another iff it is ranked higher according to every probability in the set. Thus, while both Bewley's model and the Maxmin Expected Utility Model (Gilboa & Schmeidler, 1989) feature sets of prior beliefs, the role of the prior beliefs in the two models differ, and a bridge between the two models is provided by Gilboa et al. (2010).

Bewley (2002) further assumes that there exists a "status quo" option, and if the DM cannot directly compare two options - they simply select the status quo option. As there is no apparent "status quo" in our setting, we amend this model and assume that the DM chooses to randomize over options when they are not comparable. Given this assumption, we can obtain simple predictions for the interval reported by a DM for one shape based on their observed interval for the other shape.

Suppose the DM reports an interval $[a, b]$, with $0 \leq a < b \leq 1$, when asked for PEs of one shape being larger than a second shape. If we assume that the DM randomizes iff they do not have strict preference over the two options, then $[a, b]$ is their set of probabilities (percentage chances) of the one shape being larger according to our amended version of the Bewley (2002) model. Thus, their set of probabilities (percentage chances) of the second shape being larger than the first shape is $[1 - b, 1 - a]$, and we can predict that they would report this interval of PEs for the other shape being larger (ignoring discreteness issues for now). We call this the **Bewley prediction** for PEs. Interestingly, this prediction is very different than what would be predicted by Multiple Prior Preferences (MPP) when assuming the DM isolates the three choice problems. Theorem 1 imposes that the upper bound chosen for the other shape (which has not had its reported interval observed yet) is less than or equal to $1 - b$ (see Theorem 1, ignoring discreteness issues for now), and thus under isolation MPP predict a set of intervals instead of a specific interval, and the single interval that is the Bewley prediction is not in the set of predicted intervals as long as $a < b$ (a non-degenerate interval is observed).

As is shown in Section D.8, these set predictions of MPP under isolation do quite poorly, but Bewley predictions, as can be surmised from Figure 8, actually perform extremely well on average, and, as predicted, the horizontal line at 0.5 is highly populated. If we look at rounds where subjects report a non-degenerate interval for the left shape then the average error of the Bewley predictions for the lower and upper bounds for the right shape are -0.0126 and 0.0138 respectively, and if we look at rounds where subjects report a non-degenerate interval for the right shape then the average error of the Bewley predictions for the lower and upper bounds for the left shape are 0.0004 and 0.0101 respectively.[49] These average errors are tiny, and thus the Bewley predictions are quite accurate on average. Further, as can be observed from Figure 8, the errors do not seem to be bi-modal.

Some readers may have noticed that the Bewley predictions are similar to the predictions made by our generalization of MPP under integration (see Theorem 7 for the result in

[49]In rounds with a non-degenerate interval for the left shape the average absolute value of the Bewley prediction error for the lower and upper bounds for the right shape are 0.1164 and 0.1090 respectively, and in rounds with a non-degenerate interval for the right shape the average absolute value of the Bewley prediction error for the lower and upper bounds for the left shape are 0.1023 and 0.1217 respectively.

the continuous case), but there is one key distinction. Under integration, our generalization of MPP predicts that the intervals of PEs reported by the DM should not overlap (no part of either interval should be contained in the interior of the other interval, see Theorem 7). In contrast, if the DM reports an interval $[a, b]$ with $a < 0.5 < b$ for one shape, then the Bewley prediction would say that we should expect an overlap between the DM's two intervals of PEs.

The Bewley predictions do much better than the MPP model under integration in this regard. If a subject reports a non-degenerate interval for the left (right) shape and the amended Bewley model predicts an overlap between the two intervals, we find an overlap 78% (82%) of the time in our data. If a subject reports a non-degenerate interval for the left (right) shape and the amended Bewley model predicts no overlap between the intervals we find an overlap 15% (14%) of the time.

D.10 Lottery Rounds

The Lottery rounds are similar to the Big-Shape rounds except for two differences. First, L or R results in the subject winning the prize with a chance that is equal to the proportion of the circle covered by the shape. Second, instead of asking what is the chance of each shape being larger, we ask what percent of the circle they believe is covered by each shape.

The goal of the Big-Shape rounds is to test the behavior for consistency with general models of complete preferences, which is possible to accomplish in a rigorous way because the two probability equivalent questions are for complementary events. Since each shape in the Lottery rounds represents a lottery, it is significantly harder to rule out complementarity between acts and convexity of preferences.

The Lottery rounds are, however, more similar to the type of decision problems that have featured in previous experiments. A typical experiment features choices between two or more options that have values that are not perfectly negatively correlated: all options can have high or low realized values simultaneously, unlike the bets on shapes in the Big-Shape rounds where one bet being beneficial means the other is not. As such, these rounds provide a natural bridge between the stark experimental environment in the Big-Shape rounds and previous experiments.

For the prevalent reporting of non-degenerate intervals in the Big-Shape rounds to be of interest to the broader field of economics it needs to be established that the non-degenerate intervals are not a consequence of the complementarity of the events on which the bets are defined, which is exactly the goal of the Lottery rounds.

We find that subjects are more likely to choose a non-degenerate interval in the Lottery rounds compared to the Big-Shape rounds, reporting a non-degenerate interval in 87% of the former. The intervals reported in the Lottery rounds are smaller on average, but still large, with an average size of 18 when a non-degenerate interval is reported, and this difference in average size makes sense given the differences in the expected incentives. Thus, the main conclusion that should be drawn from the Lottery rounds is that subjects' frequent desire to report intervals in the Big-Shape rounds is not being driven by the perfect negative correlation between the value of L and the value of R that is, for our purposes, a crucial feature of the Big-Shape rounds.

E Experimental Interface: Instructions and Quizzes

The following pages are a sample of the instructions and quizzes that may be seen by subjects in the experiment. There are 4 possible combinations, as detailed in Section C.2. What follows is a mixture that cannot be seen by any single subject but allows the reader to get a sense of all possible subject experiences. Here, we start with the Big-Shape rounds with the binary choice question at the top, followed by the Lottery rounds with the binary choice question at the bottom.
Note: while the sliders themselves are not visible in these screenshots (due to technical difficulties), they were visible to the subjects, as they are in Figures 4 and 5.

Consent Form

Who is conducting the study?
The principal investigators are Yoram Halevy, David Walker-Jones, and Lanny Zrill.

Who is funding this study?
This study is run with external funds from SSHRC administered by the University of Toronto.

Why are we doing this study?
Making choices under uncertainty when information about the consequences of one's choices is available plays a central role in economic models. The purpose of this study is to observe how decisions are made under different information and feedback conditions.

How is the study done?
In this study you will be asked to make many decisions about how confident you are in your perception of which of two shapes is larger. We expect the experiment to take up to 60 minutes to complete, but you can take as much time as you want to finish it. Please note that, as in all experiments in economics, there is no deception in the study: we are not trying to trick you, and all payments are real.

What will we do with the study results?
We expect that the results of the study will be published in academic journals.

Could participating in the study be bad for you?
We do not think there is anything in this study that could harm you or be bad for you.

What are the benefits of participating?
You will be paid for your participation in the study (see below). In addition, the decisions you will be asked to make could be interesting, and further, you may learn something about different learning heuristics.

How will your privacy be maintained?
Your performance and decisions will be kept strictly confidential. You will never be identified by name or any other identifying feature with relation to this study. All electronic data from this experiment will be analyzed anonymously and will be kept on a password protected secure server.

Will you be paid for taking part in the study?
As long as you have online banking with a Canadian bank so that you can receive Interac e-transfers, you will be paid between $5 and $10 for completing the training portion of the experiment, and in addition the experiment gives you the chance to win a monetary prize once you are done with the experiment. You will receive all payments as Interac e-transfers after the experiment.

Who can you contact if you have questions about this study?
If you have any questions or concerns, or desire additional information about this study, please contact David Walker-Jones at david.walker.jones@mail.utoronto.ca, or Yoram Halevy at yoram.halevy@utoronto.ca.

Who can you contact if you have complaints or concerns about the study?
If you have any concerns or complaints about your rights as a research participant and/or your experiences while participating in this study, contact the Office of Research Ethics at ethics.reviews@utoronto.ca or 416-946-3273.

Consent: Information / feedback Taking part in this study is entirely up to you. You have the right to refuse to participate in this study. If you decide to take part, you may choose to pull out of the study at any time without giving a reason and without any negative impact on your class standing. When you type in the field below 'I consent', it serves as an electronic signature and indicates that you consent to participate in this experiment. If you wish, you can print this page for your own records.

Please enter your age in years:

Please enter the day of the month:

Please select the month:

Please select the year:

Please enter your student number:

Please enter the email with which you would like to receive the Interac e-transfer of any money you earn:

Please enter your full name:

If you wish to consent to participating in the experiment, please type 'I consent':

Next

Welcome to this TEEL online experiment!

Thank you for participating in this study of confidence in choice under uncertainty. **Please read the following instructions carefully**. Understanding what is going on could help you earn more money (**you can earn up to $40 in this experiment**).

The experiment is divided into two sections which have 12 rounds of questions each. Each section begins with some training and a quiz to confirm your understanding. There will be 5 questions in each quiz. You must answer each question correctly before you can move on to the next question. When you complete each quiz you will earn a $2.50 training payment. In addition, for each quiz question you answer correctly on the first attempt you will earn another $0.5 training bonus payment once you finish the quiz. This means **you can earn up to $10 just by doing well during the training.**

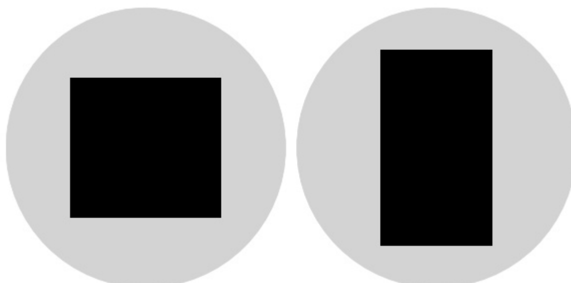
When you click the 'Next' button and move on from one page to the next you will not be able to go back, so **make sure you read everything carefully before moving on**. Please do not try to use the back button or the refresh button during the experiment; this could cause the experiment to crash.

You have until 6 PM Toronto time (EST) to complete the experiment. If you have questions or require any kind of assistance please use the Zoom link that was included in the invitation you received to participate and we will be happy to help.

Next

Training Part 1 (page 1 of 3):

The experiment consists of **24 rounds.** Each round has an **image of two circles with shapes inside** of them. **Below is an example**:



In each round one of the two shapes inside of the circles is larger (has the bigger area), either the shape in the left circle (the "left shape" for short) or the shape in the right circle (the "right shape" for short). The left circle and right circle are the same size as each other in each round. Even if the images in two rounds look similar it does not imply that they are the same image.

Your goal is to try to determine the relative sizes of the two shapes and how confident you are in your judgement. Choosing the larger shape and deciding how confident you are that you chose correctly will increase your chance of winning the **$30 prize**, which is in addition to whatever you earn from the training. How the payment of the $30 prize is determined will be explained over the course of the training.

In each round, **the chance of the shape in the left circle being larger than the shape in the right circle is exactly 50%,** which is the same as the chance of the shape in the right circle being larger than the shape in the left circle.

In each round, except for Round 1, **we will show you the image of the shapes for the round**.

In each round, **you will answer 3 questions** about the relative sizes of the shapes. You must spend at least **45 seconds** answering these questions before you can move on to the next round. In Round 1 the image of the shapes is hidden from you, so you must answer the 3 questions without seeing the shapes.

The 3 questions that you answer in each of the first 12 rounds and the 3 questions that you answer in each of the last 12 rounds differ from each other slightly, however. You will be trained on the questions in the the last 12 rounds after you finish the first 12 rounds.

In the first 12 rounds you care about which of the two shapes in the circles is larger, and the 3 questions you answer are:
Question 1: Would you rather bet on the shape in the right circle, the shape in the left circle, or randomize over the two options? (Betting on the larger shape increases your chance of winning the $30 prize, you will learn more about this soon)
Question 2: What do you think is the chance that the shape in the right circle is larger?
Question 3: What do you think is the chance that the shape in the left circle is larger?

Your chance of winning the $30 prize is determined by your answer to one of the 3 questions in one of the 24 rounds. This question, which we say is **used for payment**, has already been randomly selected by the computer, so your decisions do not impact which question is used for payment. You do not know which question is used for payment, however, so **you may answer each question as if it is the question being used for payment.**

Behind the scenes there is a **random number** between 0 and 100. The random number has already been randomly selected by the computer, so your decisions do not impact it.
When you answer questions 2 and 3 in each of the first 12 rounds you essentially tell us, for each potential value of the random number, if you would rather win the $30 prize if the shape in the relevant circle is larger or win the $30 prize with a percent chance that is equal to the random number. You will see the specifics of how all of this works in the coming pages.

If you want to stop the experiment before completing the 24 rounds then you have the **option to exit** below the "Next" button at the bottom of each page with the 3 questions for a round. If you exit before the question that is used for payment, however, then you will not be able win the $30 prize, and you will only get what you earned from the training.

Next

Training Part 1 (page 2 of 3):

Question 1 asks you if you would you rather bet on the shape in the right circle, the shape in the left circle, or randomize over the two options.

You can **randomize** by assigning each of **100 different balls** to betting on the left or right shape (you will see how this works below). **The computer will draw 1 of the 100 balls at random,** each with the same chance. If the drawn ball is red it places a bet for you on the right shape, and if the drawn ball is blue it places a bet for you on the left shape.

Question 1 is the same in all 24 rounds, but the result of betting on a shape is different in the first 12 rounds than in the last 12 rounds. If a Question 1 from the first 12 rounds is used for payment then you win the $30 prize if the drawn ball places a bet on the larger shape.

Remember: In each of the first 12 rounds, except for Round 1, you will be able to see the image of the shapes when you answer Question 1.

The first 12 rounds:

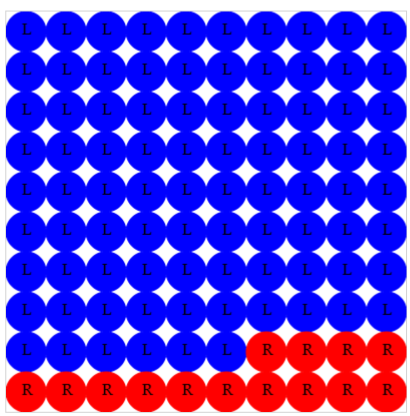
In each of the first 12 rounds you will answer Question 1 below. If such a question is used for payment, then you win the $30 prize if the ball that is drawn places a bet on the larger of the two shapes in the circles.

You answer the question using the slider below the diagram of the balls, and your answer appears just above the slider. As you move the slider the text above and below the slider will be updated. The text below the slider explains how your answer to the question determines your chance of winning the $30 prize if the question is used for payment.

The initial position of the slider for the question is randomly determined by the computer and has nothing to do with the image or how you have answered previous questions. The initial position of the slider is in no way meant to be a suggestion about how you should answer the question.

You should try moving the slider and see how the text above and below the slider changes.

Question 1: Would you rather bet on the shape in the right circle, the shape in the left circle, or randomize over the two options?



I would like to assign 14 balls to betting on the shape in the right circle

and I would like to assign 86 balls to betting on the shape in the left circle

☐

If this question is used for payment:

You have a 14% chance of betting on the shape in the right circle, in which case you win the $30 prize if the shape in the right circle is larger than the shape in the left circle and you do not win the prize if is not.

You have a 86% chance of betting on the shape in the left circle, in which case you win the $30 prize if the shape in the left circle is larger than the shape in the right circle and you do not win the prize if is not.

Before you move to the next page:
Answer Question 1 so that 27 balls are assigned to betting on the shape in the left circle.

Next

Training Part 1 (page 3 of 3):

In each of the first 12 rounds you will answer the following two questions:
Question 2: What do you think is the chance that the shape in the right circle is larger than the shape in the left circle?
Question 3: What do you think is the chance that the shape in the left circle is larger than the shape in the right circle?

You answer the questions using two double-sliders and your answer appears just above each double-slider. As you move each slider the text above and below the double-slider will be updated. When you answer a question the sliders can coincide (touch) if you want them to, or there can be a gap between the sliders if you want to report a range of confidences.

Remember: In each of the first 12 rounds, except for Round 1, you will be able to see the image of the shapes when you answer questions 2 and 3.

The text below each question explains how your answer to the question determines your chance of winning the $30 prize if the question is used for payment, and the "fair digital coin" has a 50% chance of coming up both heads and tails:

You should try moving the sliders and see how the text above and below each double-slider changes.

Question 2: What do you think is the chance that the shape in the **right** circle is larger?

The chance of the shape being larger is between 0% and 100%

0 25 50 75 100

If the random number is below 0 and this question is used for payment:
You bet that the shape in the right circle is larger. This means that you win the $30 prize if the shape in the right circle is larger than the shape in the left circle and you do not win the prize if the shape in the right circle is smaller than the shape in the left circle.

If the random number is above 100 and this question is used for payment:
You bet on the random number. This means that you win the $30 prize with a percent chance that is equal to the random number.

If the random number is equal to or between 0 and 100 and this question is used for payment:
A fair digital coin is flipped for you that determines if you bet on the random number or bet that the shape in the right circle is larger.
If the coin comes up heads you bet that the shape in the right circle is larger: you win the $30 prize if the right shape is larger.
If the coin comes up tails you bet on the random number: you win the $30 prize with a percent chance that is equal to the random number.

Question 3: What do you think is the chance that the shape in the **left** circle is larger?

The chance of the shape being larger is between 60% and 80%

0 25 50 75 100

If the random number is below 60 and this question is used for payment:
You bet that the shape in the left circle is larger. This means that you win the $30 prize if the shape in the left circle is larger than the shape in the right circle and you do not win the prize if the shape in the left circle is smaller than the shape in the right circle.

If the random number is above 80 and this question is used for payment:
You bet on the random number. This means that you win the $30 prize with a percent chance that is equal to the random number.

If the random number is equal to or between 60 and 80 and this question is used for payment:
A fair digital coin is flipped for you that determines if you bet on the random number or bet that the shape in the left circle is larger.

If the coin comes up heads you bet that the shape in the left circle is larger: you win the $30 prize if the left shape is larger.
If the coin comes up tails you bet on the random number: you win the $30 prize with a percent chance that is equal to the random number.

Before you move to the next page:
Answer Question 3 above so that if it were used for payment you would bet that the left shape is larger if the random number is smaller than 60, bet on the random number if the random number is larger than 80, and for all other values of the random number a fair digital coin is flipped for you that determines if you bet on the random number or bet that the shape in the left circle is larger.

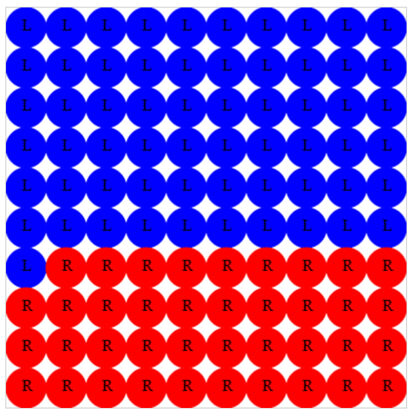
Next

Quiz time! Quiz 1, Question 1

It is now time to do the quiz. **Remember:** to move on to the next question you must first answer the current question correctly. By completing the quiz you will earn a $2.5 training payment, and in addition to the training payment, you can earn another $0.5 for each question you answer correctly on your first attempt.

If you are in one of the first 12 rounds and **you know for sure that the shape in the right circle is larger**, how should you answer Question 1, Question 2, and Question 3 below if one of them were used for payment and you want to maximize your chance of winning the $30 prize?

Question 1: Would you rather bet on the shape in the right circle, the shape in the left circle, or randomize over the two options?



I would like to assign 39 balls to betting on the shape in the right circle

and I would like to assign 61 balls to betting on the shape in the left circle

If this question is used for payment:

You have a 39% chance of betting on the shape in the right circle, in which case you win the $30 prize if the shape in the right circle is larger than the shape in the left circle and you do not win the prize if is not.

You have a 61% chance of betting on the shape in the left circle, in which case you win the $30 prize if the shape in the left circle is larger than the shape in the right circle and you do not win the prize if is not.

Question 2: What do you think is the chance that the shape in the **right** circle is larger?

The chance of the shape being larger is between 0% and 100%

0 25 50 75 100

If the random number is below 0 and this question is used for payment:
You bet that the shape in the right circle is larger. This means that you win the $30 prize if the shape in the right circle is larger than the shape in the left circle and you do not win the prize if the shape in the right circle is smaller than the shape in the left circle.

If the random number is above 100 and this question is used for payment:
You bet on the random number. This means that you win the $30 prize with a percent chance that is equal to the random number.

If the random number is equal to or between 0 and 100 and this question is used for payment:
A fair digital coin is flipped for you that determines if you bet on the random number or bet the shape in the right circle is larger.
If the coin comes up heads you bet the shape in the right circle is larger: you win the $30 prize if the right shape is larger.
If the coin comes up tails you bet on the random number: you win the $30 prize with a percent chance that is equal to the random number.

Question 3: What do you think is the chance that the shape in the **left** circle is larger?

The chance of the shape being larger is between 0% and 100%

☐ ☐

0 25 50 75 100

If the random number is below 0 and this question is used for payment:
You bet that the shape in the left circle is larger. This means that you win the $30 prize if the shape in the left circle is larger than the shape in the right circle and you do not win the prize if the shape in the left circle is smaller than the shape in the right circle.

If the random number is above 100 and this question is used for payment:
You bet on the random number. This means that you win the $30 prize with a percent chance that is equal to the random number.

If the random number is equal to or between 0 and 100 and this question is used for payment:
A fair digital coin is flipped for you that determines if you bet on the random number or bet the shape in the left circle is larger.
If the coin comes up heads you bet the shape in the left circle is larger: you win the $30 prize if the left shape is larger.
If the coin comes up tails you bet on the random number: you win the $30 prize with a percent chance that is equal to the random number.

Next

Quiz time! Quiz 1, Question 2

If you are in one of the first 12 rounds and **you cannot see the image of the shapes for the round** how should you answer Question 2 and Question 3 below if either of them were used for payment and you want to maximize your chance of winning the $30 prize?

Question 2: What do you think is the chance that the shape in the **right** circle is larger?

The chance of the shape being larger is between 0% and 100%

☐ ☐

0 25 50 75 100

If the random number is below 0 and this question is selected for payment:
You bet that the shape in the right circle is larger. This means that you win the $30 prize if the shape in the right circle is larger than the shape in the left circle and you do not win the prize if the shape in the right circle is smaller than the shape in the left circle.

If the random number is above 100 and this question is selected for payment:
You bet on the random number. This means that you win the $30 prize with a percent chance that is equal to the random number.

If the random number is equal to or between 0 and 100 and this question is selected for payment:
A fair digital coin is flipped for you that determines if you bet on the random number or bet the shape in the right circle is larger.
If the coin comes up heads you bet the shape in the right circle is larger: you win the $30 prize if the right shape is larger.
If the coin comes up tails you bet on the random number: you win the $30 prize with a percent chance that is equal to the random number.

Question 3: What do you think is the chance that the shape in the **left** circle is larger?

The chance of the shape being larger is between 0% and 100%

☐ ☐

0 25 50 75 100

If the random number is below 0 and this question is selected for payment:
You bet that the shape in the left circle is larger. This means that you win the $30 prize if the shape in the left circle is larger than the shape in the right circle and you do not win the prize if the shape in the left circle is smaller than the shape in the right circle.

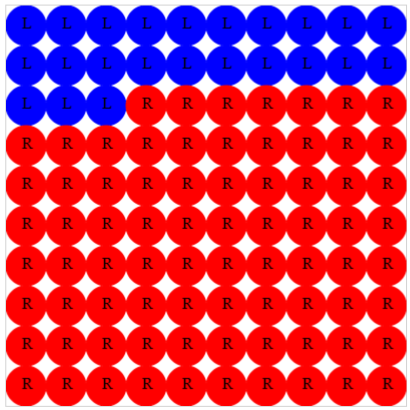
If the random number is above 100 and this question is selected for payment:
You bet on the random number. This means that you win the $30 prize with a percent chance that is equal to the random number.

If the random number is equal to or between 0 and 100 and this question is selected for payment:
A fair digital coin is flipped for you that determines if you bet on the random number or bet the shape in the left circle is larger.
If the coin comes up heads you bet the shape in the left circle is larger: you win the $30 prize if the left shape is larger.
If the coin comes up tails you bet on the random number: you win the $30 prize with a percent chance that is equal to the random number.

Next

Quiz time! Quiz 1, Question 3

Question 1: Would you rather bet on the shape in the right circle, the shape in the left circle, or randomize over the two options?



I would like to assign 77 balls to betting on the shape in the right circle

and I would like to assign 23 balls to betting on the shape in the left circle

☐

If this question is used for payment:

You have a 77% chance of betting on the shape in the right circle, in which case you win the $30 prize if the shape in the right circle is larger than the shape in the left circle and you do not win the prize if is not.

You have a 23% chance of betting on the shape in the left circle, in which case you win the $30 prize if the shape in the left circle is larger than the shape in the right circle and you do not win the prize if is not.

If a Question 1 from the first 12 rounds like the one above is used for payment, which of the following is true about your chance of winning the $30 prize?

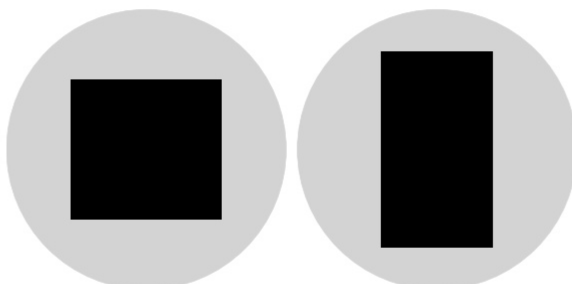
Next

Quiz time! Quiz 1, Question 4

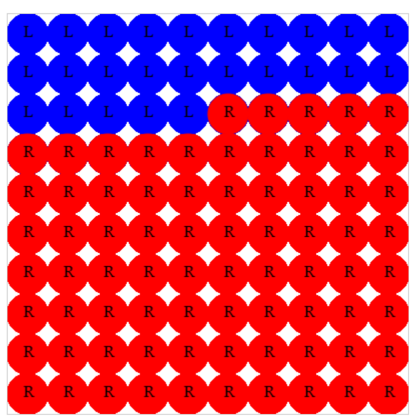
Please look at the image of shapes below and answer the questions about your confidence in which is larger:

Remember: you must spend at least 45 seconds on this page. The "Next" button will not appear until 45 seconds pass.

This round is for practice only. However, you will be rewarded for correct responses to the quiz questions on the following pages.



Question 1: Would you rather bet on the shape in the right circle, the shape in the left circle, or randomize over the two options?



I would like to assign 75 balls to betting on the shape in the right circle

and I would like to assign 25 balls to betting on the shape in the left circle

☐

If this question is used for payment:

You have a 75% chance of betting on the shape in the right circle, in which case you win the $30 prize if the shape in the right circle is larger than the shape in the left circle and you do not win the prize if is not.

You have a 25% chance of betting on the shape in the left circle, in which case you win the $30 prize if the shape in the left circle is larger than the shape in the right circle and you do not win the prize if is not.

Question 2: What do you think is the chance that the shape in the **right** circle is larger?

The chance of the shape being larger is between 45% and 60%

☐ ☐

0 25 50 75 100

If the random number is below 45 and this question is used for payment:
You bet that the shape in the right circle is larger. This means that you win the $30 prize if the shape in the right circle is larger than the shape in the left circle and you do not win the prize if the shape in the right circle is smaller than the shape in the left circle.

If the random number is above 60 and this question is used for payment:
You bet on the random number. This means that you win the $30 prize with a percent chance that is equal to the random number.

If the random number is equal to or between 45 and 60 and this question is used for payment:
A fair digital coin is flipped for you that determines if you bet on the random number or bet the shape in the right circle is larger.
If the coin comes up heads you bet the shape in the right circle is larger: you win the $30 prize if the right shape is larger.
If the coin comes up tails you bet on the random number: you win the $30 prize with a percent chance that is equal to the random number.

Question 3: What do you think is the chance that the shape in the **left** circle is larger?

The chance of the shape being larger is between 40% and 55%

☐ ☐

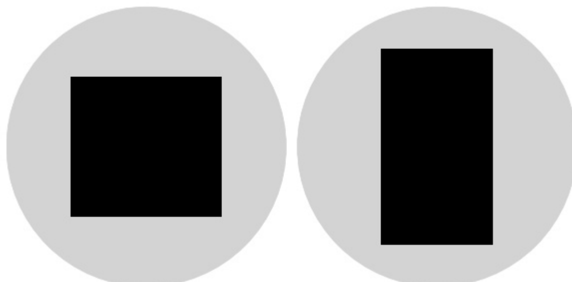
0 25 50 75 100

If the random number is below 40 and this question is used for payment:
you bet that the shape in the left circle is larger. This means that you win the $30 prize if the shape in the left circle is larger than the shape in the right circle and you do not win the prize if the shape in the left circle is smaller than the shape in the right circle.

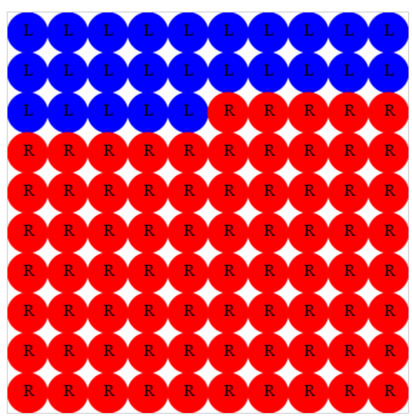
If the random number is above 55 and this question is used for payment:
You bet on the random number. This means that you win the $30 prize with a percent chance that is equal to the random number.

If the random number is equal to or between 40 and 55 and this question is used for payment:
A fair digital coin is flipped for you that determines if you bet on the random number or bet the shape in the left circle is larger.
If the coin comes up heads you bet the shape in the left circle is larger: you win the $30 prize if the left shape is larger.
If the coin comes up tails you bet on the random number: you win the $30 prize with a percent chance that is equal to the random number.

Quiz time! Quiz 1, Question 4



Previously, you selected the following in response to the question: "Would you rather bet on the shape in the right circle, the shape in the left circle, or randomize over the two options?"



I would like to assign 75 balls to betting on the shape in the right circle

and I would like to assign 25 balls to betting on the shape in the left circle

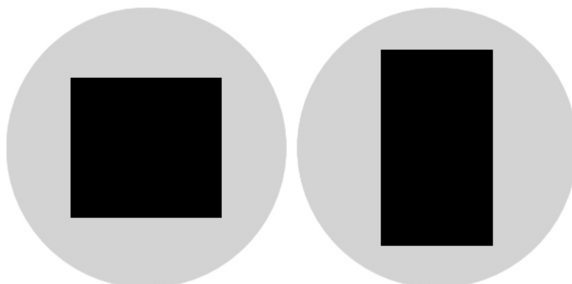
If this question is used for payment:

Suppose the shape in the right circle is bigger. In this case, you will have a 75% chance of winning the $30 prize as this is the chance that a red ball is randomly drawn.

On the other hand, suppose the shape in the left circle is bigger. In this case, what is the chance of winning the $30 prize?

Next

Quiz time! Quiz 1, Question 5



Previously, you selected the following in response to the question: "What do you think is the chance that the shape in the **right** circle is larger?"

The chance of the shape being larger is between 0% and 100%

☐ ☐

0 25 50 75 100

If the random number is equal to or between 0 and 100, for example 39, and this question is used for payment, which of the following statements is true?

Next

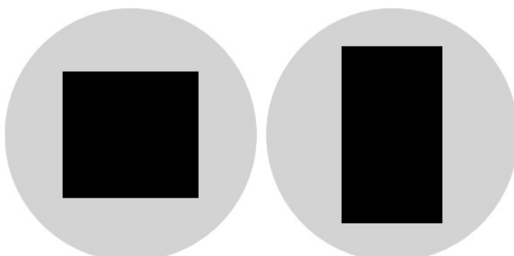
Congratulations: you have completed the quiz!

Up next: 12 rounds of perceptual decision problems.

Next

Training Part 1 (page 1 of 3):

The experiment consists of **24 rounds.** Each round has an **image of two circles with shapes inside** of them. **Below is an example**:



In each round one of the two shapes inside of the circles is larger (has the bigger area), either the shape in the left circle (the "left shape" for short) or the shape in the right circle (the "right shape" for short). The left circle and right circle are the same size as each other in each round. Even if the images in two rounds look similar it does not imply that they are the same image.

Your goal is to try to determine the relative sizes of the two shapes and how confident you are in your judgement. Choosing the larger shape and deciding how confident you are that you chose correctly will increase your chance of winning the **$30 prize**, which is in addition to whatever you earn from the training. How the payment of the $30 prize is determined will be explained over the course of the training.

In each round, **the chance of the shape in the left circle being larger than the shape in the right circle is exactly 50%,** which is the same as the chance of the shape in the right circle being larger than the shape in the left circle.

In each of the first 12 rounds **we will show you the image of the shapes for the round**.

In each round, **you will answer 3 questions** about the relative sizes of the shapes. You must spend at least **45 seconds** answering these questions before you can move on to the next round.

The 3 questions that you answer in each of the first 12 rounds and the 3 questions that you answer in each of the last 12 rounds differ from each other slightly, however. You will be trained on the questions in the the last 12 rounds after you finish the first 12 rounds.

In the first 12 rounds you care about how much of the circles are covered by the shapes, and the 3 questions you answer are:
Question 1: What percent of the right circle do you think is covered by the right shape?
Question 2: What percent of the left circle do you think is covered by the left shape?
Question 3: Would you rather bet on the shape in the right circle, the shape in the left circle, or randomize over the two options? (Betting on the larger shape increases your chance of winning the $30 prize, you will learn more about this soon)

Your chance of winning the $30 prize is determined by your answer to one of the 3 questions in one of the 24 rounds. This question, which we say is **used for payment**, has already been randomly selected by the computer, so your decisions do not impact which question is used for payment. You do not know which question is used for payment, however, so **you may answer each question as if it is the question being used for payment.**

Behind the scenes there is a **random number** between 0 and 100. The random number has already been randomly selected by the computer, so your decisions do not impact it.
When you answer questions 1 and 2 in each of the first 12 rounds you essentially tell us, for each potential value of the random number, if you would rather win the $30 prize with a chance that is equal to the percent of the relevant circle that is covered by its shape or win the $30 prize with a percent chance that is equal to the random number. You will see the specifics of how all of this works in the coming pages.

If you want to stop the experiment before completing the 24 rounds then you have the **option to exit** below the "Next" button at the bottom of each page with the 3 questions for a round. If you exit before the question that is used for payment, however, then you will not be able win the $30 prize, and you will only get what you earned from the training.

Next

Training Part 1 (page 2 of 3):

In each of the first 12 rounds you answer the following two questions:
Question 1: What percent of the right circle do you think is covered by the right shape?
Question 2: What percent of the left circle do you think is covered by the left shape?

You answer the questions using two double-sliders, and your answer appears just above each double-slider. As you move each slider the text above and below the double-slider will be updated. When you answer a question the sliders can coincide (touch) if you want them to, or there can be a gap between the sliders if you want to report a range of percentages.

Remember: In each of the first 12 rounds you will be able to see the image of the shapes when you answer questions 1 and 2.

The text below each question explains how your answer to the question determines your chance of winning the $30 prize if the question is used for payment, and the "fair digital coin" has a 50% chance of coming up both heads and tails:

You should try moving the sliders and see how the text above and below each double-slider changes.

Question 1: What percent of the **right** circle do you think is covered by the shape in the **right** circle?

The percent of the circle covered is between 0% and 100%

☐ ☐

0 25 50 75 100

If the random number is below 0 and this question is used for payment:
You bet on the percent of the right circle covered by its shape. This means that you win the $30 prize with a chance that is equal to the percent of the right circle that is covered by its shape.

If the random number is above 100 and this question is used for payment:
You bet on the random number. This means that you win the $30 prize with a percent chance that is equal to the random number.

If the random number is equal to or between 0 and 100 and this question is used for payment:
A fair digital coin is flipped for you that determines if you bet on the random number or on the percent of the right circle covered by its shape.
If the coin comes up heads you bet on the percent of the right circle covered by its shape: You win the $30 prize with a chance that is equal to the percent of the right circle that is covered by its shape.
If the coin comes up tails you bet on the random number: you win the $30 prize with a percent chance that is equal to the random number.

Question 2: What percent of the **left** circle do you think is covered by the shape in the **left** circle?

The percent of the circle covered is between 0% and 100%

☐ ☐

0 25 50 75 100

If the random number is below 0 and this question is used for payment:
You bet on the percent of the left circle covered by its shape. This means that you win the $30 prize with a chance that is equal to the percent of the left circle that is covered by its shape.

If the random number is above 100 and this question is used for payment:
You bet on the random number. This means that you win the $30 prize with a percent chance that is equal to the random number.

If the random number is equal to or between 0 and 100 and this question is used for payment:
A fair digital coin is flipped for you that determines if you bet on the random number or on the percent of the left circle covered by its

shape.
If the coin comes up heads you bet on the percent of the left circle covered by its shape: You win the $30 prize with a chance that is equal to the percent of the left circle that is covered by its shape.
If the coin comes up tails you bet on the random number: you win the $30 prize with a percent chance that is equal to the random number.

Before you move to the next page:
Answer Question 1 above so that if it were used for payment you would bet on the percent of the right circle covered by its shape if the random number is smaller than 30, bet on the random number if the random number is larger than 35, and for all other values of the random number a fair digital coin is flipped for you that determines if you bet on the random number or bet on the percent of the right circle covered by its shape.

Next

Training Part 1 (page 3 of 3):

Question 3 asks you if you would you rather bet on the shape in the right circle, the shape in the left circle, or randomize over the two options.

You can **randomize** by assigning each of **100 different balls** to betting on the left or right shape (you will see how this works below). **The computer will draw 1 of the 100 balls at random,** each with the same chance. If the drawn ball is red it places a bet for you on the right shape, and if the drawn ball is blue it places a bet for you on the left shape.

Question 3 is the same in all 24 rounds, but the result of betting on a shape is different in the first 12 rounds than in the last 12 rounds. If a Question 3 from the first 12 rounds is used for payment, then you win the $30 prize with a chance that is equal to the percent of the circle that is covered by the shape that the drawn ball places a bet on.

Remember: In each of the first 12 rounds you will be able to see the image of the shapes when you answer Question 3.

The first 12 rounds:

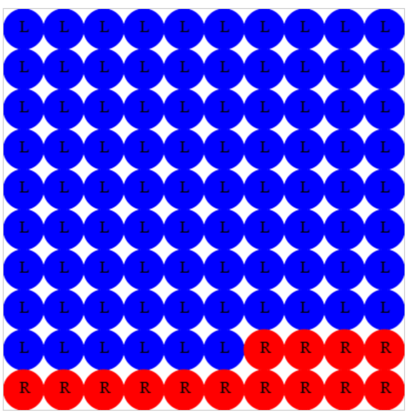
In each of the first 12 rounds you will answer Question 3 below. If such a question is used for payment, then you win the $30 prize with a chance that is equal to the percent of the circle that is covered by the shape that the drawn ball places a bet on.

You answer the question using the slider below the diagram of the balls, and your answer appears just above the slider. As you move the slider the text above and below the slider will be updated. The text below the slider explains how your answer to the question determines your chance of winning the $30 prize if the question is used for payment.

The initial position of the slider for the question is randomly determined by the computer and has nothing to do with the image or how you have answered previous questions. The initial position of the slider is in no way meant to be a suggestion about how you should answer the question.

You should try moving the slider and see how the text above and below the slider changes.

Question 3: Would you rather bet on the shape in the right circle, the shape in the left circle, or randomize over the two options?



I would like to assign 14 balls to betting on the shape in the right circle

and I would like to assign 86 balls to betting on the shape in the left circle

If this question is used for payment:

You have a 14% chance of betting on the shape in the right circle, in which case you win the $30 prize with a chance that is equal to the percent of the right circle that is covered by the shape in the right circle.

You have a 86% chance of betting on the shape in the left circle, in which case you win the $30 prize with a chance that is equal to the percent of the left circle that is covered by the shape in the left circle.

Before you move to the next page:
Answer Question 3 so that 27 balls are assigned to betting on the shape in the left circle.

Next

Quiz time! Quiz 1, Question 1

It is now time to do the quiz. **Remember:** to move on to the next question you must first answer the current question correctly. By completing the quiz you will earn a $2.5 training payment, and in addition to the training payment, you can earn another $0.5 for each question you answer correctly on your first attempt.

Question 1: What percent of the **right** circle do you think is covered by the shape in the **right** circle?

The percent of the circle covered is between 55% and 65%

0 25 50 75 100

If the random number is below 55 and this question is used for payment:
You bet on the percent of the right circle covered by its shape. This means that you win the $30 prize with a chance that is equal to the percent of the right circle that is covered by its shape.

If the random number is above 65 and this question is used for payment:
You bet on the random number. This means that you win the $30 prize with a percent chance that is equal to the random number.

If the random number is equal to or between 55 and 65 and this question is used for payment:
A fair digital coin is flipped for you that determines if you bet on the random number or on the percent of the right circle covered by its shape.
If the coin comes up heads you bet on the percent of the right circle covered by its shape: You win the $30 prize with a chance that is equal to the percent of the right circle that is covered by its shape.
If the coin comes up tails you bet on the random number: you win the $30 prize with a percent chance that is equal to the random number.

If a Question 1 from the first 12 rounds is used for payment, and you answer it as is done above, what is true about your probability of winning the $30 prize?

Next

Quiz time! Quiz 1, Question 2

If you are in one of the first 12 rounds and you know for sure that the shape in the right circle takes up 35% of its circle and the shape in the left circle takes up 60% of its circle, how should you answer Question 1, Question 2, and Question 3 below if one of them were used for payment and you want to maximize your chance of winning the $30 prize?

Question 1: What percent of the **right** circle do you think is covered by the shape in the **right** circle?

The percent of the circle covered is between 0% and 100%

☐ ☐

0 25 50 75 100

If the random number is below 0 and this question is used for payment:
You bet on the percent of the right circle covered by its shape. This means that you win the $30 prize with a chance that is equal to the percent of the right circle that is covered by its shape.

If the random number is above 100 and this question is used for payment:
You bet on the random number. This means that you win the $30 prize with a percent chance that is equal to the random number.

If the random number is equal to or between 0 and 100 and this question is used for payment:
A fair digital coin is flipped for you that determines if you bet on the random number or on the percent of the right circle covered by its shape.
If the coin comes up heads you bet on the percent of the right circle covered by its shape: You win the $30 prize with a chance that is equal to the percent of the right circle that is covered by its shape.
If the coin comes up tails you bet on the random number: you win the $30 prize with a percent chance that is equal to the random number.

Question 2: What percent of the **left** circle do you think is covered by the shape in the **left** circle?

The percent of the circle covered is between 0% and 100%

☐ ☐

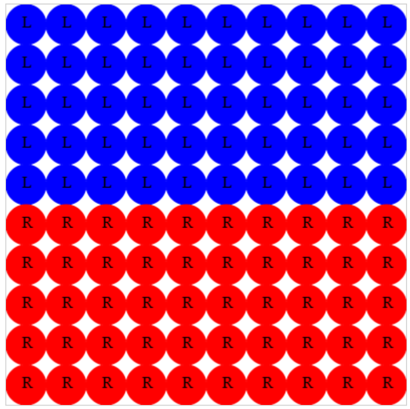
0 25 50 75 100

If the random number is below 0 and this question is used for payment:
You bet on the percent of the left circle covered by its shape. This means that you win the $30 prize with a chance that is equal to the percent of the left circle that is covered by its shape.

If the random number is above 100 and this question is used for payment:
You bet on the random number. This means that you win the $30 prize with a percent chance that is equal to the random number.

If the random number is equal to or between 0 and 100 and this question is used for payment:
A fair digital coin is flipped for you that determines if you bet on the random number or on the percent of the left circle covered by its shape.
If the coin comes up heads you bet on the percent of the left circle covered by its shape: You win the $30 prize with a chance that is equal to the percent of the left circle that is covered by its shape.
If the coin comes up tails you bet on the random number: you win the $30 prize with a percent chance that is equal to the random number.

Question 3: Would you rather bet on the shape in the right circle, the shape in the left circle, or randomize over the two options?



I would like to assign 50 balls to betting on the shape in the right circle

and I would like to assign 50 balls to betting on the shape in the left circle

☐

If this question is used for payment:

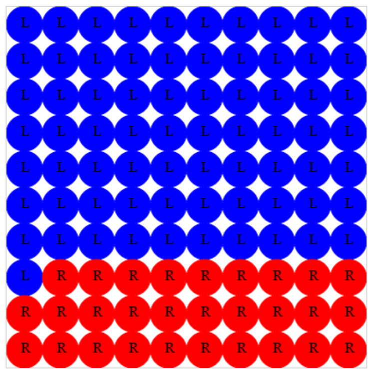
You have a 50% chance of betting on the shape in the right circle, in which case you win the $30 prize with a chance that is equal to the percent of the right circle that is covered by the shape in the right circle.

You have a 50% chance of betting on the shape in the left circle, in which case you win the $30 prize with a chance that is equal to the percent of the left circle that is covered by the shape in the left circle.

Next

Quiz time! Quiz 1, Question 3

Question 3: Would you rather bet on the shape in the right circle, the shape in the left circle, or randomize over the two options?



I would like to assign 29 balls to betting on the shape in the right circle

and I would like to assign 71 balls to betting on the shape in the left circle

☐

If this question is used for payment:

You have a 29% chance of betting on the shape in the right circle, in which case you win the $30 prize with a chance that is equal to the percent of the right circle that is covered by the shape in the right circle.

You have a 71% chance of betting on the shape in the left circle, in which case you win the $30 prize with a chance that is equal to the percent of the left circle that is covered by the shape in the left circle.

If a Question 3 from the first 12 rounds like the one above is used for payment, which of the following is true about your chance of winning the $30 prize?

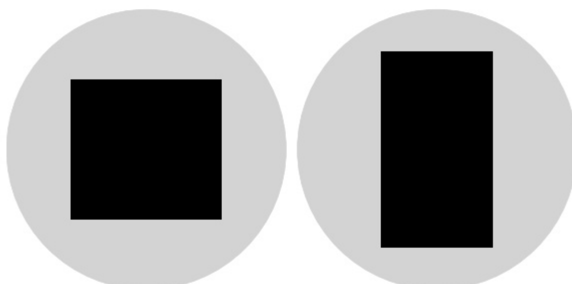
Next

Quiz time! Quiz 1, Question 4

Please look at the image of shapes below and answer the questions about your confidence in which is larger:

Remember: you must spend at least 45 seconds on this page. The "Next" button will not appear until 45 seconds pass.

This round is for practice only. However, you will be rewarded for correct responses to the quiz questions on the following pages.



Question 1: What percent of the **right** circle do you think is covered by the shape in the **right** circle?

The percent of the circle covered is between 0% and 100%

☐ ☐

0 25 50 75 100

If the random number is below 0 and this question is used for payment:
You bet on the percent of the right circle covered by its shape. This means that you win the $30 prize with a chance that is equal to the percent of the right circle that is covered by its shape.

If the random number is above 100 and this question is used for payment:
You bet on the random number. This means that you win the $30 prize with a percent chance that is equal to the random number.

If the random number is equal to or between 0 and 100 and this question is used for payment:
A fair digital coin is flipped for you that determines if you bet on the random number or on the percent of the right circle covered by its shape.

If the coin comes up heads you bet on the percent of the right circle covered by its shape: You win the $30 prize with a chance that is equal to the percent of the right circle that is covered by its shape.
If the coin comes up tails you bet on the random number: you win the $30 prize with a percent chance that is equal to the random number.

Question 2: What percent of the **left** circle do you think is covered by the shape in the **left** circle?

The percent of the circle covered is between 0% and 100%

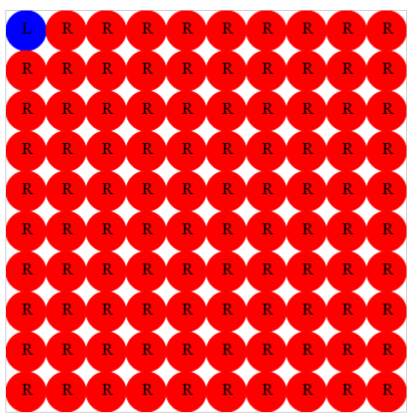
0 25 50 75 100

If the random number is below 0 and this question is used for payment:
You bet on the percent of the left circle covered by its shape. This means that you win the $30 prize with a chance that is equal to the percent of the left circle that is covered by its shape.

If the random number is above 100 and this question is used for payment:
You bet on the random number. This means that you win the $30 prize with a percent chance that is equal to the random number.

If the random number is equal to or between 0 and 100 and this question is used for payment:
A fair digital coin is flipped for you that determines if you bet on the random number or on the percent of the left circle covered by its shape.
If the coin comes up heads you bet on the percent of the left circle covered by its shape: You win the $30 prize with a chance that is equal to the percent of the left circle that is covered by its shape.
If the coin comes up tails you bet on the random number: you win the $30 prize with a percent chance that is equal to the random number.

Question 3: Would you rather bet on the shape in the right circle, the shape in the left circle, or randomize over the two options?



I would like to assign 99 balls to betting on the shape in the right circle

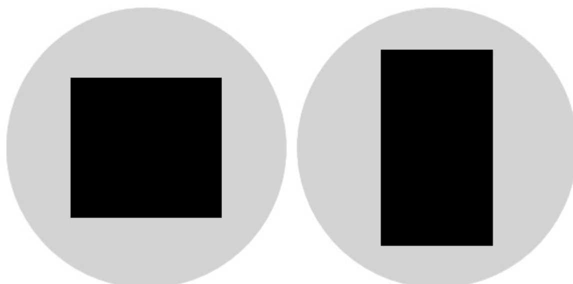
and I would like to assign 1 balls to betting on the shape in the left circle

If this question is used for payment:

You have a 99% chance of betting on the shape in the right circle, in which case you win the $30 prize with a chance that is equal to the percent of the right circle that is covered by the shape in the right circle.

You have a 1% chance of betting on the shape in the left circle, in which case you win the $30 prize with a chance that is equal to the percent of the left circle that is covered by the shape in the left circle.

Quiz time! Quiz 1, Question 4



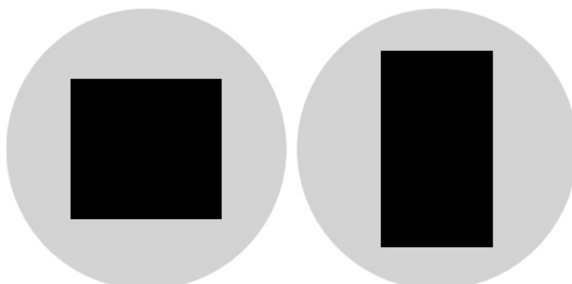
Previously, you selected the following in response to the question: "What percent of the **left** circle do you think is covered by the shape in the **left** circle?"

Suppose a random number is selected which is below 47, for example 15, and this question is used for payment:
You bet on the percent of the left circle covered by its shape. This means that you win the $30 prize with a chance that is equal to the percent of the left circle that is covered by its shape.

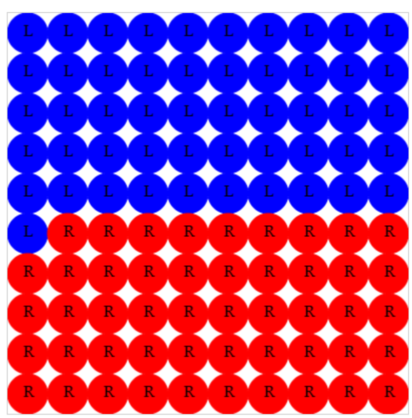
On the other hand, if the random number is above 50, for example 59, and this question is used for payment, which of the following statements is true?

Finally, if the random number is equal to or between 47 and 50, for example 49, and this question is used for payment:
A fair digital coin is flipped for you that determines if you bet on the random number or on the percent of the left circle covered by its shape.
If the coin comes up heads you bet on the percent of the left circle covered by its shape: you win the $30 prize with a chance that is equal to the percent of the left circle that is covered by its shape.
If the coin comes up tails you bet on the random number: you win the $30 prize with a 49 percent chance, a chance equal to the random number.

Quiz time! Quiz 1, Question 5



Previously, you selected the following in response to the question: "Would you rather bet on the shape in the right circle, the shape in the left circle, or randomize over the two options?"



I would like to assign 49 balls to betting on the shape in the right circle

and I would like to assign 51 balls to betting on the shape in the left circle

If this question is used for payment:

You have a 49% chance of betting on the shape in the right circle, as this is the chance that a red ball is randomly drawn. If you bet on the shape in the right circle, what would be the chance that you win the $30 prize?

A chance that is equal to the percent of the right

You have a 51% chance of betting on the shape in the left circle, as this is the chance that a blue ball is randomly drawn. If you bet on the shape in the left circle, you would win the $30 prize with a chance that is equal to the percent of the left circle that is covered by the shape in the left circle.

Next